CHAPTER 3

EXERCISES 3.2

$$1-\lim_{x\to 1}\frac{x-1}{x+x^2-3}=\frac{1-1}{1+1^2-3}=\frac{0}{1+1-3}=\frac{0}{2-3}=\frac{0}{-1}=0$$

$$2-\lim_{x\to 4}\frac{3^{-1}-(2x-5)^{-1}}{4-x}=\frac{3^{-1}-(2\times 4-5)^{-1}}{4-4}=\frac{3^{-1}-(8-5)^{-1}}{0}$$
$$=\frac{3^{-1}-(3)^{-1}}{0}=\frac{0}{0}$$

نحتاج إلى تغير شكل الداله

$\lim_{x\to 4}\frac{3^{-1}(3)(2x-5)-(2x-5)^{-1}(3)(2x-5)}{(4-x)(3)(2x-5)}=\lim_{x\to 4}\frac{(2x-5)-(3)}{(4-x)(3)(2x-5)}=$
$\lim_{x\to 4}\frac{2x-5-3}{(4-x)(3)(2x-5)}=\lim_{x\to 4}\frac{2x-8}{(4-x)(3)(2x-5)}=\lim_{x\to 4}\frac{-2(-x+4)}{(4-x)(3)(2x-5)}=$
$\lim_{x\to 4}\frac{-2(4-x)}{(4-x)(3)(2x-5)}=\lim_{x\to 4}\frac{-2}{(3)(2x-5)}=\frac{-2}{3\times(2\times 4-5)}=\frac{-2}{3(8-5)}=\frac{-2}{3\times 3}=$
$\frac{-2}{9}$

Or by L'hospital Role:

$$\lim_{x\to 4}\frac{3^{-1}-(2x-5)^{-1}}{4-x}=\lim_{x\to 4}\frac{0-(-1)(2x-5)^{-1-1}(2)}{-1}$$
$$=\lim_{x\to 4}\frac{+2(2x-5)^{-2}}{-1}=\frac{+2(2\times 4-5)^{-2}}{-1}=\frac{+2(8-5)^{-2}}{-1}$$
$$=\frac{2(3)^{-2}}{-1}=\frac{2}{-1(3)^2}=-\frac{2}{9}$$

$$3-\lim_{x\to 2}\frac{x-2}{2x-x^2}=\frac{2-2}{2(2)-2^2}=\frac{0}{4-4}=\frac{0}{0}$$

$$\lim_{x\to 2}\frac{(x-2)}{-x(x-2)}=\lim_{x\to 2}\frac{1}{-x}=\frac{1}{-2}=-\frac{1}{2}$$

Or by L'hospital Role:

$\lim_{x\to 2}\frac{x-2}{2x-x^2}=\lim_{x\to 2}\frac{1-0}{2-2x}=\frac{1}{2-2(2)}=\frac{1}{2-4}=\frac{1}{-2}=-\frac{1}{2}$

$$4-\lim_{x\to -2}\frac{1-(x+4)^{-2}}{x-2}=\frac{1-(-1+4)^{-2}}{-1-2}=\frac{1-(3)^{-2}}{-3}=\frac{1-\frac{1}{3^2}}{-3}$$

$$=\frac{1-\frac{1}{9}}{-3}=\frac{\frac{9}{9}-\frac{1}{9}}{-3}=\frac{\frac{8}{9}}{-3}=-\frac{8}{9(3)}=-\frac{8}{27}$$

$$5-\lim_{x\to 1}\left[\frac{x^2-2}{x+4}+x^2-2x\right]=\left[\frac{1^2-2}{1+4}+1^2-2\times 1\right]=$$

$$\left[\frac{1-2}{5}+1-2\right]=\frac{-1}{5}-1=\frac{-1}{5}-\frac{5}{5}=\frac{-6}{5}$$

