Exercises 4.4

$$1)\ y = 2u - 3\ ,u = 3x^2 \Rightarrow y = 2(3x^2) - 3 = 6x^2 - 3$$

$$\Rightarrow \frac{dy}{dx} = 6(2x) - 0 = 12x$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (2)(6x) = 12x$$

$$2)\ y = \cos u\ ,u = \tan x \Rightarrow y = \cos(\tan x)$$

$$\Rightarrow \frac{dy}{dx} = -\sin(\tan x)\,(sec^2 x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = (-\sin u)(sec^2 x) = -\sin(\tan x)\,(sec^2 x)$$

$$3)\ y = \cot u\ ,u = e^x\ \Rightarrow y = \cot e^x$$

$$\Rightarrow \frac{dy}{dx} = -csc^2 e^x (e^x) = -e^x csc^2 e^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -csc^2 u(e^x) = -e^x csc^2 u = -e^x csc^2 e^x$$

$$4)\ y = \csc u\ ,u = x^2 - 1 \Rightarrow y = \csc(x^2 - 1)$$

$$\Rightarrow \frac{dy}{dx} = -\csc(x^2 - 1)\cot(x^2 - 1)\,(2x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\csc u \cot u\,(2x) = -\csc(x^2 - 1)\cot(x^2 - 1)\,(2x)$$

$$5)\ y = \tan u\ ,u = 10x - 2\ \Rightarrow y = \tan(10x - 2)$$

$$\Rightarrow \frac{dy}{dx} = sec^2(10x - 2)(10) = 10sec^2(10x - 2)$$

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = sec^2(u)(10) = sec^2(10x - 2)(10)$$
$$= 10sec^2(10x - 2)$$

$$6)\ y = (1 - x^2)^9 \Rightarrow y' = 9(1 - x^2)^8(0 - 2x) = 9(-2x)(1 - x^2)^8$$
$$= -18x(1 - x^2)^8$$

$$7)\ y=\cos(e^{2x}) \Rightarrow y' = -\sin(e^{2x})\, e^{2x}(2) = -2e^{2x}\sin(e^{2x})$$

$$8)\ y=\tan(\sin x) \Rightarrow y' = sec^2(\sin x)(\cos x) = \cos x\, sec^2(\sin x)$$

$$9)\ y=\sin(x^2-3x+2) \Rightarrow y' = \cos(x^2-3x+2)\,(2x-3)$$
$$= (2x-3)\cos(x^2-3x+2)$$

$$10)\ y = sec^{5/4}(x+5)$$

$$\Rightarrow y' = \frac{5}{4} sec^{\frac{5}{4}-1}(x+5)[\sec(x+5)\tan(x+5)](1)$$
$$= \frac{5}{4} sec^{\frac{5}{4}-1}(x+5)\sec(x+5)\tan(x+5)$$
$$= \frac{5}{4} sec^{\frac{5}{4}-1+1}(x+5)\tan(x+5)$$
$$= \frac{5}{4} sec^{\frac{5}{4}}(x+5)\tan(x+5)$$

$$11)\ y = x^{7/2} \Rightarrow y' = \frac{7}{2}x^{7/2-1} = \frac{7}{2}x^{\frac{7}{2}-\frac{2}{2}} = \frac{7}{2}x^{5/2}$$

$$12)\ y = \sqrt[3]{6x} = (6x)^{\frac{1}{3}} \Rightarrow y' = \frac{1}{3}x^{1/3-1}(6) = \frac{1}{3}x^{\frac{1}{3}-\frac{3}{3}} = \frac{6}{3}x^{-2/3}$$
$$= 2x^{-2/3} = \frac{2}{x^{2/3}} = \frac{2}{\sqrt[3]{x^2}}$$

$$13)\ y = \sqrt{x^3-3} \Rightarrow y' = \frac{1}{2\sqrt{x^3-3}}(3x^2) = \frac{3x^2}{2\sqrt{x^3-3}}$$

