MATH 110

EXERCISES 4.1

$$1)\ f(x) = 3x^2 \Rightarrow\ f'(x) = 3(2)x = 6x$$

$$\Longrightarrow f'(2) = 6(2) = 12$$

$$\Longrightarrow\ f'(0) = 6(0) = 0$$

$$\Longrightarrow f'(3) = 6(3) = 18$$

$$2)\ g(x) = \sqrt{x+4} \Rightarrow\ f'(x) = \frac{1}{2\sqrt{x+4}} \cdot (1) = \frac{1}{2\sqrt{x+4}}$$

$$\Longrightarrow g'(5) = \frac{1}{2\sqrt{5+4}} = \frac{1}{2\sqrt{9}} = \frac{1}{2(3)} = \frac{1}{9}$$

$$\Longrightarrow\ g'(0) = \frac{1}{2\sqrt{0+4}} = \frac{1}{2\sqrt{4}} = \frac{1}{2(2)} = \frac{1}{4}$$

$$\Longrightarrow g'(-3) = \frac{1}{2\sqrt{-3+4}} = \frac{1}{2\sqrt{1}} = \frac{1}{2}$$

$$3)\ h(x) = x^2 - 5 \Rightarrow\ f'(x) = 2x - 0 = 2x$$

$$\Longrightarrow h'(3) = 2(3) = 6$$

$$\Longrightarrow\ h'(0) = 2(0) = 0$$

$$\Longrightarrow h'(-1) = 2(-1) = -2$$

$$4)\ f(x) = \frac{1}{x^2} = x^{-2} \Rightarrow\ f'(x) = -2x^{-3} = \frac{-2}{x^3}$$

$$\Longrightarrow f'(-1) = \frac{-2}{(-1)^3} = \frac{-2}{-1} = 2$$

$$\Longrightarrow\ f'(1) = \frac{-2}{1^3} = \frac{-2}{1} = -2$$

$$\Longrightarrow f'(3) = \frac{-2}{3^3} = \frac{-2}{27}$$

$$5)\ g(x) = 2x^3 - 1 \Rightarrow f'(x) = 2(3)x^2 = 6x^2$$

$$\Rightarrow g'(0) = 6(0)^2 = 0$$

$$\Rightarrow g'(2) = 6(2)^2 = 6(4) = 24$$

$$\Rightarrow g'(-2) = 6(-2)^2 = 6(4) = 24$$

$$6)\ F(z) = \frac{1-2z}{z} \Rightarrow f'(x) = \frac{(0-2)(z)-(1-2z)(1)}{z^2} = \frac{-2z-1+2z}{z^2} = \frac{1}{z^2}$$

$$\Rightarrow F'(-1) = \frac{1}{(-1)^2} = \frac{1}{1} = 1$$

$$\Rightarrow F'(2) = \frac{1}{2^2} = \frac{1}{4}$$

$$\Rightarrow F'(\sqrt{2}) = \frac{1}{\sqrt{2}^2} = \frac{1}{2}$$

$$7)\ f(3) = -1, f'(3) = 5, y = f(x) at\ x = 3 \Rightarrow x_1 = 3, y_1 = -1, m = 5$$

The equation of tangent line is:

$$\frac{y-y_1}{x-x_1} = m \Rightarrow \frac{y-(-1)}{x-3} = 5$$

$$\Rightarrow y+1 = 5(x-3) \Rightarrow y+1 = 5x-5(3) \Rightarrow y = 5x-15-1$$

$$\Rightarrow y = 5x-16$$

$$8)\ f(-1) = 2, f'(-1) = 4, y = f(x) at\ x = -1 \Rightarrow x_1 = -1, y_1 = 2, m = 4$$

The equation of tangent line is:

$$\frac{y-y_1}{x-x_1} = m \Rightarrow \frac{y-(2)}{x-(-1)} = 4$$

$$\Rightarrow y-1 = 4(x+1) \Rightarrow y-1 = 4x+4(1) \Rightarrow y = 4x+4+1 \Rightarrow y = 4x+5$$

$$9)\ f(0) = 1, f'(0) = 2, y = f(x) at\ x = 0 \Rightarrow x_1 = 0, y_1 = 1, m = 2$$

The equation of tangent line is:

$$\frac{y-y_1}{x-x_1}=m \Rightarrow \frac{y-(1)}{x-0}=2$$

$$\Rightarrow y-1=2(x) \Rightarrow y=2x+1$$

$$10)\ y=2x-3 \ \Rightarrow \frac{dy}{dx}=2(1)-0=2$$

$$11)\ y=2x^3-3x+1 \ \Rightarrow \frac{dy}{dx}=2(3x^2)-3(1)+0=6x^2-3$$

$$12)\ y=\frac{1}{x}=x^{-1} \ \Rightarrow \frac{dy}{dx}=-1x^{-2}=-x^{-2}=\frac{-1}{x^2}$$

$$13)\ y=\frac{1}{\sqrt{x-4}}=\frac{1}{(x-4)^{1/2}}=(x-4)^{-1/2}$$

$$\Rightarrow \frac{dy}{dx}=-\frac{1}{2}(x-4)^{\frac{-1}{2}-1}(1)=-\frac{1}{2}(x-4)^{\frac{-1}{2}-\frac{2}{2}}=-\frac{1}{2}(x-4)^{\frac{-3}{2}}=\frac{-1}{2\sqrt{(x-4)^3}}$$

$$14)\ y=x+\frac{1}{x}=x+x^{-1} \Rightarrow \frac{dy}{dx}=1-1x^{-2}=1-\frac{1}{x^2}$$