$$6-\lim_{x\to 2}\frac{2^{-1}-(3x-4)^{-1}}{2-x}=\frac{2^{-1}-(3\times 2-4)^{-1}}{2-2}=\frac{2^{-1}-(6-4)^{-1}}{0}$$

$$=\frac{2^{-1}-(2)^{-1}}{0}=\frac{0}{0}$$

$$\lim_{x\to 2}\frac{2^{-1}(2)(3x-4)-(3x-4)^{-1}(2)(3x-4)}{(2-x)(2)(3x-4)}=\lim_{x\to 2}\frac{(3x-4)-(2)}{(2-x)(2)(3x-4)}=$$

$$\lim_{x\to 2}\frac{3x-4-2}{(2-x)(2)(3x-4)}=\lim_{x\to 2}\frac{3x-6}{(2-x)(2)(3x-4)}=\lim_{x\to 2}\frac{-3(-x+2)}{(2-x)(2)(3x-4)}=$$

$$\lim_{x\to 2}\frac{-3(2-x)}{(2-x)(2)(3x-4)}=\lim_{x\to 2}\frac{-3}{(2)(3x-4)}=\frac{-3}{2\times(3\times 2-4)}=\frac{-3}{2(6-4)}=\frac{-3}{2\times 2}=$$

$$\frac{-3}{4}=-\frac{3}{4}$$

Or by L'hospital Role:

$$\lim_{x\to 2}\frac{2^{-1}-(3x-4)^{-1}}{2-x}=\lim_{x\to 2}\frac{0-(-1)(3x-4)^{-1-1}(3)}{-1}$$

$$=\lim_{x\to 2}\frac{+3(3x-4)^{-2}}{-1}=\frac{3(3\times 2-4)^{-2}}{-1}=\frac{3(6-4)^{-2}}{-1}$$

$$=\frac{3(2)^{-2}}{-1}=\frac{3}{-1(2)^2}=-\frac{3}{4}$$

$$7-\lim_{x\to 1}(x^2+3x-5)^4=(1^2+3(1)-5)^4=(1+3-5)^4$$

$$=(4-5)^4=(-1)^4=1$$

$$9-\lim_{x\to 3}\frac{x^2-8x+15}{x^2+4x-21}=\frac{3^2-8(3)+15}{3^2+4(3)-21}=\frac{9-24+15}{9+12-21}=\frac{24-24}{21-21}=\frac{0}{0}$$

$$x^2-8x+15=(x-3)(x-5)$$

$$x^2+4x-21=(x-3)(x+7)$$

$$\lim_{x\to 3}\frac{(x-3)(x-5)}{(x-3)(x+7)}=\lim_{x\to 3}\frac{(x-5)}{(x+7)}=\frac{3-5}{3+7}=\frac{-2}{10}=-\frac{1}{5}$$

Or by L'hospital rule

$$\lim_{x\to 3}\frac{x^2-8x+15}{x^2+4x-21}=\lim_{x\to 3}\frac{2x-8}{2x+4}=\frac{2\times 3-8}{2\times 3+4}=\frac{6-8}{6+4}=\frac{-2}{10}=-\frac{1}{5}$$

$$11-\lim_{x\to 4}\frac{x^2-6x+8}{x^2+x-20}=\frac{4^2-6(4)+8}{4^2+4-20}=\frac{16-24+8}{16+4-20}=\frac{24-24}{20-20}=\frac{0}{0}$$

نحلل البسط والمقام

$$x^2-6x+8=(x-2)(x-4)$$

$$x^2+x-20=(x-4)(x+5)$$

$$\lim_{x\to 4}\frac{(x-2)(x-4)}{(x-4)(x+5)}=\lim_{x\to 4}\frac{(x-2)}{(x+5)}=\frac{4-2}{4+5}=\frac{2}{9}$$

