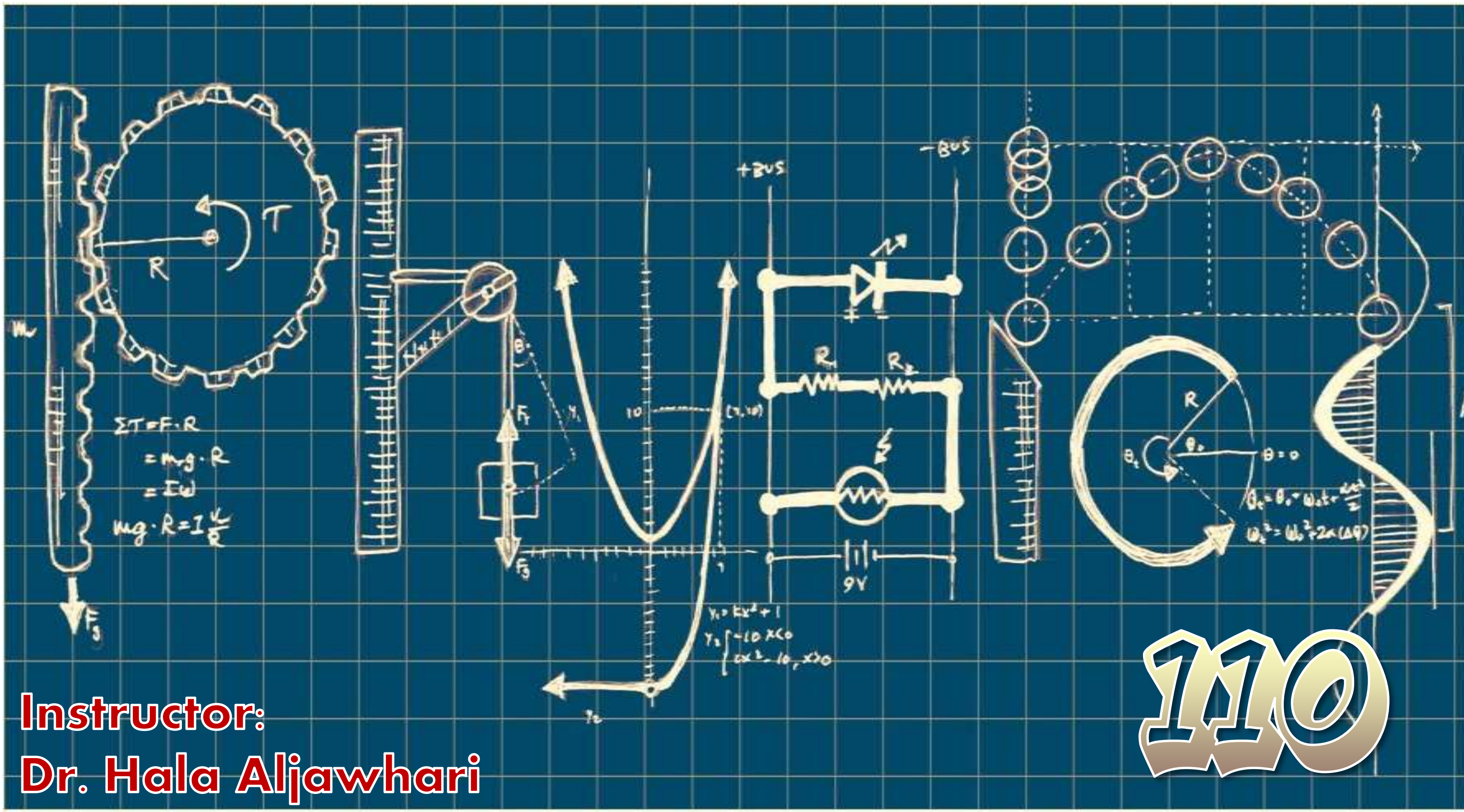
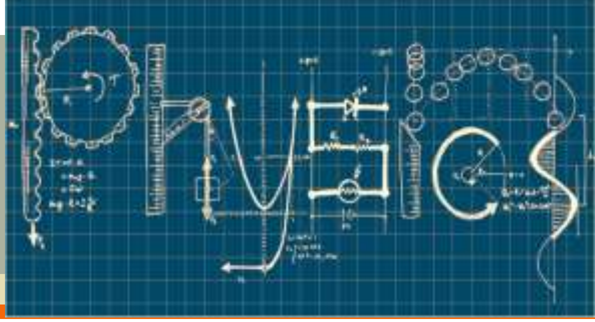
بسم الله الرحمن الرحيم



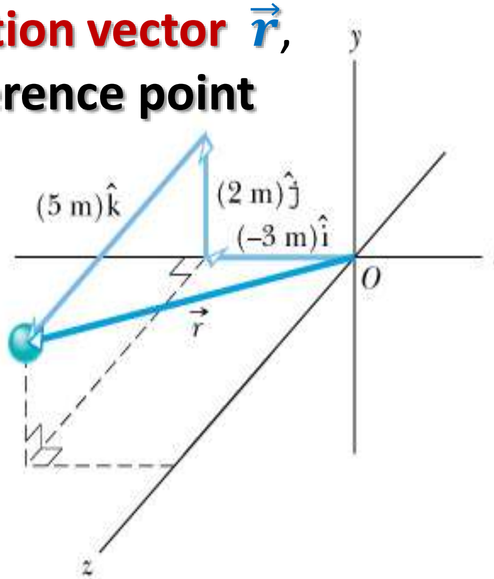
Instructor:
Dr. Hala Aljawhari

Motion in Two And Three Dimensions



Position Vector :

To locate an object means to find it's **position vector $\vec{r}$**, which **is a vector that extends from a reference point (usually the origin)to the particle**.



