MATH 110

Exercises 4.2

$$\mathbf{1)}\ y = 3x^3 \Rightarrow y' = 3(3x^{3-1}) = 3(3x^2) = 9x^2$$

$$\mathbf{2)}\ y = x^7 + 3x^4 - 5 \Longrightarrow y' = 7x^{7-1} + 3(4x^{4-1}) - 0 = 7x^6 + 12x^3$$

$$\mathbf{3)}\ y = \frac{1}{3}(x^3 + 3x - 1) = \frac{1}{3}x^3 + \left(\frac{1}{3}\right)3x - \left(\frac{1}{3}\right)1 = \frac{1}{3}x^3 + x - \left(\frac{1}{3}\right)$$

$$\Longrightarrow y' = \frac{1}{3}(3)(x)^{3-1} + 1 - 0 = x^2 + 1$$

Or

$$y = \frac{1}{3}(x^3 + 3x - 1)$$

$$\Longrightarrow y' = \frac{1}{3}(3x^{3-1} + 3 - 0) = \frac{1}{3}(3x^2 + 3) = \left(\frac{1}{3}\right)3x^2 + \left(\frac{1}{3}\right)3 = x^2 + 1$$

$$\mathbf{4)}\ y = \frac{x^{10} + 5x + 4}{5} = \frac{x^{10}}{5} + \frac{5x}{5} + \frac{4}{5} \Longrightarrow y' = \frac{10x^{10-1}}{5} + \frac{5(1)}{5} + 0 = 2x^9 + 1$$

Or

$$y = \frac{x^{10} + 5x + 4}{5} = \frac{1}{5}(x^{10} + 5x + 4)$$

$$\Longrightarrow y' = \frac{1}{5}(10x^{10-1} + 5 + 0) = \frac{1}{5}(10x^9 + 5) = \left(\frac{1}{5}\right)10x^9 + \left(\frac{1}{5}\right)5 = 2x^9 + 1$$

$$\mathbf{5)}\ y = \sqrt{3}x^3 - \frac{1}{\sqrt{3}} \Longrightarrow y' = \sqrt{3}(3)x^{3-1} - 0 = \sqrt{3}(3)x^2 = 3\sqrt{3}x^2$$

$$\mathbf{6)}\ y = -\frac{1}{4}x^{-12} \Longrightarrow = \left(-\frac{1}{4}\right)(-12)x^{-12-1} = +\frac{12}{4}x^{-13} = 3x^{-13}$$

$$\mathbf{7)}\ y = 2x^2 - 3x + 1 \Longrightarrow y' = 2(2)x^{2-1} - 3 + 0 = 4x - 3 \Longrightarrow y'' = 4$$

$$8)\ y = -x^4 + 2x^2 - 3 \Longrightarrow\ y' = -4x^{4-1} + 2(2)x^{2-1} - 0 = -4x^3 + 4x$$

$$\Longrightarrow y'' = -4(3)x^{3-1} + 4 = -12x^2 + 4$$

$$9)\ y = 1 - t^2 + 4t^4 \Longrightarrow y' = 0 - 2t^{2-1} + 4(4)t^{4-1} = -2t + 16t^3$$

$$\Longrightarrow \mathrm{y}'' = -2 + 16(3)\mathrm{t}^{3-1} = -2 + 48\mathrm{t}^2 = 48\mathrm{t}^2 - 2$$

$$10)\ y = -s^{-2} + \frac{1}{s} = -s^{-2} + s^{-1} \Longrightarrow y' = -(-2)s^{-2-1} - 1s^{-1-1} = 2s^{-3} - s^{-2}$$

$$\Longrightarrow y'' = 2(-3)s^{-3-1} - (-2)s^{-2-1} = -6s^{-4} + 2s^{-3} = 2s^{-3} - 6s^{-4}$$

$$11)\ y = \frac{1}{2}x^4 + \frac{2}{3}x^3 + \frac{1}{2}x^2 + \frac{2}{5}$$

$$\Longrightarrow y' = \frac{1}{2}(4)x^{4-1} + \frac{2}{3}(3)x^{3-1} + \frac{1}{2}(2)x^{2-1} + 0 = 2x^3 + 2x^2 + x$$