Or by L'hospital rule

$$\lim_{x\to 4}\frac{x^2-6x+8}{x^2+x-20}=\lim_{x\to 4}\frac{2x-6}{2x+1}=\frac{2\times 4-6}{2\times 4+1}=\frac{8-6}{8+1}=\frac{2}{9}$$

$$12-\lim_{x\to -2}\frac{4x^2+6x-4}{2x^2-8}=\frac{4(-2)^2+6(-2)-4}{2(-2)^2-8}=\frac{4(4)-12-4}{2(4)-8}=\frac{16-12-4}{8-8}=\frac{16-16}{0}=\frac{0}{0}$$

$$4x^2+6x-4=2(2x^2+3x-2)=2(2x-1)(x+2)$$

$$\left\{\begin{matrix} 2x & +2 \\ x & -1 \end{matrix}\right\}_{-2x}^{+2x} \Rightarrow 0x \quad \text{or} \quad \left\{\begin{matrix} 2x & -1 \\ x & +2 \end{matrix}\right\}_{+4x}^{-x} \Rightarrow 3x$$

$$2x^2-8=2(x^2-4)=2(x^2-2^2)=2(x-2)(x+2)$$

$$\lim_{x\to-2}\frac{4x^2+6x-4}{2x^2-8}=\lim_{x\to-2}\frac{2(2x-1)(x+2)}{2(x-2)(x+2)}=\lim_{x\to-2}\frac{(2x-1)}{(x-2)}$$
$$=\frac{2(-2)-1}{-2-2}=\frac{-4-1}{-4}=\frac{-5}{-4}=\frac{5}{4}$$

Or by L'hospital rule

$$\lim_{x\to-2}\frac{4x^2+6x-4}{2x^2-8}\lim_{x\to-2}\frac{8x+6}{4x}=\frac{8(-2)+6}{4(-2)}=\frac{-16+6}{-8}=\frac{-10}{-8}$$
$$=\frac{10}{8}=\frac{5}{4}$$

$$13-\lim_{x\to-2}\frac{x^3+8}{x^2-x-6}=\frac{(-2)^3+8}{(-2)^2-(-2)-6}=\frac{8-8}{4+2-6}=\frac{0}{0}$$

$$x^3+8=x^3+2^3=(x+2)(x^2-2x+2^2)=(x+2)(x^2-2x+4)$$

$$x^2-x-6=(x-3)(x+2)$$

$$\lim_{x\to-2}\frac{x^3+8}{x^2-x-6}=\lim_{x\to-2}\frac{(x+2)(x^2-2x+4)}{(x-3)(x+2)}=\lim_{x\to-2}\frac{(x^2-2x+4)}{(x-3)}$$
$$=\frac{(-2)^2-2(-2)+4}{-2-3}=\frac{4+4+4}{-5}=-\frac{12}{5}$$

Or by L'hospital rule

$$\lim_{x\to-2}\frac{x^3+8}{x^2-x-6}=\lim_{x\to-2}\frac{3x^2+0}{2x-1}=\frac{3(-2)^2}{2(-2)-1}=\frac{3(4)}{-5}=-\frac{12}{5}$$

$$14-\lim_{x\to-1}\frac{x^2-2x-3}{x^5-x^3}=\frac{(-1)^2-2(-1)-3}{(-1)^5-(-1)^3}=\frac{1+2-1}{-1-(-1)}=\frac{0}{0}$$

$$x^2-2x-3=(x-3)(x+1)$$

$$x^5-x^3=x^3(x^2-1)=x^3(x-1)(x+1)$$

$$\lim_{x\to-1}\frac{x^2-2x-3}{x^5-x^3} = \lim_{x\to-1}\frac{(x-3)(x+1)}{x^3(x-1)(x+1)}$$

$$= \lim_{x\to-1}\frac{(x-3)}{x^3(x-1)} = \frac{-1-3}{(-1)^3(-1-1)} = \frac{-4}{(-1)(-2)} = \frac{-4}{+2}$$

$$= -2$$

Or by L'hospital role:

$$\lim_{x\to-1}\frac{x^2-2x-3}{x^5-x^3} = \lim_{x\to-1}\frac{2x-2}{5x^4-3x^2} = \frac{2(-1)-2}{5(-1)^4-3(-1)^2} = \frac{-2-2}{5-3}$$

$$= \frac{-4}{+2} = -2$$

$$15-\lim_{x\to-3}\frac{\mathrm{x}^2+2\mathrm{x}-3}{2\mathrm{x}^2+5\mathrm{x}-3} = \frac{(-3)^2+2(-3)-3}{2(-3)^2+5(-3)-3}$$

$$= \frac{9-6-3}{2(9)-15-3} = \frac{9-9}{18-18} = \frac{0}{0}$$

$$\mathrm{x}^2+2\mathrm{x}-3 = (x+3)(x-1)$$

$$2\mathrm{x}^2+5\mathrm{x}-3 = (2x-1)(x+3)$$

$$\begin{Bmatrix} 2x & +3 \\ x & -1 \end{Bmatrix}^{3x}_{-2x} \Rightarrow x \qquad \text{or} \begin{Bmatrix} 2x & -1 \\ x & +3 \end{Bmatrix}^{-x}_{6x} \Rightarrow 5$$

$$\lim_{x\to-3}\frac{\mathrm{x}^2+2\mathrm{x}-3}{2\mathrm{x}^2+5\mathrm{x}-3} = \lim_{x\to-3}\frac{(x+3)(x-1)}{(2x-1)(x+3)}$$

$$= \lim_{x\to-3}\frac{(x-1)}{(2x-1)} = \frac{-3-1}{2(-3)-1} = \frac{-4}{-6-1} = \frac{-4}{-7} = \frac{4}{7}$$

Or by L'hospital role:

$$\lim_{x\to -3}\frac{x^2+2x-3}{2x^2+5x-3}=\lim_{x\to -3}\frac{2x+2}{2(2x)+5}=\frac{2(-3)+2}{4(-3)+5}$$

$$=\frac{-6+2}{-12+5}=\frac{-4}{-7}=\frac{4}{7}$$

$$16-\lim_{x\to 3}\left(\frac{\sqrt{2x+1}}{2x+3}\right)\left(\frac{x^2-9}{x-3}\right)=\lim_{x\to 3}\left(\frac{\sqrt{2x+1}}{2x+3}\right)\cdot\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$$

$$\lim_{x\to 3}\left(\frac{\sqrt{2x+1}}{2x+3}\right)=\left(\frac{\sqrt{2(3)+1}}{2(3)+3}\right)=\left(\frac{\sqrt{6+1}}{6+3}\right)=\left(\frac{\sqrt{7}}{9}\right)$$

$$\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)=\left(\frac{3^2-9}{3-3}\right)=\frac{9-9}{0}=\frac{0}{0}$$

$$\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)=\lim_{x\to 3}\left(\frac{x^2-3^2}{x-3}\right)=\lim_{x\to 3}\frac{(x-3)(x+3)}{(x-3)}$$

$$=\lim_{x\to 3}x+3=3+3=6$$

Or $\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)=\lim_{x\to 3}\frac{2x}{1}=2(3)=6$

Then,

$$\lim_{x\to 3}\left(\frac{\sqrt{2x+1}}{2x+3}\right)\cdot\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)=\left(\frac{\sqrt{7}}{9}\right)(6)=\frac{6\sqrt{7}}{9}=\frac{2\sqrt{7}}{3}$$

$$17-\lim_{x\to -3}\frac{x^3+27}{x^2-3x-18}=\frac{(-3)^3+27}{(-3)^2-3(-3)-18}=\frac{-27+27}{9+9-18}=\frac{0}{0}$$