For example: $\vec{r} = \left(-3\hat{i} + 2\hat{j} + 5\hat{k}\right)m$

displacement Vector :

If the position vector changes from $\vec{r}_1$, to $\vec{r}_2$, during a certain time interval, then the particle's **displacement vector** is

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

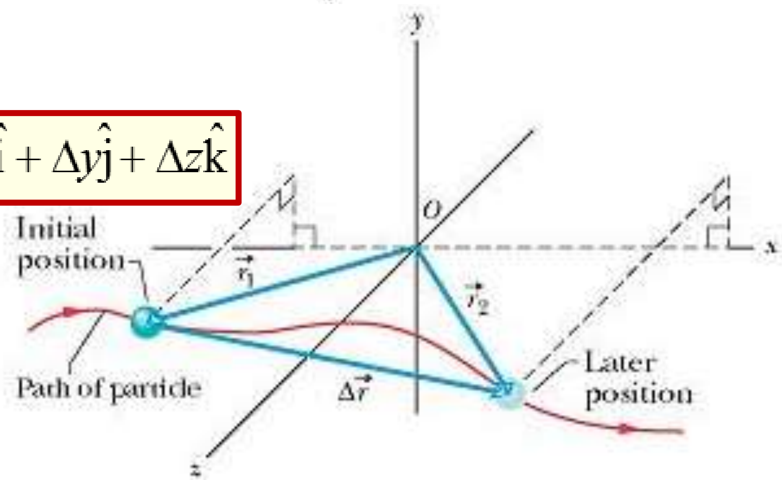
$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

where:

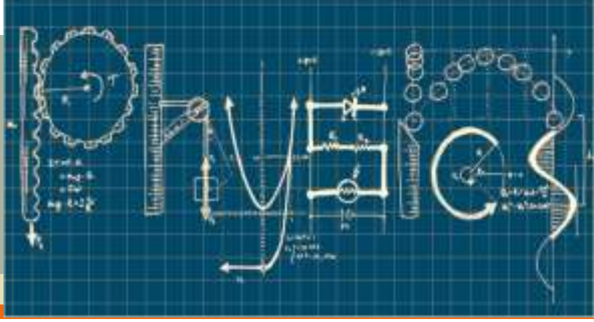
$$\Delta x = x_2 - x_1$$

$$\Delta y = y_2 - y_1$$

$$\Delta z = z_2 - z_1$$



Motion in Two And Three Dimensions



Sample Problem 4-1

In Fig. 4-2, the position vector for a particle initially is

$$\vec{r}_1 = (-3.0\text{ m})\hat{\mathrm{i}} + (2.0\text{ m})\hat{\mathrm{j}} + (5.0\text{ m})\hat{\mathrm{k}}$$

and then later is

$$\vec{r}_2 = (9.0\text{ m})\hat{\mathrm{i}} + (2.0\text{ m})\hat{\mathrm{j}} + (8.0\text{ m})\hat{\mathrm{k}}.$$

What is the particle's displacement $\Delta\vec{r}$ from $\vec{r}_1$ to $\vec{r}_2$?

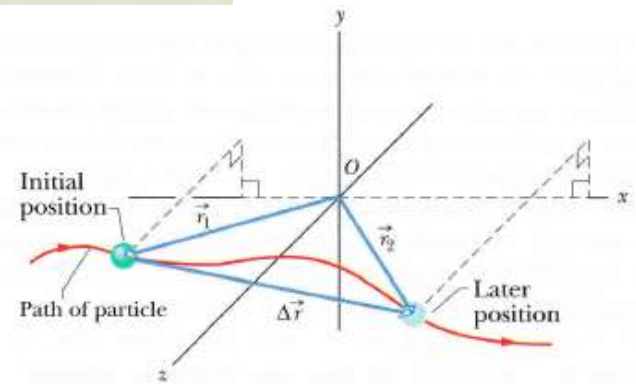
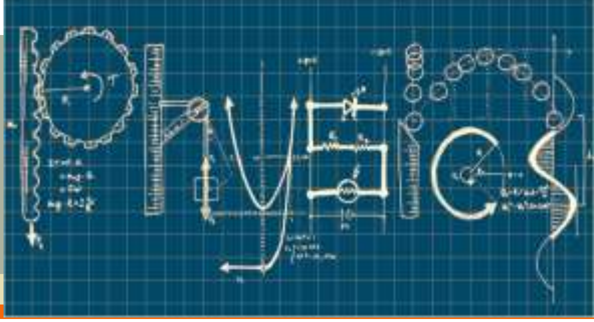


FIG. 4-2 The displacement $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ extends from the head of the initial position vector $\vec{r}_1$ to the head of the later position vector $\vec{r}_2$.