$$\Longrightarrow y'' = 2(3)x^{3-1} + 2(2)x^{2-1} + 1 = 6x^2 + 4x + 1$$

$$12)\ y = 4\theta - \frac{3}{\theta} + \frac{1}{\theta^2} = 4\theta - 3\theta^{-1} + \theta^{-2}$$

$$\Longrightarrow y' = 4 - 3(-1)\theta^{-1-1} + (-2)\theta^{-2-1} = 4 + 3\theta^{-2} - 2\theta^{-3}$$

$$\Longrightarrow y'' = 0 + 3(-2)\theta^{-2-1} - 2(-3)\theta^{-3-1} = -6\theta^{-3} + 6\theta^{-4}$$

$$13)\ y = \frac{1}{3}x^{-3} + 2\sqrt{x} = \frac{1}{3}x^{-3} + 2x^{1/2}$$

$$\Rightarrow y' = \frac{1}{3}(-3)x^{-3-1} + 2\frac{1}{2}x^{1/2-1} = -x^{-4} + x^{-1/2}$$

$$\Longrightarrow y'' = -(-4)x^{-4-1} - \frac{1}{2}x^{-1/2-1} = 4x^{-5} - \frac{1}{2}x^{-3/2} = \frac{4}{x^5} - \frac{1}{2\sqrt{x^3}}$$

$$14)\ y = x + \frac{1}{x} - \frac{3}{x^2} = x + x^{-1} - 3x^{-2}$$

$$\Longrightarrow y' = 1 + (-1)x^{-1-1} - 3(-2)x^{-2-1} = 1 - x^{-2} + 6x^{-3}$$

$$\Longrightarrow y'' = 0 - (-2)x^{-2-1} + 6(-3)x^{-3-1} = +2x^{-3} - 18x^{-4}$$

$$15)\ y = (2x+1)^3$$

$$\Longrightarrow y' = 3(2x+1)^2(2) = 6(2x+1)^2$$

$$\Longrightarrow y'' = 6(2)(2x+1)(2) = 24(2x+1)$$

$$16)\ y = \left(2x^5 + x^2\right)^2$$

$$\Longrightarrow y' = 2\left(2x^5 + x^2\right)(2(5)x^4 + 2x) = 2\left(2x^5 + x^2\right)(10x^4 + 2x)$$

$$\begin{aligned}\Longrightarrow y'' &= 2(2(5)x^4 + 2x)(10x^4 + 2x) + 2\left(2x^5 + x^2\right)(10(4)x^3 + 2) \\ &= 2(10x^4 + 2x)(10x^4 + 2x) + 2\left(2x^5 + x^2\right)(40x^3 + 2) \\ &= 2(10x^4 + 2x)^2 + 2\left(2x^5 + x^2\right)(40x^3 + 2)\end{aligned}$$

$$17)\ y = \frac{1}{x-1} \Longrightarrow \frac{dy}{dx} = \frac{0(x-1) - 1(1-0)}{(x-1)^2} = \frac{0-1}{(x-1)^2} = \frac{-1}{(x-1)^2}$$

Or,

$$y = (x-1)^{-1} \Longrightarrow y' = -1(x-1)^{-1-1}(1) = -(x-1)^{-2} = \frac{-1}{(x-1)^2}$$

$$\frac{dy}{dx}|_{x=2} = \frac{-1}{(-2-1)^2} = \frac{-1}{(-3)^2} = \frac{-1}{9}$$

$$18)\ y = \frac{2x}{\sqrt{x}-1} \Longrightarrow \frac{dy}{dx} = \frac{2\left(\sqrt{x}-1\right) - (2x)\left(\frac{1}{2\sqrt{x}} - 0\right)}{(\sqrt{x}-1)^2}$$

$$= \frac{2\left(\sqrt{x}-1\right) - (2x)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x}-1)^2} = \frac{2\left(\sqrt{x}-1\right) - \left(\frac{2x}{2\sqrt{x}}\right)}{(\sqrt{x}-1)^2}$$