$x^3+27=x^3+3^3=(x+3)(x^2-3x+3^2)=(x+3)(x^2-3x+9)$

$x^2-3x-18=(x-6)(x+3)$

$$\lim_{x\to -3}\frac{x^3+27}{x^2-3x-18}=\lim_{x\to -3}\frac{(x+3)(x^2-3x+9)}{(x-6)(x+3)}$$

$$= \lim_{x\to -3} \frac{(x^2 - 3x + 9)}{(x - 6)}$$

$$= \frac{(-3)^2 - 3(-3) + 9}{-3 - 6} = \frac{9 + 9 + 9}{-9} = \frac{27}{-9} = -3$$

Or by L'hospital rule:

$$\lim_{x\to -3} \frac{x^3 + 27}{x^2 - 3x - 18} = \lim_{x\to -3} \frac{3x^2}{2x - 3} = \frac{3(-3)^2}{2(-3) - 3} = \frac{(3)(9)}{-6 - 3} = \frac{(3)(9)}{-9}$$

$$= -3$$

$$18 - \lim_{x\to 3} \frac{-4x^2 + 6x - 4}{x^2 - 8} = \frac{-4(3)^2 + 6(3) - 4}{3^2 - 8} = \frac{-4(9) + 18 - 4}{9 - 8}$$

$$= \frac{-36 + 18 - 4}{1} = -40 + 18 = -22$$

$$19 - \lim_{x\to 4} \frac{x^2 + 2x - 3}{x^2 + 5x - 3} = \frac{4^2 + 2(4) - 3}{(4)^2 + 5(4) - 3} = \frac{16 + 8 - 3}{16 + 20 - 3} = \frac{24 - 3}{36 - 3}$$

$$= \frac{21}{33}$$

$$20 - \lim_{x \to 1/2} \frac{(x+3)(x-4)(2x-1)}{(5x-3)(1-2x)(3x-1)}$$

$$= \frac{\left(1/2+3\right)\left(1/2-4\right)(2\,1/2-1)}{\left(5\,1/2-3\right)\left(1-2\,1/2\right)(3\,1/2-1)}$$

$$= \frac{\left(1/2+3\right)\left(1/2-4\right)(1-1)}{\left(5\,1/2-3\right)(1-1)(3\,1/2-1)}$$

$$= \frac{\left(1/2+3\right)\left(1/2-4\right)(0)}{\left(5\,1/2-3\right)(0)(3\,1/2-1)} = \frac{0}{0}$$

$$\lim_{x \to 1/2} \frac{(x+3)(x-4)(2x-1)}{(5x-3)(1-2x)(3x-1)}$$

$$= \lim_{x \to 1/2} \frac{(x+3)(x-4)(-1)(1-2x)}{(5x-3)(1-2x)(3x-1)}$$

$$= \lim_{x \to 1/2} \frac{(x+3)(x-4)(-1)}{(5x-3)(3x-1)}$$

$$= \frac{\left(1/2+3\right)\left(1/2-4\right)(-1)}{\left(5\,1/2-3\right)(3\,1/2-1)} = \frac{\left(\frac{1+6}{2}\right)\left(\frac{1-8}{2}\right)(-1)}{\left(\frac{5}{2}-3\right)\left(\frac{3}{2}-1\right)}$$

$$= \frac{\left(\frac{7}{2}\right)\left(\frac{-7}{2}\right)(-1)}{\left(\frac{5-6}{2}\right)\left(\frac{3-2}{2}\right)} = \frac{\frac{49}{4}}{\left(\frac{-1}{2}\right)\left(\frac{1}{2}\right)} = \frac{\frac{49}{2}}{\left(\frac{-1}{4}\right)} = \frac{49}{-1} = -49$$

$$21-\lim_{x\to 0}\frac{2-\sqrt{x+4}}{x}=\frac{2-\sqrt{0+4}}{0}=\frac{2-\sqrt{4}}{0}=\frac{2-2}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{2-\sqrt{x+4}}{x}\times\frac{2+\sqrt{x+4}}{2+\sqrt{x+4}}=\lim_{x\to 0}\frac{2^2-(\sqrt{x+4})^2}{x(2+\sqrt{x+4})}$$