Motion in Two And Three Dimensions



Average velocity $\vec{v}_{avg}$

The ratio of displacement that occurs during a particular time interval to that interval

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1}$$

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

Instantaneous velocity $\vec{v}$

It is the derivative of $\vec{r}$ with respect to **t**.

$$\vec{v} = \frac{d\vec{r}}{dt}$$

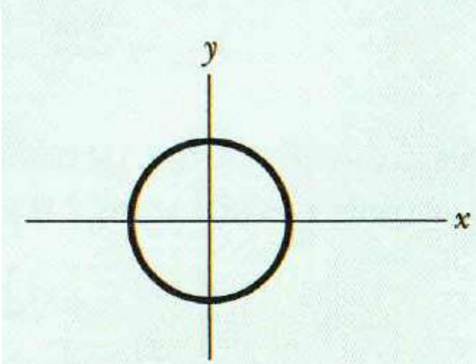
But, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

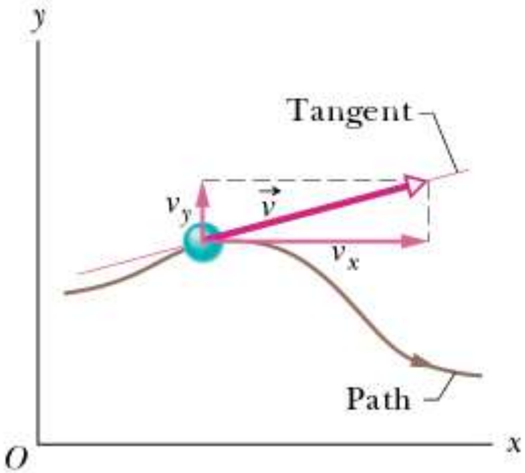
Then, $\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$ or $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

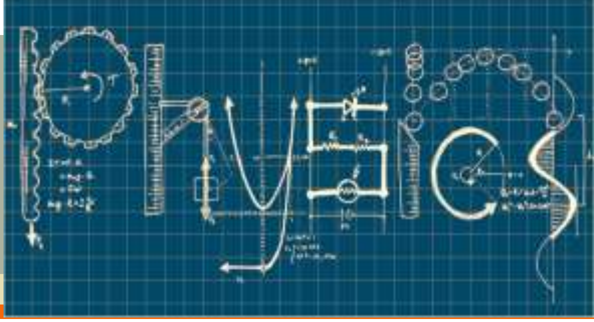
Motion in Two And Three Dimensions

The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

CHECKPOINT 1 The figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v} = (2\text{ m/s})\hat{i} - (2\text{ m/s})\hat{j}$, through which quadrant is the particle moving at that instant if it is traveling (a) clockwise and (b) counterclockwise around the circle? For both cases, draw $\vec{v}$ on the figure.







Sample Problem 4-2

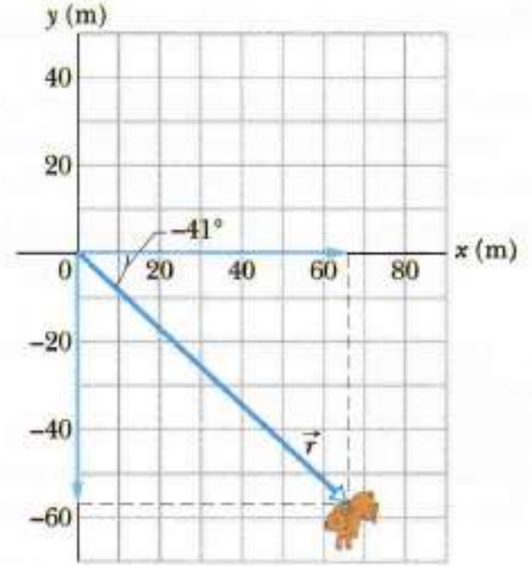
A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28$$

and

$$y = 0.22t^2 - 9.1t + 30.$$

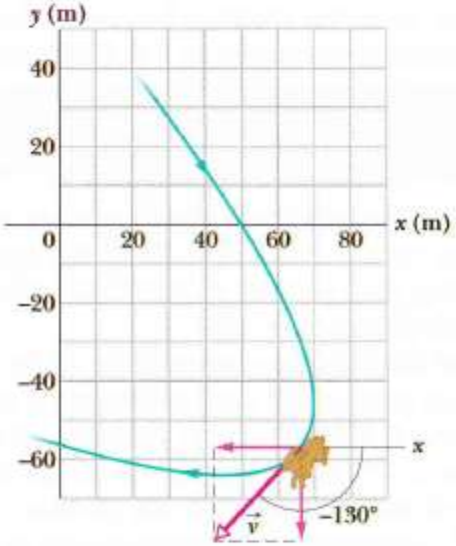
(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?



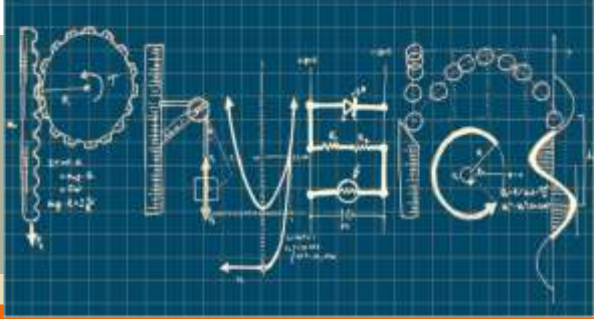
Motion in Two And Three Dimensions

Sample Problem 4-3

For the rabbit in Sample Problem 4-2 find the velocity $\vec{v}$ at time $t = 15$ s.



Motion in Two And Three Dimensions



Average acceleration $\vec{a}_{avg}$

The ratio of change of velocity that occurs during a particular time interval to that interval

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1}$$

Instantaneous acceleration $\vec{a}$

It is the derivative of $\vec{v}$ with respect to **t**.

