

EXERCISES 2.4:

$$1 - \lim_{x \rightarrow 3^+} \frac{6x - 1}{x^2 - 9} = \frac{6(3) - 1}{3^2 - 9} = \frac{18 - 1}{9 - 9} = \frac{17}{0} = \infty$$

Choose 3.1 is the right of 3 to test the sign

$$\frac{6(3.1) - 1}{3.1^2 - 9} = \frac{+}{9.61 - 9} = \frac{+}{+}$$

$$2 - \lim_{x \rightarrow 0^+} \frac{3x - 1}{\sqrt{2x}} = \frac{3(0) - 1}{\sqrt{0}} = \frac{0 - 1}{0} = \frac{-1}{0} = -\infty$$

Choose 0.1 is the right of 0 to test the sign

$$\frac{3(0.1) - 1}{\sqrt{0.1}} = \frac{0.3 - 1}{+} = \frac{-}{+} = -$$

$$3 - \lim_{x \rightarrow -3^-} \frac{x + 1}{x^2 - 9} = \frac{-3 + 1}{(-3)^2 - 9} = \frac{-2}{9 - 9} = \frac{-2}{0} = -\infty$$

Choose -3.1 is the left of 3 to test the sign

$$\frac{-3.1 + 1}{(-3.1)^2 - 9} = \frac{-}{9.61 - 9} = \frac{-}{+} = -\infty$$

$$4 - \lim_{x \rightarrow 0^-} \frac{x + 1}{\sqrt{-x}} = \frac{(0) + 1}{\sqrt{0}} = \frac{1}{0} = +\infty$$

Choose -0.1 is the left of 0 to test the sign

$$\frac{(-0.1) + 1}{\sqrt{-(-0.1)}} = \frac{+}{+} = \frac{+}{+} = +$$

$$5 - \lim_{x \rightarrow 0^-} \frac{5x - 7}{2x - x^2} = \frac{5(0) - 7}{0 - 0} = \frac{-7}{0} = +\infty$$

Choose -0.1 is the left of 0 to test the sign

$$\frac{5(-0.1) - 7}{2(-0.1) - (-0.1)^2} = \frac{-0.5 - 7}{-0.2 - 0.01} = \frac{-}{-} = +$$

$$7 - \lim_{x \rightarrow (-\pi/2)} [1 - \sec x] = 1 - \sec(-\pi/2) = 1 - \frac{1}{\cos(-\pi/2)} = 1 - \frac{1}{\cos(3\pi/2)} = 1 - \frac{1}{0} = 1 - (-\infty) = 1 + \infty = \infty$$

عندما نكتب زوايه بالسالب هذا يعني اننا نمشي مع عقارب الساعه في دائرة الوحدہ

يسار الزاويه المذكوره يعني الزاويه التي قبلها بمعنى يسار الزاويه ٢٧٠ يعني في الربع الثالث ولذلك ستكون الناتج سالب

$$8 - \lim_{x \rightarrow -3^+} \frac{(x + 5)^{-2}}{x + 3} = \lim_{x \rightarrow -3^+} \frac{1}{(x + 3)(x + 5)^2} = \frac{1}{(-3 + 3)(-3 + 5)^2} = \frac{1}{(0)(2)^2} = \frac{1}{0} = \infty$$

Choose 3.1 is the right of 3 to test the sign

$$\frac{1}{(3.1 + 3)(3.1 + 5)^2} = \frac{+}{+} = +$$

$$9 - \lim_{x \rightarrow (3\pi/2)^-} \tan(x) = \tan \frac{3\pi}{2} = \frac{\sin(3\pi/2)}{\cos(3\pi/2)} = \frac{-1}{0} = +\infty$$

يسار الزاويه ٢٧٠ يعني اصغر من ٢٧٠ يعني في الربع الثالث فستكون الداله موجب

$$10 - \lim_{x \rightarrow -2^-} \frac{x^2 + 4}{2x^2 - 8} = \frac{(-2)^2 + 4}{2(-2)^2 - 8} = \frac{4 + 4}{8 - 8} = \frac{8}{0} = \infty$$

Choose -2.1 is the left of -2 to test the sign

$$\frac{(-2.1)^2 + 4}{2(-2.1)^2 - 8} = \frac{+}{8.82 - 8} = \frac{+}{+} = +$$

$$15 - \lim_{x \rightarrow 0^-} \frac{2 - x}{\sqrt{x}} = \frac{2 - 0}{\sqrt{0}} = \frac{2}{0} = \text{does not exist}$$

Choose -0.1 is the left of 0 to test the sign

$$\frac{2 - (-0.1)}{\sqrt{-0.1}} = \frac{+}{\text{undefined}} = \text{does not exist}$$

او من البدايه مجال الداله في المقام اكبر من او يساوي صفر ولانه يريد يسار الصفر او اقل من الصفر يعني خارج المجال وبالتالي لا يوجد رسم فالنهايه غير موجوده

$$18 - \lim_{x \rightarrow 1^-} \frac{3}{(x - 1)^3} = \frac{3}{(1 - 1)^3} = \frac{3}{0} = -\infty$$

Choose 0.9 is the left of 1 to test the sign

$$\frac{3}{(0.9 - 1)^3} = \frac{+}{-^3} = \frac{+}{-} = -$$

$$f(x) = \frac{1}{|x + 1|}$$

$$|x + 1| = \begin{cases} (x + 1) & \text{if } x + 1 > 0 \\ -(x + 1) & \text{if } x + 1 < 0 \end{cases}$$

$$|x + 1| = \begin{cases} (x + 1) & \text{if } x > -1 \\ -(x + 1) & \text{if } x < -1 \end{cases}$$