$$=\lim_{x\to 0}\frac{4-(x+4)}{x(2+\sqrt{x+4})}$$

$$=\lim_{x\to 0}\frac{4-x-4}{x(2+\sqrt{x+4})}=\lim_{x\to 0}\frac{x}{x(2+\sqrt{x+4})}$$

$$=\lim_{x\to 0}\frac{1}{(2+\sqrt{x+4})}=\frac{1}{2+\sqrt{0+4}}=\frac{1}{2+\sqrt{4}}=\frac{1}{2+2}=\frac{1}{4}$$

$$22-\lim_{x\to 0}\frac{(x+1)^2-1}{x}=\frac{(0+1)^2-1}{0}=\frac{(1)^2-1}{0}=\frac{1-1}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{(x+1)^2-1}{x}=\lim_{x\to 0}\frac{(x^2+2x(1)+1^2)-1}{x}$$

$$=\lim_{x\to 0}\frac{x^2+2x+1-1}{x}=\lim_{x\to 0}\frac{x^2+2x}{x}=\lim_{x\to 0}\frac{x(x+2)}{x}$$

$$=\lim_{x\to 0}x+2=0+2=+2$$

$$23-\lim_{x\to 1}\frac{\sqrt{3-2x}-1}{x-1}=\frac{\sqrt{3-2(1)}-1}{1-1}=\frac{\sqrt{1}-1}{0}=\frac{1-1}{0}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{\sqrt{3-2x}-1}{x-1}\times\frac{\sqrt{3-2x}+1}{\sqrt{3-2x}+1}=\lim_{x\to 1}\frac{\left(\sqrt{3-2x}\right)^2-1^2}{(x-1)\left(\sqrt{3-2x}+1\right)}$$

$$=\lim_{x\to 1}\frac{3-2x-1}{(x-1)\left(\sqrt{3-2x}+1\right)}==\lim_{x\to 1}\frac{-2x+2}{(x-1)\left(\sqrt{3-2x}+1\right)}$$

$$=\lim_{x\to 1}\frac{-2(x-1)}{(x-1)\left(\sqrt{3-2x}+1\right)}==\lim_{x\to 1}\frac{-2}{\left(\sqrt{3-2x}+1\right)}$$

$$=\frac{-2}{\left(\sqrt{3-2(1)}+1\right)}=\frac{-2}{\left(\sqrt{1}+1\right)}=\frac{-2}{(1+1)}=\frac{-2}{2}=-1$$

$$24-\lim_{x\to 0}\frac{(x+2)^3+2(x+2)^2-16}{x}=\frac{(0+2)^3+2(0+2)^2-16}{0}$$

$$=\frac{(2)^3+2(2)^2-16}{0}=\frac{8+8-16}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{(x+2)^3+2(x+2)^2-16}{x}$$

$$=\lim_{x\to 0}\frac{x^3+3x^2(2)+3x(2)^2+2^3+2[x^2+2x(2)+2^2]-16}{x}$$

$$=\lim_{x\to 0}\frac{x^3+6x^2+3x(4)+8+2x^2+8x+8-16}{x}$$

$$=\lim_{x\to 0}\frac{x^3+6x^2+12x+8+2x^2+8x+8-16}{x}$$

$$=\lim_{x\to 0}\frac{x^3+6x^2+12x+2x^2+8x}{x}=$$

$$=\lim_{x\to 0}\frac{x^3+8x^2+20x}{x}=\lim_{x\to 0}\frac{x(x^2+8x+20)}{x}$$

$$=\lim_{x\to 0}x^2+8x+20=0^2+8\times 0+20=20$$

$$25-\lim_{x\to 1}\frac{\sqrt{2x+2}-2}{\sqrt{3x-2}-1}=\frac{\sqrt{2(1)+2}-2}{\sqrt{3(1)-2}-1}=\frac{\sqrt{2+2}-2}{\sqrt{1}-1}$$