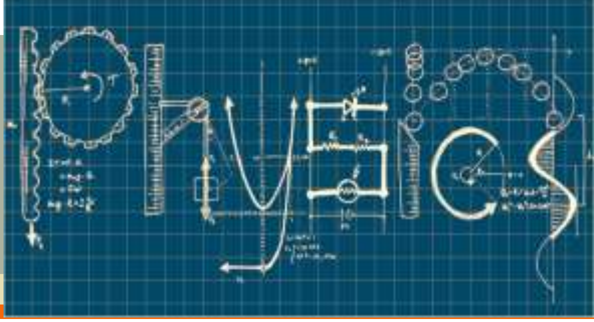
$$\vec{a} = \frac{d\vec{v}}{dt}$$

But, $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

Then, $$\vec{a} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

or $$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Motion in Two And Three Dimensions



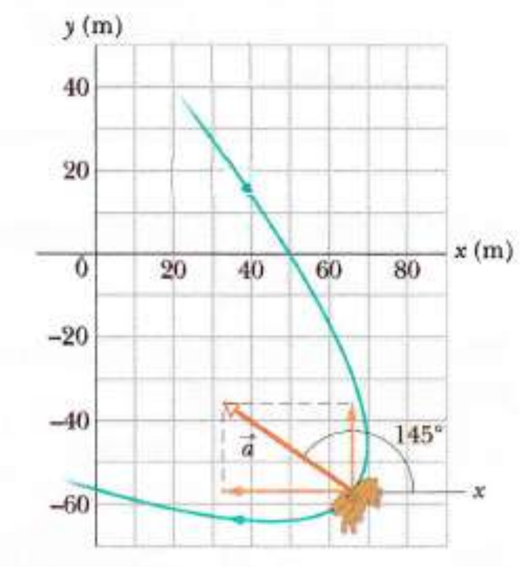
Sample Problem 4-4

A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28$$

and $$y = 0.22t^2 - 9.1t + 30.$$

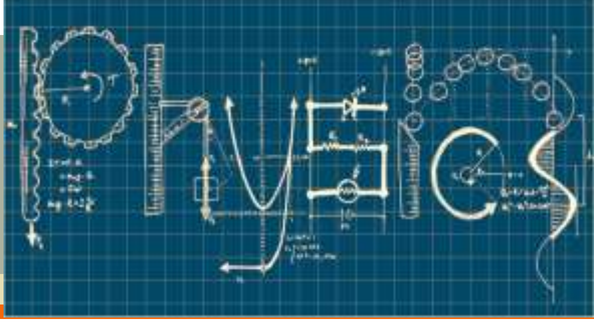
Find the acceleration $\vec{a}$ at **t=15** s.



Sample Problem 4-5

A particle with velocity $\vec{v}_0 = -2.0\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}$ (in meters per second) at $t = 0$ undergoes a constant acceleration $\vec{a}$ of magnitude $a = 3.0\ \mathrm{m/s^2}$ at an angle $\theta = 130°$ from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at $t = 5.0$ s?

Motion in Two And Three Dimensions

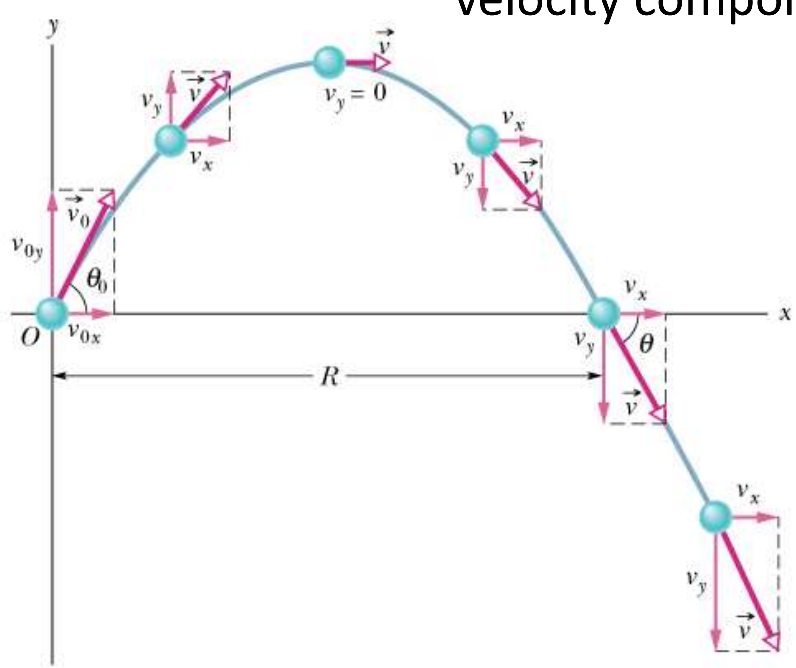


Projectile Motion

The motion of an object in a vertical plane under the influence of gravitational force is known as "**projectile motion**."

