# CHAPTER 1
# Part 1

# Combinatorial Probability

# Contents

# A) The basic principle of counting

- **Rule1**: suppose that two experiments are to be performed . Then if experiment 1 can result in any one of $m$ possible outcomes and if experiment 2 can result in any one of $n$ possible outcomes ,then together there are $mn$ possible outcomes of two experiments .

# *A) The basic principle of counting*

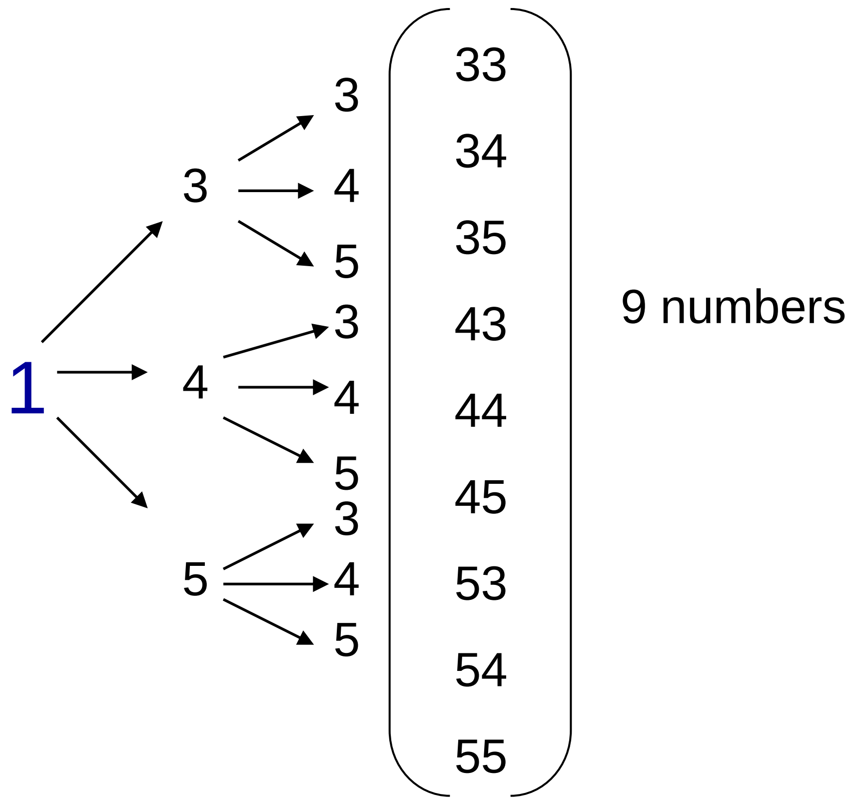
**Ex. 1** : How many different ways are possible to choose two numbers from 3,4,5 when

1- with replacement.

**Solution:**

m=3 , n=3

Total=3*3=9



# A) The basic principle of counting

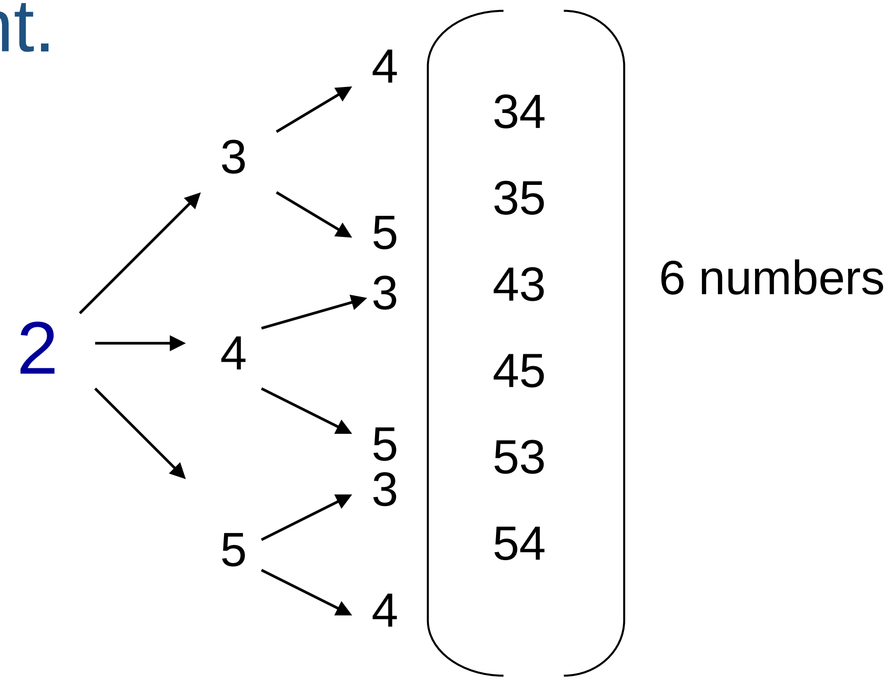
**Ex. 1** **:** How many different ways are possible to choose two numbers from 3,4,5 when

2- without replacement.

**solution:**

m=3 , n=3

Total=3*2=6



## *A) The basic principle of counting*

**Ex. 2 :** A small community consists of 10 women , each of women has 3 children , if one women and one of her children are to be chosen as mother and child of the year , how many different choices are possible ?

**Solution:**

m=10, n=3

choices=10*3=30.

