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Co.No. _____
Serial No. _____

Final Exam

Math 204

King Abdulaziz University
Faculty of Science
Mathematics Department
8/7/1433 Second term

(Q1) Choose the correct answer of the following; [10 marks]

(1) For the equation $M(x, y)dx + (\sec^2 y + 3xy^2)dy = 0$ to be exact, $M(x, y)$ must equal;

- (a) $y^3 + g(y)$ (b) $y^3 + g(x)$
(c) $y^2x + g(x)$
-

(2) If $y_1 = \frac{1}{4} \sin 2t$ is a solution of $y'' + 2y' + 4y = \cos 2t$ and
 $y_2 = \frac{t}{4}$ is a solution of $y'' + 2y' + 4y = t$ then the solution to
 $y'' + 2y' + 4y = 11t - 12 \cos 2t$ is;

- (a) $y = 11t - 12 \sin 2t$ (b) $y = \frac{4}{11}t - 12 \cos 2t$
(c) $y = \frac{11}{4}t - 3 \sin 2t$
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(3) $\mathcal{L}\{e^{3t} \cos 2t\} =$

- (a) $\frac{s+3}{s^2-2s+13}$ (b) $\frac{s-3}{(s+3)^2+4}$
(c) $\frac{s-3}{s^2-6s+13}$
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(4) $\mathcal{L}^{-1}\left\{\frac{3s}{s^2+2s+10}\right\} =$

- (a) $e^{-t} \cos 3t + e^{-t} \sin 3t$ (b) $e^{-t} \sin 3t + 3e^{-t} \cos 3t$
(c) $3e^{-t} \cos 3t - e^{-t} \sin 3t$
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(5) $\mathcal{L}\{f''\} =$

- (a) $s^2 \mathcal{L}\{f\} - sf'(0) - f(0)$ (b) $s^2 \mathcal{L}\{f\} - sf(0) - f(0)$
(c) $s^2 \mathcal{L}\{f\} - sf(0) - f'(0)$
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(6) $\mathcal{L}\{\sin^2 t\} =$

- (a) $\frac{1}{2s} + \frac{s}{2(s^2+4)}$ (b) $\frac{1}{s} - \frac{s}{(s^2+4)}$
(c) $\frac{1}{2s} - \frac{s}{2(s^2+4)}$
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(7) The O.D.E, $\frac{dy}{dx} = \frac{xy}{x^2-1}$ is;

- (a) *Linear equation* (b) *Homogeneous equation*
(c) *Both*

(8) The Auxiliary equation of $y''' + 2y'' - 4y' - 8y = 0$ has the roots;

- (a) $\{-2, 2, 2\}$ (b) $\{2, -2, -2\}$
 (c) $\{2, 1, -2\}$
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(9) $\mathcal{L}^{-1}\left\{\frac{14}{(s+5)^3}\right\} =$

- (a) $14e^{-5t}t^3$ (b) $14e^{5t}t^3$
 (c) $7e^{-5t}t^2$
-

(10) $\mathcal{L}\{\sin 7t \cos 3t\} =$

- (a) $\frac{5}{s^2 + 100} + \frac{2}{s^2 + 16}$ (b) $\frac{10}{s^2 + 100} + \frac{4}{s^2 + 16}$
 (c) $\frac{2}{s^2 + 10} + \frac{5}{s^2 + 4}$
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(Q2) Mark true (T) or false (F) of the following ; (1 mark each)

(1) The differential equation $\frac{1}{x} \frac{dy}{dx} - \frac{2y}{x^2} = xy^2$ is a linear in y . []

(2) $y \frac{dy}{dx} = x$, $y(1) = 0$ has unique solution []

(3) The function $y = 2x^3$ is a solution to $x \frac{dy}{dx} = 3y$ []

(4) We can solve $y'' - y = \sin^2 x + \cos^2 x$ by undetermined coefficient method . []

(5) y_1 and y_2 are linearly independent if and only if
 neither of them is constant multiple of the other []

(6) If $\frac{1}{N} \left(\frac{\partial M}{\partial y} - \left(\frac{\partial N}{\partial x} \right) \right) = 5$, then the integrating factor is $e^{\int dy}$ []

Q(3) Determine the form of a particular solution of $y'' + 9y = e^{3x} + x \sin 3x + \cos 4x$ (2.5 marks)

Q(4) Find the general solution of the equation ;

$$x \frac{dy}{dx} + y = \frac{1}{y^2} \quad (4 \text{ marks})$$

Q(5) Show that the following equation is Exact ,then Solve it ,

$$3y \, dx + (3x - y - 1)dy = 0 \quad (4 \text{ marks})$$

Q(6) Find the particular solution of the equation(by undetermined coefficients);
 $y'' - 2y' + 2y = 2x - 2 \quad y(0) = 0 \quad , y'(0) = 2 \quad (5 \text{ marks})$

Q(7) a) prove that $\mathcal{L}\{te^{4t} \cosh 3t\} = \frac{(s-4)^2 + 9}{[(s-4)^2 - 9]^2}$ **(3 marks)**

b) Find $\mathcal{L}\{(t+1)^2 e^{5t}\}$ **by the table (1.5marks)**

Q8) Solve the initial value problem by using the method of laplace transforms;

$$y'' + 6y = t^2 - 1$$

$$y(0) = 0, y'(0) = -1$$

(6 marks)