$$=\frac{\sqrt{4}-2}{1-1}\frac{2-2}{0}=\frac{0}{0}$$

$$\lim_{x\to 1}\frac{\sqrt{2x+2}-2}{\sqrt{3x-2}-1}\times\frac{\sqrt{2x+2}+2}{\sqrt{2x+2}+2}\times\frac{\sqrt{3x-2}+1}{\sqrt{3x-2}+1}$$

$$=\lim_{x\to 1}\frac{\left[\left(\sqrt{2x+2}\right)^2-(2)^2\right]\left(\sqrt{3x-2}+1\right)}{\left[\left(\sqrt{3x-2}\right)^2-(1)^2\right]\left(\sqrt{2x+2}+2\right)}$$

$$=\lim_{x\to 1}\frac{[2x+2-4]\left(\sqrt{3x-2}+1\right)}{[3x-2-1]\left(\sqrt{2x+2}+2\right)}=\lim_{x\to 1}\frac{[2x-2]\left(\sqrt{3x-2}+1\right)}{[3x-3]\left(\sqrt{2x+2}+2\right)}$$

$$\lim_{x\to 1}\frac{2[x-1]\left(\sqrt{3x-2}+1\right)}{3[x-1]\left(\sqrt{2x+2}+2\right)}=\lim_{x\to 1}\frac{2\left(\sqrt{3x-2}+1\right)}{3\left(\sqrt{2x+2}+2\right)}$$

$$=\frac{2\left(\sqrt{3(1)-2}+1\right)}{3\left(\sqrt{2(1)+2}+2\right)}=\frac{2\left(\sqrt{1}+1\right)}{3\left(\sqrt{4}+2\right)}=\frac{2(1+1)}{3(2+2)}=\frac{(2)(2)}{(3)(4)}$$

$$=\frac{1}{3}$$

$$26-\lim_{x\to 0}\frac{(x+1)^3-1}{x}=\frac{(0+1)^3-1}{0}=\frac{1-1}{0}=\frac{0}{0}$$

$$\lim_{x\to 0}\frac{(x+1)^3-1}{x}=\lim_{x\to 0}\frac{x^3+3x^2(1)+3x(1)^2+1^3-1}{x}$$

$$=\lim_{x\to 0}\frac{x^3+3x^2+3x+1-1}{x}=\lim_{x\to 0}\frac{x^3+3x^2+3x}{x}$$

$$=\lim_{x\to 0}\frac{x[x^2+3x++3]}{x}=\lim_{x\to 0}[x^2+3x++3]$$

$$=0+0+3=3$$

$$27-\lim_{x\to 2}\frac{3-\sqrt{2x+5}}{x-2}=\frac{3-\sqrt{2(2)+5}}{2-2}=\frac{3-\sqrt{4+5}}{0}=\frac{3-\sqrt{9}}{0}$$

$$=\frac{3-3}{0}=\frac{0}{0}$$

$$=\lim_{x\to 2}\frac{9-(2x+5)}{(x-2)\left(3+\sqrt{2x+5}\right)}=\lim_{x\to 2}\frac{9-2x-5}{(x-2)\left(3+\sqrt{2x+5}\right)}$$

$$=\lim_{x\to 2}\frac{-2x+4}{(x-2)\left(3+\sqrt{2x+5}\right)}=\lim_{x\to 2}\frac{-2(x-2)}{(x-2)\left(3+\sqrt{2x+5}\right)}$$

$$=\lim_{x\to 2}\frac{-2}{\left(3+\sqrt{2x+5}\right)}=\frac{-2}{\left(3+\sqrt{2(2)+5}\right)}=\frac{-2}{\left(3+\sqrt{4+5}\right)}$$