## *A) The basic principle of counting*

- **Rule 2 : The generalized basic principle of counting:**

**If $r$ experiments that are to be performed are such that the first one way result in any of $n1$ possible outcomes, and if for each of these $n1$ possible outcomes there are $n2$ possible outcomes of the second experiment, and if for each of the possible outcomes of the first two experiments there are $n3$ possible outcomes of the third experiment, and if . . . , then there is a total of $n1, n2, . . . , nr$ possible outcomes of r experiments.**

# *A) The basic principle of counting*

- **Ex. 3** :How many different ways are possible to choose four number from 1, 2, 3, 4, 5 when

1- With replacement

**Solution:**

$$n_1 = 5,\ \ n_2 = 5,\ n_3 = 5,\ \ n_4 = 5.$$

$$\text{Total} = 5\times5\times5\times5 = 625.$$

2- Without replacement .

Solution:

$$n_1 = 5,\ \ n_2 = 4,\ n_3 = 3,\ \ n_4 = 2.$$

$$\text{Total} = 5\times4\times3\times2 = 120.$$

# *A) The basic principle of counting*

- **Ex. 4** :A college planning committee consists of 3 fresh man. 4 sophomores, 5 juniors, and 2 seniors, a sub committee of 4, consisting a single representative from each of the classes is to be chosen. How many different subcommittees are possible?

  **Solution:**

$$n_1 = 3,\ n_2 = 4,\ n_3 = 5,\ n_4 = 2.$$

$$\text{Total} = 3 \times 4 \times 5 \times 2 = 120.$$

## *A) The basic principle of counting*

- **Ex. 5** :How many different 7-place license plates are possible if the first 3 places are to be occupied by letters and the final 4 by numbers?

1- With replacement.

**Solution**:

$$n_1 = 26,\ n_2 = 26,\ n_3 = 26,\ n_4 = 10,\ n_5 = 10,\ n_6 = 10,\ n_7 = 10.$$

$$\text{Total} = 26 \times 26 \times 26 \times 10 \times 10 \times 10 \times 10 = 175{,}760{,}000.$$

2- Without replacement.

**Solution**:

$$n_1 = 26,\ n_2 = 25,\ n_3 = 24,\ n_4 = 10,\ n_5 = 9,\ n_6 = 8,\ n_7 = 7.$$

$$\text{Total} = 26 \times 25 \times 24 \times 10 \times 9 \times 8 \times 7 = 78{,}624{,}000.$$

# B) Permutation

**Rule 1**

**There are Pn**

**n! = n (n - 1) ... 3 * 2 * 1 Possible linear orderings of n items**

$$P_n^n = (n)_n = n!$$

$$P_r^n = (n)_r = \frac{n!}{(n-r)!}$$

# B) Permutation

❖ **Ex. 6** :How many different arrangements are possible to arrange 10 people?

**Solution**:

$$10! = 10 \times 92 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 3628800.$$

❖ **Ex. 7** : How many different batting orders are possible for a baseball team consisting of 9 players?

**Solution**:

$$9! = 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 362880.$$

# B) Permutation

- **Rule 2 : There are**

$$\binom{n}{n_1 \ \ n_2 \ \ n_3 \ldots \ n_k} = \frac{n!}{n_1!\ n_2! \ldots n_k!}$$

**different permutations of n objects, where $n = n_1 + n_2 + \ldots + n_k$, which n1 are alike , $n_2$ are alike , … , $n_k$ are alike .**

# B) Permutation

- **Ex. 10** : How many different letter arrangements can be formed using the letters P E P P E R?

**Solution:**

$$n_1 = P = 3,$$

$$n_2 = E = 2,$$

$$n_3 = R = 1.$$

$$n = n_1 + n_2 + n_3 = 3 + 2 + 1 = 6.$$

Then, there are

$$\binom{6}{3\ \ 2} = \frac{6!}{3!\,2!} = 60 \text{ possible letter arrangement.}$$

# B) Permutation

- **Ex. 11** : How many different signals, each consisting of 9 flags hung in a line, can be made from a set of 4 white flags,3 red flags, and 2 blue flags if all flags of the same color are identical?

**Solution:**

$$n_1 = \text{white flags} = 4,$$

$$n_2 = \text{red flags} = 3,$$

$$n_3 = \text{blue flags} = 2.$$

$$n = n_1 + n_2 + n_3 = 4 + 3 + 2 = 9.$$

$$\text{Then, there are}$$

$$\binom{9}{4\ 3\ 2} = \frac{9!}{4!\,3!\,2!} = 1260 \ \text{ different signals.}$$

## C) Combination

The number of different subgroups of size **r** that can be chosen from a set of size **n** is given by

$$C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P_r^n}{r!} \quad , \quad r \le n$$

# C) Combination

- **Ex. 12** :A committee of 3 is to be performed from a group of 20 people. How many different committees are possible?

**Solution**:

There are

$$\binom{20}{3} = \frac{20!}{3!\ 17!} = 1140 \text{ possible committees.}$$

# C) Combination

- **<u>Ex. 13</u>** :From a group of 5 women and 7 men, How many different committees consisting of 2 women and 3 men can be performed ?

**<u>Solution</u>:**

There are

$$C_2^5 \times C_3^7 = \frac{5!}{2!\,3!} \times \frac{7!}{3!\,4!} = 350 \text{ possible committees.}$$

# Sampling

The number of samples
( size n )
that can be made from
a population
( size N ) is

without replacement

$$P_n^N$$

with replacement

$$N^n$$

# Binomial Theorem

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

# Thank You！