The projectile is launched with an initial velocity $\vec{v}_0$

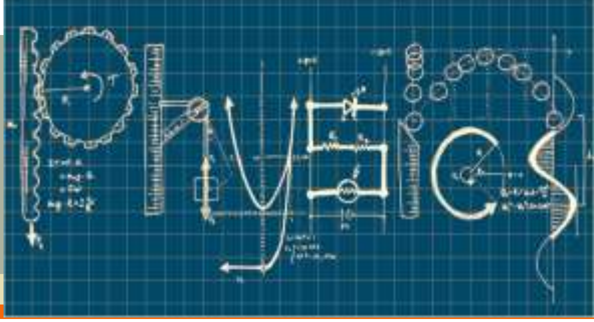
From the fig. we can find that the horizontal and vertical velocity components are: $v_{0x} = v_0 \cos\theta_0$ $v_{0y} = v_0 \sin\theta_0$



Projectile motion can be divided into a horizontal and a vertical motion along the *x*- and *y*-axes, respectively.

These two motions are **independent** of each other. Motion along the *x*-axis has $a_y = 0$. Motion along the *y*-axis has uniform acceleration $a_y = -g$.

Motion in Two And Three Dimensions



the particle's equations of motion along the horizontal x axis and vertical y axis are

Horizontal Motion

No Force ➡ $a_x = 0$ ➡ $v_x = v_{0x}$ ➡ $x = v_{0x}t$

Vertical Motion

Gravity

⬇

$$a_y = -g$$

⬇

$$v_y = v_{0y} - g\,t$$

⬇

$$y = v_{0y}t - \frac{1}{2}gt^2$$

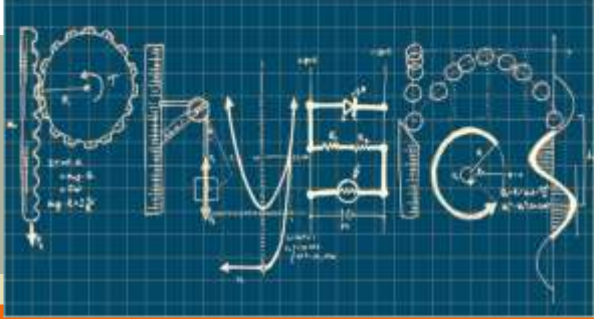
$$x - x_0 = (v_0 \cos\theta_0)t,$$

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2,$$

$$v_y = v_0 \sin\theta_0 - gt,$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

Motion in Two And Three Dimensions



Maximum Height H

$\vec{v} = v_x$

$v_y = 0$

$$H = \frac{(v_0 \sin \theta_0)^2}{2g}$$

The Range R

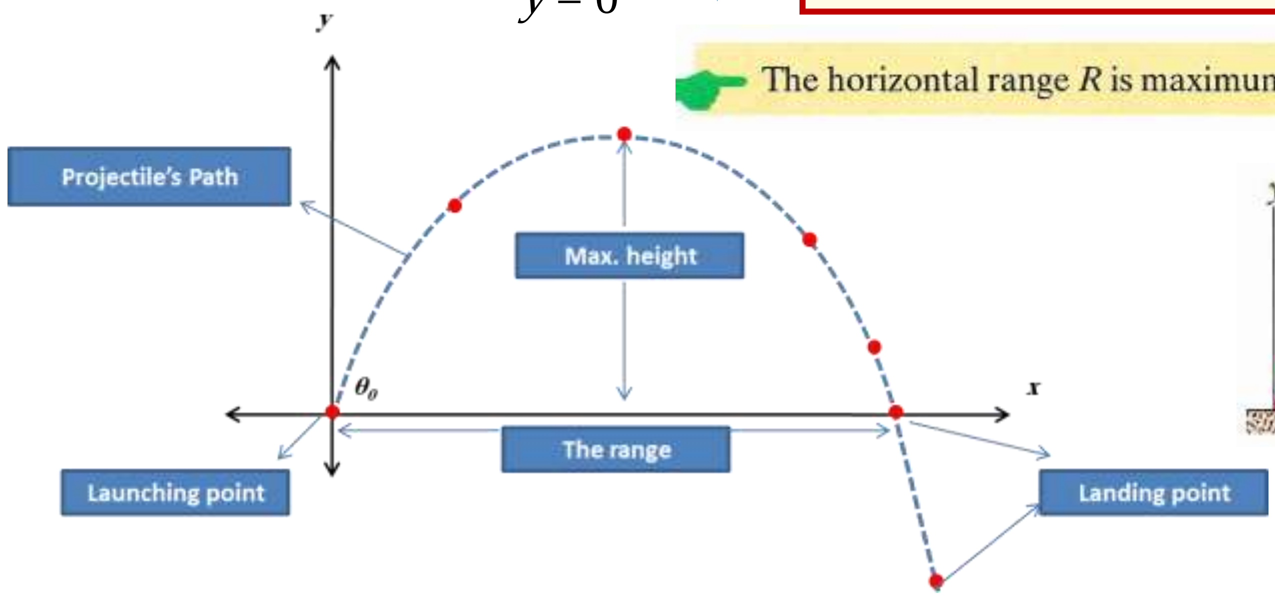
$x = R$

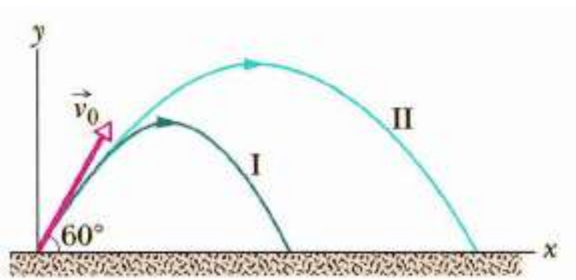
$y = 0$

$$R = \frac{v_0^2}{g} \sin 2\theta_0.$$

$\theta_0 = 45^0 \rightarrow R_{max} = \frac{v_0^2}{g}$

The horizontal range R is maximum for a launch angle of 45°.