$$14)\ F(x) = (x^3-2x-1)^3 = 3(x^3-2x-1)^2(3x^2-2)$$
$$= 3(3x^2-2)(x^3-2x-1)^2$$

$$15)\ G(x) = (1+x^5)^{\frac{3}{2}}$$

$$\Rightarrow y' = \frac{3}{2}(1+x^5)^{\frac{3}{2}-1}(5x^4) = \frac{3}{2}(5x^4)(1+x^5)^{\frac{3}{2}-\frac{2}{2}} = \frac{15}{2}(x^4)(1+x^5)^{\frac{1}{2}}$$
$$= \frac{15}{2}x^4\sqrt{1+x^5}$$

$$16)\ H(t) = \frac{1}{(t^3+1)^4} = (t^3+1)^{-4}$$

$$\Rightarrow y' = -4(t^3+1)^{-4-1}(3t^2) = -4(3t^2)(t^3+1)^{-5}$$

$$= -12(t^2)(t^3+1)^{-5} = \frac{-12(t^2)}{(t^3+1)^5}$$

$$17)\ y = \tan(n\theta)\ \Rightarrow y' = sec^2(n\theta)(n) = nsec^2(n\theta)$$

$$18)\ y = (2x-5)^3(3x^2+1)^{-4}$$

$$\Rightarrow y' = 3(2x-5)^2(2)(3x^2+1)^{-4}$$

$$+ (2x-5)^3(-4)(3x^2+1)^{-4-1}(6x)$$

$$= 6(2x-5)^2(3x^2+1)^{-4} - 24x(2x-5)^3(3x^2+1)^{-5}$$

Or by logarithmic differentiable:

$$y = (2x-5)^3(3x^2+1)^{-4}$$

$$ln\, y = ln[(2x-5)^3(3x^2+1)^{-4}]$$

$$lny = ln(2x-5)^3 + ln(3x^2+1)^{-4}$$

$$lny = 3\ln(2x-5) - 4ln(3x^2+1)$$

$$\frac{1}{y}y' = 3\frac{1}{(2x-5)}(2) - 4\frac{1}{(3x^2+1)}(6x) = \frac{1}{(2x-5)} - \frac{24}{(3x^2+1)}$$

$$y' = y\left[\frac{6}{(2x-5)} - \frac{24x}{(3x^2+1)}\right]$$

$$y' = (2x-5)^3(3x^2+1)^{-4}\left[\frac{6}{(2x-5)} - \frac{24x}{(3x^2+1)}\right]$$

$$= \left[\frac{6(2x-5)^3(3x^2+1)^{-4}}{(2x-5)} - \frac{24x(2x-5)^3(3x^2+1)^{-4}}{(3x^2+1)}\right]$$

$$== 6(2x-5)^2(3x^2+1)^{-4} - 24x(2x-5)^3(3x^2+1)^{-5}$$

$$19)\ y = \left(\frac{x^2-1}{x^3+1}\right)^4$$

$$lny = ln\left(\frac{x^2-1}{x^3+1}\right)^4 = 4ln\left(\frac{x^2-1}{x^3+1}\right) = 4[\ln(x^2-1) - \ln(x^3+1)]$$

$$lny = 4\ln(x^2-1) - 4\ln(x^3+1)$$

$$\frac{y'}{y} = \frac{4(2x)}{x^2-1} - \frac{4(3x^2)}{x^3+1}$$

$$\Rightarrow y' = y\left[\frac{(8x)}{x^2-1} - \frac{(12x^2)}{x^3+1}\right] = \left(\frac{x^2-1}{x^3+1}\right)^4 \left[\frac{(8x)}{x^2-1} - \frac{(12x^2)}{x^3+1}\right]$$
$$= \frac{(x^2-1)^4}{(x^3+1)^4}\left[\frac{(8x)(x^3+1) - 12x^2(x^2-1)}{(x^2-1)(x^3+1)}\right]$$
$$= \frac{(x^2-1)^4}{(x^3+1)^4}\left[\frac{-4x^4+12x^2+8x}{(x^2-1)(x^3+1)}\right]$$
$$= \frac{(x^2-1)^4}{(x^3+1)^5}[-4x(x^3-3x-2)]$$