$$15)\ f(t)=3t^2+1 \ \Rightarrow f'(t)=3(2t)+0=6t$$

$$16)\ A=\pi r^2 \Rightarrow \frac{dA}{dr}=2\pi r$$

$$17)\ z=\frac{2}{\sqrt{w-1}}=\frac{2}{(w-1)^{1/2}}=2(w-1)^{-1/2}$$

$$\Rightarrow \frac{dz}{dw} = 2\left(\frac{-1}{2}\right)(w-1)^{-\frac{1}{2}-1} = -(w-1)^{\frac{-3}{2}} = \frac{-1}{\sqrt{(w-1)^3}}$$

$$18)\ x = vt + \frac{1}{2}gt^2 \ \Rightarrow \frac{dx}{dt} = v + \frac{1}{2}(2)gt = v + gt$$

$$19)\ y = (x+1)^5 \ \Rightarrow \frac{dy}{dx} = 5(x+1)^4(1) = 5(x+1)^4$$

The slope of tangent line at x=-2 is

$$\left.\frac{dy}{dx}\right|_{x=-2} = 5(-2+1)^4 = 5(-1)^4 = 5(1) = 5$$

$$20)\ f(t) = t + \frac{t}{2} \Rightarrow \ f(t)' = 1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$$

The slope of tangent line at t=2 is

$$\left.\frac{df}{dt}\right|_{t=2} = \frac{3}{2}$$

$$21)\ y = (x+2)^{-3} \ \Rightarrow \frac{dy}{dx} = -3(x+2)^{-4}(1) = \frac{-3}{(x+2)^4}$$

The slope of tangent line at x=-1 is

$$\left.\frac{dy}{dx}\right|_{x=-1} = \frac{-3}{(-1+2)^4} = \frac{-3}{(1)^4} = \frac{-3}{1} = -3$$

$$22)\ f(x) = \sqrt{x^2-3x+6} \ \Rightarrow \frac{df}{dx} = \frac{1}{2\sqrt{x^2-3x+6}}(2x-3) = \frac{2x-3}{2\sqrt{x^2-3x+6}}$$

The slope of tangent line at x=2 is

$$\left.\frac{df}{dx}\right|_{x=2} = \frac{2(2)-3}{2\sqrt{(2)^2-3(2)+6}} = \frac{4-3}{2\sqrt{4-6+6}} = \frac{1}{2\sqrt{4}} = \frac{1}{2(2)} = \frac{1}{4}$$

$$23)\ f(x) = 1-2x^3 \ \Rightarrow \ \frac{dy}{dx} = 0 - 2(3)x^2 = -6x^2$$

$$f(1) = 1 - 2(1)^3 = 1 - 2 = -1$$

$$f'(1) = -6(1)^2 = -6$$

$$\Rightarrow x_1 = 1, y_1 = -1, m = -6$$

The equation of tangent line is:

$$\frac{y - y_1}{x - x_1} = m \Rightarrow \frac{y - (-1)}{x - (1)} = -6$$

$$\Rightarrow y + 1 = -6(x - 1) \Rightarrow y + 1 = -6x - 6(-1) \Rightarrow y = -6x + 6 - 1 \Rightarrow y = -6x + 5$$

$$24)\ x(t) = \frac{1}{2}t^2 + 3t \Rightarrow \frac{dx}{dt} = \frac{1}{2}(2t) + 3 = t + 3$$

$$x(2) = \frac{1}{2}2^2 + 3(2) = \frac{1}{2}(4) + 6 = 2 + 6 = 8 \Rightarrow \left.\frac{dx}{dt}\right|_{t=2} = 2 + 3 = 6$$

$$\Rightarrow x_1 = 2, y_1 = 8, m = 6$$

The equation of tangent line is:

$$\frac{y - y_1}{x - x_1} = m \Rightarrow \frac{y - (8)}{x - (2)} = 6$$

$$\Rightarrow y - 8 = 6(x - 2) \Rightarrow y - 8 = 6x - 6(2) \Rightarrow y = 6x - 12 + 8 \Rightarrow y = 6x - 4$$

$$25)\ g(x) = \sqrt{1 - x^3} \Rightarrow \frac{dg}{dx} = \frac{1}{2\sqrt{1 - x^3}}(0 - 3x^2) = \frac{-3x^2}{2\sqrt{1 - x^3}}$$

$$g(-1) = \sqrt{1 - (-1)^3} = \sqrt{1 + 1} = \sqrt{2} \Rightarrow \left.\frac{dg}{dx}\right|_{x=-1} = \frac{-3(-1)^2}{2\sqrt{1 - (-1)^3}} = \frac{-3(1)}{2\sqrt{1 + 1}} = \frac{-3}{2\sqrt{2}}$$

$$\Rightarrow x_1 = -1, y_1 = \sqrt{2}, m = \frac{-3}{2\sqrt{2}}$$

The equation of tangent line is:

$$\frac{y - y_1}{x - x_1} = m \Rightarrow \frac{y - \left(\sqrt{2}\right)}{x - (-1)} = \frac{-3}{2\sqrt{2}}$$

$$\Rightarrow 2\sqrt{2}y - 2\sqrt{2}\sqrt{2} = -3(x + 1) \Rightarrow 2\sqrt{2}y - 2(2) = -3x - 3(1)$$

$$\Rightarrow 2\sqrt{2}y = -3x - 3 + 4 \Rightarrow 2\sqrt{2}y = -3x + 1 \Rightarrow y = \frac{-3x + 1}{2\sqrt{2}}$$