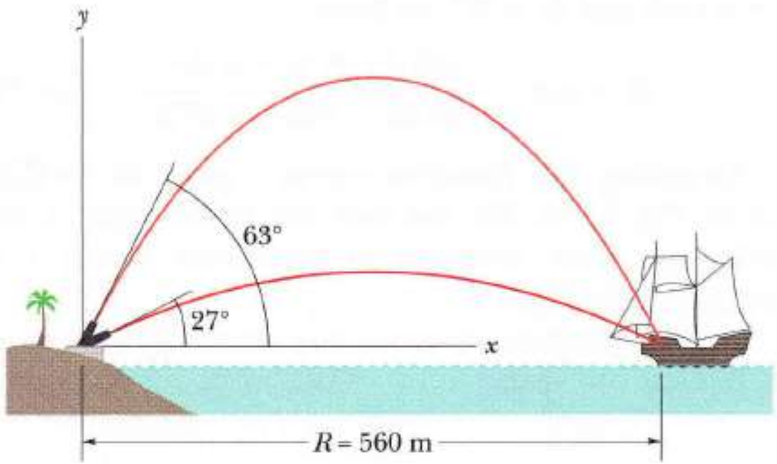


Sample Problem 4-7

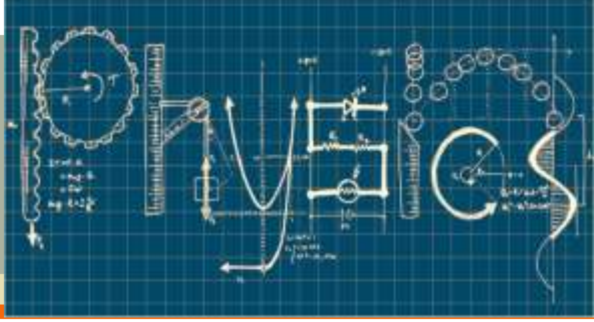
Figure 4-16 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?

(b) What is the maximum range of the cannonballs?



Motion in Two And Three Dimensions



Problem 4-21

A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air?

$$y = v_0 \sin\theta_0 \, t - \frac{1}{2} g t^2$$

$$-45 = 250\sin(0) - \tfrac{1}{2} 9.8 t^2$$

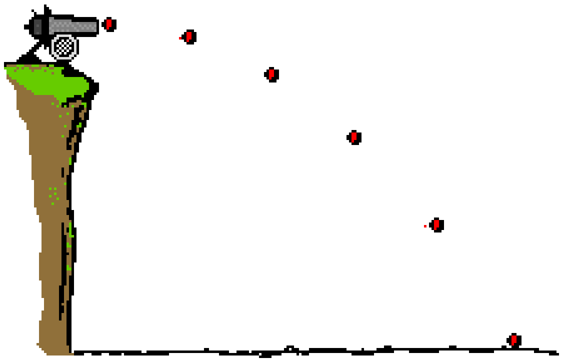
$$t^2 = \frac{45}{4.9} = 9.18 \quad \Rightarrow \boxed{t = 3.03s}$$

$$x - x_0 = (v_0 \cos\theta_0)t,$$

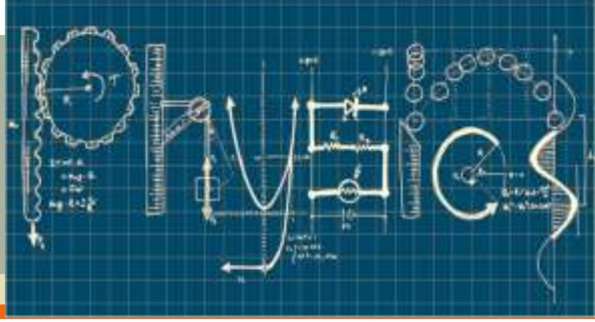
$$\boxed{y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2} g t^2,}$$

$$v_y = v_0 \sin\theta_0 - gt,$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$



Motion in Two And Three Dimensions



Problem 4-21

(b) At what horizontal distance from the firing point does it strike the ground?

$$x = v_0 \cos\theta_0\, t$$

$$x = 250\cos(0)(3.03)$$

$$x = 757.5m$$

(c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

$$v_y = v_0 \sin\theta_0 - g\,t$$

$$v_y = 250\sin(0) - 9.8(3.03)$$

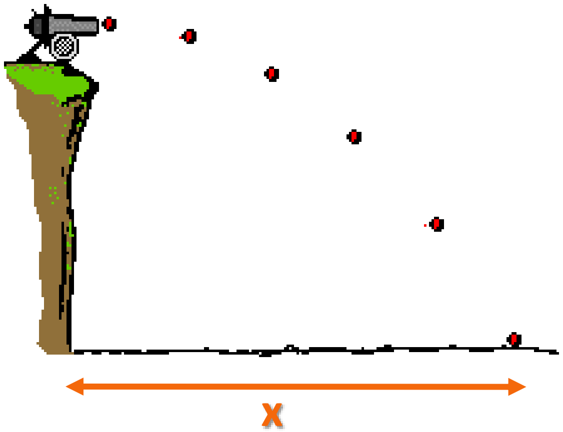
$$v_y = -29.7m/s$$

$$x - x_0 = (v_0 \cos\theta_0)t,$$

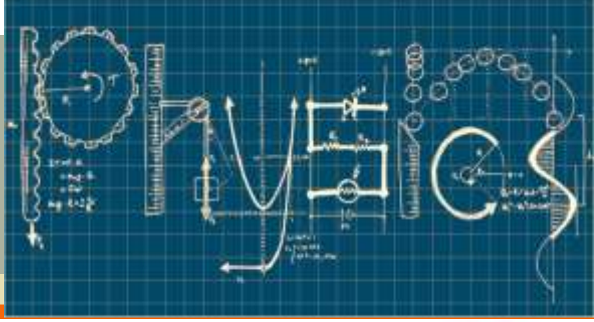
$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2,$$