$$= \frac{2\left(\sqrt{x}-1\right) - \left(\frac{x}{\sqrt{x}}\right)}{(\sqrt{x}-1)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \frac{2\left(\sqrt{2}-1\right) - \left(\frac{2}{\sqrt{2}}\right)}{(\sqrt{2}-1)^2}$$

$$19)\ y=\frac{x-3}{x+2}$$

$$\Rightarrow \frac{dy}{dx}=\frac{(1-0)(x+2)-(x-3)(1+0)}{(x+2)^2}=\frac{(1)(x+2)-(x-3)(1)}{(x+2)^2}$$
$$=\frac{(x+2)-(x-3)}{(x+2)^2}=\frac{x+2-x+3}{(x+2)^2}=\frac{5}{(x+2)^2}$$

$$\Rightarrow y''(2)=\frac{5}{(2+2)^2}=\frac{5}{(4)^2}=\frac{5}{16}$$

$$20)\ y=\frac{x^2-4x+3}{x+1}\Rightarrow$$

$$y'=\frac{(2x-4+0)(x+1)-(x^2-4x+3)(1)}{(x+1)^2}$$
$$=\frac{(2x-4)(x+1)-(x^2-4x+3)}{(x+1)^2}$$

$$\Rightarrow y'(2)=\frac{(2(2)-4)(2+1)-(2^2-4(2)+3)}{(2+1)^2}=\frac{(4-4)(3)-(4-8+3)}{(3)^2}$$
$$=\frac{(0)(3)-(-1)}{9}=\frac{0+1}{9}=\frac{1}{9}$$

$$21)\ y=\sqrt{x}+\frac{1}{x}=\sqrt{x}+x^{-1}$$

$$\Rightarrow y'=\frac{1}{2\sqrt{x}}-1x^{-1-1}=\frac{1}{2\sqrt{x}}-x^{-2}=\frac{1}{2\sqrt{x}}-\frac{1}{x^2}$$

$$\Rightarrow y'(2)=\frac{1}{2\sqrt{2}}-\frac{1}{2^2}=\frac{1}{2\sqrt{2}}-\frac{1}{4}$$

$$22)\ y=\frac{1}{4}x^4-4x^2-3$$

$$\Rightarrow\ y'=\frac{1}{4}(4x^3)-4(2x)-0=x^3-8x$$

$$\mathbf{23)}\ y = \frac{1}{360}x^6 - \frac{1}{120}x^5 + \frac{1}{12}x^4$$

$$\Rightarrow y' = \frac{1}{360}(6x^5) - \frac{1}{120}(5x^4) + \frac{1}{12}(4x^3) = \frac{1}{60}x^5 + \frac{1}{24}x^4 + \frac{1}{3}x^3$$

$$\mathbf{24)}\ y = (2x^3 - 1)(x^2 + x + 1)$$

$$\Rightarrow y' = (2(x^2) - 0)(x^2 + x + 1) + (2x^3 - 1)(2x + 1 + 0)$$
$$= 2(x^2)(x^2 + x + 1) + (2x^3 - 1)(2x + 1)$$

$$\mathbf{25)}\ f(-2) = 1\,, f'(-2) = 3\,, g(-2) = 7\,, g'(-2) = 3$$

$$\frac{d}{dx}(fg)\Big|_{x=-2} = f'(-2)g(-2) + f(-2)g'(-2) = (3)(7) + (1)(3) = 21 + 3$$
$$= 24$$

$$\frac{d}{dx}\left(\frac{f}{g}\right)\Big|_{x=-2} = \frac{f'(-2)g(-2) + f(-2)g'(-2)}{\big(g(-2)\big)^2} = \frac{(3)(7) - (1)(3) =}{(7)^2} = \frac{21 - 3}{49}$$
$$= \frac{18}{49}$$

$$\frac{d}{dx}(2f - 3g)\Big|_{x=-2} = 2f'(x) - 3g'(x) = 2(3) + 3(3) = 6 + 9 = 15$$