Then,

$$\lim_{x \rightarrow -1^+} \frac{1}{x + 1} = \frac{1}{-1 + 1} = \frac{1}{0} = \infty$$

Choose -0.9 is the right of -1 to test the sign

$$\frac{1}{-0.9 + 1} = \frac{+}{+} = +$$

And,

$$\lim_{x \rightarrow -1^-} \frac{1}{-(x+1)} = \frac{1}{-(-1+1)} = \frac{1}{0} = \infty$$

Choose -1.1 is the left of -1 to test the sign

$$\frac{1}{-(-1.1+1)} = \frac{+}{(-)(-)} = \frac{+}{+} = +$$

But,

$$\lim_{x \rightarrow 0^+} \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = 1$$

And,

$$\lim_{x \rightarrow 0^-} \frac{1}{x+1} = \frac{1}{0+1} = \frac{1}{1} = 1$$

$$20 - \lim_{x \rightarrow 0^+} \left(\tan x + \frac{1}{x} \right) = \tan(0) + \frac{1}{0} = 0 + \frac{1}{0} = 0 + \infty = \infty$$

Choose 0.1 is the right of 0 to test the sign

$$\frac{1}{0.1} = \frac{+}{+} = +$$

$$\lim_{x \rightarrow 0^-} \left(\tan x + \frac{1}{x} \right) = \tan(0) + \frac{1}{0} = 0 + \frac{1}{0} = 0 + (-\infty) = -\infty$$

Choose -0.1 is the right of 0 to test the sign

$$\frac{1}{-0.1} = \frac{+}{-} = -$$

$$\lim_{x \rightarrow \pi/2^+} \left(\tan x + \frac{1}{x} \right) = \tan\left(\frac{\pi}{2}\right) + \frac{1}{\pi/2} = -\infty + \frac{2}{\pi} = -\infty$$

The right of $\frac{\pi}{2}$ is in second quarter in the unit circle such that tan in the second is negative

على يمين الزاويه اي اكبر منها وذلك يقع في الربع الثاني لذلك الداله ستكون سالبه

$$\lim_{x \rightarrow \pi/2} \left(\tan x + \frac{1}{x} \right) = \tan\left(\frac{\pi}{2}\right) + \frac{1}{\pi/2} = +\infty + \frac{2}{\pi} = +\infty$$

The right of $\frac{\pi}{2}$ is in first quarter in the unit circle such that tan in the first is positive

على يسار الزاويه اي اصغر منها وذلك يقع في الربع الأول لذلك الداله ستكون موجبه

$$\lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[3]{x-1}} \right) = \frac{1}{\sqrt[3]{0}} + \frac{1}{\sqrt[3]{0-1}} = \frac{1}{0} + \frac{1}{\sqrt[3]{-1}} = \infty + \frac{1}{-1} = \infty - 1 = \infty$$

Choose 0.1 is the right of 0 to test the sign

$$\frac{1}{\sqrt[3]{0.1}} = \frac{+}{+} = +$$

$$\lim_{x \rightarrow 0^-} \left(\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[3]{x-1}} \right) = \frac{1}{\sqrt[3]{0}} + \frac{1}{\sqrt[3]{0-1}} = \frac{1}{0} + \frac{1}{\sqrt[3]{-1}} = -\infty + \frac{1}{-1} = -\infty - 1 = -\infty$$

Choose -0.1 is the right of 0 to test the sign

$$\frac{1}{\sqrt[3]{-0.1}} = \frac{+}{-} = -$$

$$\lim_{x \rightarrow 1^+} \left(\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[3]{x-1}} \right) = \frac{1}{\sqrt[3]{1}} + \frac{1}{\sqrt[3]{1-1}} = \frac{1}{1} + \frac{1}{\sqrt[3]{0}} = 1 + \frac{1}{0} = 1 + \infty = \infty$$

Choose 1.1 is the right of 1 to test the sign

$$\frac{1}{\sqrt[3]{1.1-1}} = \frac{1}{\sqrt[3]{+}} = \frac{+}{+} = +$$

$$\lim_{x \rightarrow 0^-} \left(\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[3]{x-1}} \right) = \frac{1}{\sqrt[3]{1}} + \frac{1}{\sqrt[3]{1-1}} = \frac{1}{1} + \frac{1}{\sqrt[3]{0}} = 1 + \frac{1}{0} = 1 + \infty = \infty$$

Choose 0.9 is the right of 1 to test the sign

$$\frac{1}{\sqrt[3]{0.1-1}} = \frac{1}{\sqrt[3]{-}} = \frac{+}{-} = -$$

Find the vertical asymptotes of:

$$25 - y = \frac{x^2 + 2x + 1}{x - 7} = \frac{(x+1)(x+1)}{(x-7)}$$

Then the vertical asymptotes is a solution of $x - 7 = 0 \Rightarrow x = 7$

$$27 - y = \frac{x + 1}{x^2 - 3x} = \frac{(x + 1)}{x(x - 3)}$$

Then the vertical asymptotes is a solution of $x(x - 3) = 0 \Rightarrow x = 0$ or $x = 3$

$$28 - y = \frac{x^2 - 1}{x} = \frac{(x + 1)(x - 1)}{x}$$

Then the vertical asymptotes is a solution of $x = 0 \Rightarrow x = 0$

$$31 - y = \frac{(x - 1)^{-2}}{(x + 3)} = \frac{1}{(x + 3)(x - 1)^2}$$

Then the vertical asymptotes is a solution of

$$(x + 3)(x - 1)^2 = 0 \Rightarrow x + 3 = 0 \text{ or } (x - 1)^2 = 0 \Rightarrow x = 3 \text{ or } x - 1 = 0$$

$$\Rightarrow x = 3 \text{ or } x = 1$$