$$v_y = v_0 \sin\theta_0 - gt,$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$



Motion in Two And Three Dimensions



Problem 4-38

You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-38). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall?

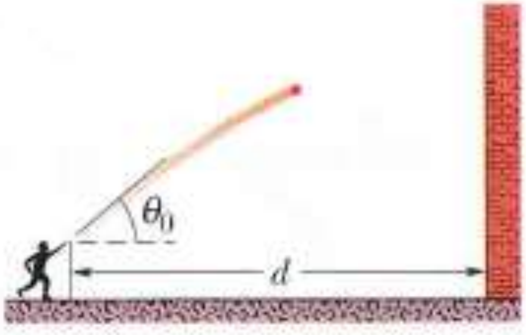


FIG. 4-38 Problem 38.

$$t = \frac{x}{v_0 \cos\theta_0}$$

$$x - x_0 = (v_0 \cos\theta_0)t,$$

$$y - y_0 = (v_0 \sin\theta_0)t - \tfrac{1}{2}gt^2,$$

$$v_y = v_0 \sin\theta_0 - gt,$$

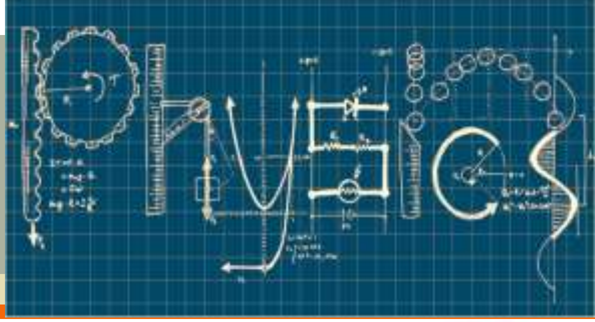
$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0).$$

The equation of the projectile path (**TRAJECTORY**)

$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0 \cos\theta_0)^2}$$

$$y = (\tan 40)22 - \frac{9.8(22)^2}{2(25\cos 40)^2} = 11.99m$$

Motion in Two And Three Dimensions



Problem 4-38

What are the (b) horizontal and
(c) vertical components of its velocity as it hits the wall?

$$v_x = v_{0x}$$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0).$$

$$v_x = 25\cos(40)$$

$$\boxed{v_x = 19.15 m/s}$$

$$v_y^2 = (25\sin 40)^2 - 2(9.8)(11.99)$$

$$v_y^2 = 23.23 \quad \Rightarrow \boxed{v_y = 4.82\text{m/s}}$$

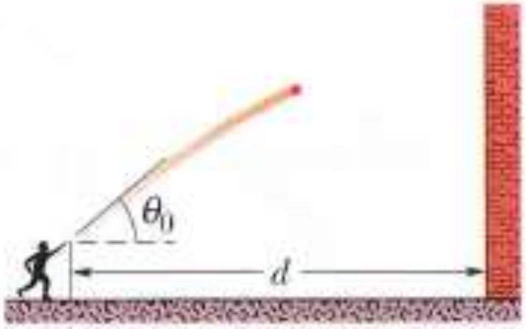


FIG. 4-38 Problem 38.

$$x - x_0 = (v_0 \cos \theta_0)t,$$

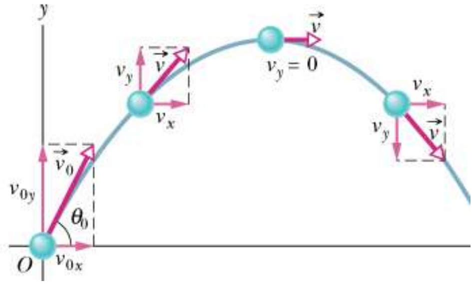
$$y - y_0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2,$$

$$v_y = v_0 \sin \theta_0 - gt,$$

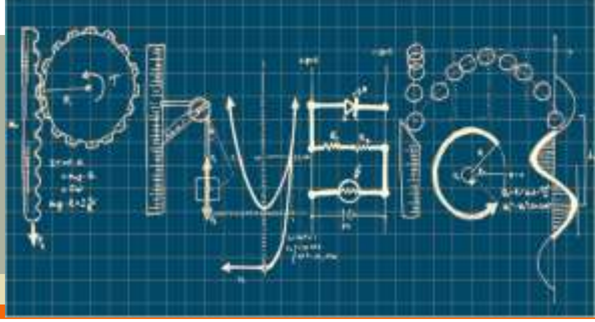
$$\boxed{v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0).}$$

(d) When it hits, has it passed the highest point on its trajectory?

Not yet , because v_y still positive.