20) $y = e^{xsinx} \Rightarrow y' = e^{xsinx}\,[(1)\sin x + xcos\; x]$
$= e^{xsinx}\,[\sin x + xcos\; x]$

21) $f(\theta) = \left(\frac{\sin\theta}{\cos\theta - 1}\right)^3 \Rightarrow f'$
$$= 3\left(\frac{\sin\theta}{\cos\theta-1}\right)^2\left[\frac{\cos\theta\,(\cos\theta-1) - \sin\theta\,(-sin\theta - 0)}{(\cos\theta-1)^2}\right]$$
$$= 3\left(\frac{\sin\theta}{\cos\theta-1}\right)^2\left[\frac{\cos^2\theta - \cos\theta + \sin^2\theta}{(\cos\theta-1)^2}\right]$$
$$= 3\frac{sin^2\theta}{(\cos\theta-1)^2}\left[\frac{1-\cos\theta}{(\cos\theta-1)^2}\right]$$
$$= 3\frac{1-cos^2\theta}{(\cos\theta-1)^2}\left[\frac{-1(\cos\theta-1)}{(\cos\theta-1)^2}\right]$$
$$= 3\frac{(1-\cos\theta)(1+\cos\theta)}{(\cos\theta-1)^2}\left[\frac{-1(\cos\theta-1)}{(\cos\theta-1)^2}\right]$$
$$= 3\frac{-(\cos\theta-1)(1+\cos\theta)}{(\cos\theta-1)^2}\left[\frac{-1}{\cos\theta-1}\right] = 3\frac{1+\cos\theta}{(\cos\theta-1)^2}$$

22) $g(t) = \tan\left(\frac{\cos t}{t}\right) \Rightarrow g' = sec^2\left(\frac{\cos t}{t}\right)\left[\frac{-t\;sin\;t - \cos t}{t^2}\right]$

23) $y = e^{x\;cos\;x} \Rightarrow y' = e^{x\;cos\;x}\,[cosx - x\sin x]$

$$\Rightarrow y'' = \frac{d}{dx}[e^{x\;cos\;x}][cosx - x\sin x] + [e^{x\;cos\;x}]\frac{d}{dx}[cosx - x\sin x]$$

$$= e^{x\cos x}\,[cosx - x\sin x][cosx - x\sin x] + e^{x\cos x}[-sinx - (sinx - xcos\,x)]$$

$$= e^{x\cos x}\big[[cosx - x\sin x]^2 + [-sinx - \sin x + x\cos x]\big]$$

$$25)\ y = \sec(2x-1) \Rightarrow y' = \sec(2x-1)\tan(2x-1)\,(2)$$
$$= 2\sec(2x-1)\tan(2x-1)$$

$$\Rightarrow y'' = 2\frac{d}{dx}[\sec(2x-1)].\tan(2x-1) + 2\sec(2x-1)\frac{d}{dx}\tan(2x-1)$$

$$= 2\sec(2x-1)\tan(2x-1)\,(2)\tan(2x-1) + 2\sec(2x-1)\,sec^2(2x-1)(2)$$

$$= (2)(2)\sec(2x-1)\,[tan^2(2x-1) + sec^2(2x-1)]$$

Use $1 + tan^2(2x-1) = sec^2(2x-1)$

$$= 4\sec(2x-1)\,[tan^2(2x-1) + tan^2(2x-1) + 1]$$

$$= 4\sec(2x-1)\,[2tan^2(2x-1) + 1]$$

$$27) y = \sin(\sin x) \quad at\ (\pi, 0)$$

$$\Rightarrow y' = \cos(sinx)\,.\cos x$$

$$\Rightarrow y'|_{x=\pi} = \cos(sin\pi)\,.\cos\pi = \cos(0)\,(-1) = (1)(-1) = -1 = m$$

The equation of tangent line is

$$y - y_1 = m(x - x_1) \Rightarrow y - 0 = -1(x - \pi) \Rightarrow y = -x + \pi \Rightarrow y + x = \pi$$