Motion in Two And Three Dimensions

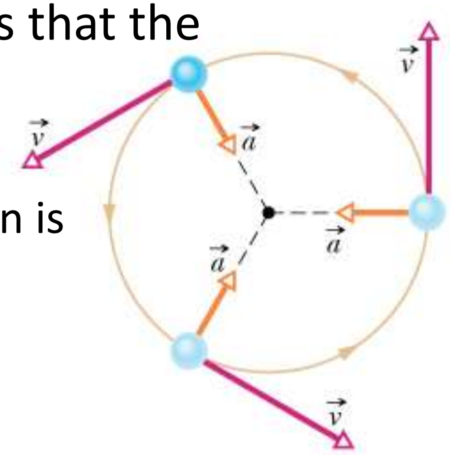


uniform circular Motion

A particle is in **uniform circular motion** if it moves on a circular path of radius **r** with constant speed **v**.

Even though **the speed** is **constant**, **the velocity** is **not**. The reason is that the direction of the velocity vector changes from point to point along the path.

The fact that the velocity changes means that the **acceleration is not zero.**



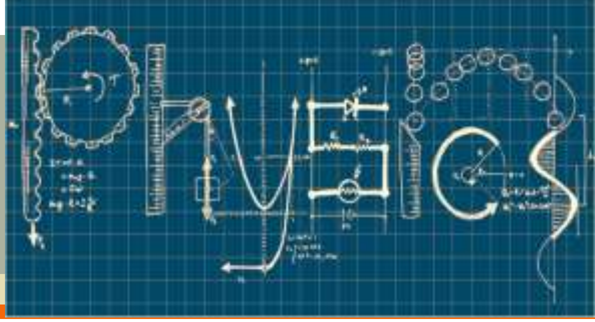
The period

The time ***T*** it takes to complete a full revolution is known as the "**period**." It is given by the equation

$$\text{Time} = \frac{\text{distance}}{\text{velocity}}$$

$$T = \frac{2\pi r}{v}.$$

Motion in Two And Three Dimensions



tangential Velocity

The velocity in uniform circular motion has the following characteristics:

1. Its vector always **tangent** to the circle in the direction of motion .

2. Its magnitude *v* is constant and given by the equation :

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

Centripetal Acceleration

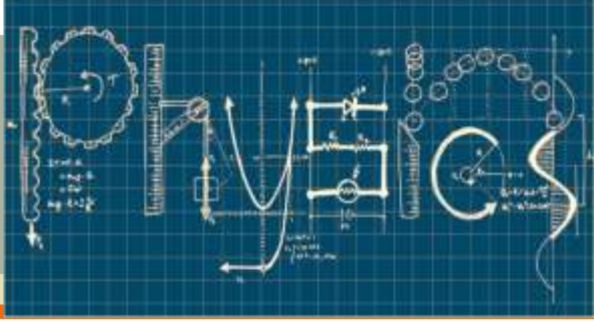
The acceleration in uniform circular motion has the following characteristics:

1. Its vector points toward the center of the circular path, thus the name "**centripetal**."

2. Its magnitude *a* is constant and given by the equation :

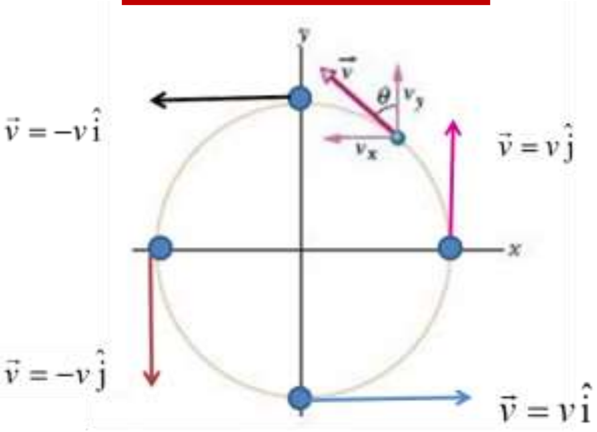
$$a = \frac{v^2}{r}.$$

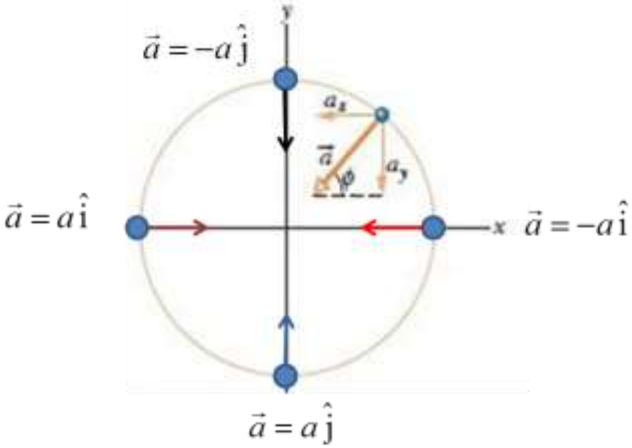
Motion in Two And Three Dimensions



$$\vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$





CHECKPOINT 5 An object moves at constant speed along a circular path in a horizontal xy plane, with the center at the origin. When the object is at $x = -2$ m, its velocity is $-(4\text{ m/s})\hat{j}$. Give the object's (a) velocity and (b) acceleration at $y = 2$ m.

Sample Problem 4-10

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{\mathrm{i}} + 500\hat{\mathrm{j}})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{\mathrm{i}} - 500\hat{\mathrm{j}})$ m/s?