$$=\frac{-2}{\left(3+\sqrt{9}\right)}=\frac{-2}{3+3}=\frac{-2}{6}=-\frac{1}{3}$$

$$29-\lim_{x\to 1}\frac{-3+\sqrt{10-x}}{x-1}=\frac{-3+\sqrt{10-1}}{1-1}=\frac{-3+\sqrt{9}}{0}=\frac{-3+3}{0}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{-3+\sqrt{10-x}}{x-1}\times\frac{-3-\sqrt{10-x}}{-3-\sqrt{10-x}}=\lim_{x\to1}\frac{(-3)^2-\left(\sqrt{10-x}\right)^2}{(x-1)\left(-3-\sqrt{10-x}\right)}$$

$$=\lim_{x\to1}\frac{9-(10-x)}{(x-1)\left(-3-\sqrt{10-x}\right)}=\lim_{x\to1}\frac{9-10+x}{(x-1)\left(-3-\sqrt{10-x}\right)}$$

$$\lim_{x\to1}\frac{(x-1)}{(x-1)\left(-3-\sqrt{10-x}\right)}=\lim_{x\to1}\frac{1}{\left(-3-\sqrt{10-x}\right)}$$

$$=\frac{1}{\left(-3-\sqrt{10-1}\right)}=\frac{1}{\left(-3-\sqrt{9}\right)}=\frac{1}{(-3-3)}=\frac{1}{-6}=-\frac{1}{6}$$

$$31—\lim_{x\to1}\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{3x+2}-\sqrt{5}}=\frac{\sqrt{1+2}-\sqrt{3}}{\sqrt{3(1)+2}-\sqrt{5}}=\frac{\sqrt{3}-\sqrt{3}}{\sqrt{5}-\sqrt{5}}=\frac{0}{0}$$

$$\lim_{x\to1}\frac{\sqrt{x+2}-\sqrt{3}}{\sqrt{3x+2}-\sqrt{5}}\times\frac{\sqrt{x+2}+\sqrt{3}}{\sqrt{x+2}+\sqrt{3}}\times\frac{\sqrt{3x+2}+\sqrt{5}}{\sqrt{3x+2}+\sqrt{5}}$$

$$=\lim_{x\to1}\frac{\left[\left(\sqrt{x+2}\right)^2-\left(\sqrt{3}\right)^2\right]\left(\sqrt{3x+2}+\sqrt{5}\right)}{\left[\left(\sqrt{3x+2}\right)^2-\left(\sqrt{5}\right)^2\right]\left(\sqrt{x+2}+\sqrt{3}\right)}$$

$$=\lim_{x\to1}\frac{[x+2-3]\left(\sqrt{3x+2}+\sqrt{5}\right)}{[3x+2-5]\left(\sqrt{x+2}+\sqrt{3}\right)}=\lim_{x\to1}\frac{[x-1]\left(\sqrt{3x+2}+\sqrt{5}\right)}{[3x-3]\left(\sqrt{x+2}+\sqrt{3}\right)}$$

$$\lim_{x\to1}\frac{[x-1]\left(\sqrt{3x+2}+\sqrt{5}\right)}{3[x-1]\left(\sqrt{x+2}+\sqrt{3}\right)}=\lim_{x\to1}\frac{\left(\sqrt{3x+2}+\sqrt{5}\right)}{3\left(\sqrt{x+2}+\sqrt{3}\right)}$$

$$=\frac{\left(\sqrt{3(1)+2}+\sqrt{5}\right)}{3\left(\sqrt{(1)+2}+\sqrt{3}\right)}=\frac{\left(\sqrt{5}+\sqrt{5}\right)}{3\left(\sqrt{3}+\sqrt{3}\right)}=\frac{\left(2\sqrt{5}\right)}{3\left(2\sqrt{3}\right)}$$

$$=\frac{\left(\sqrt{5}\right)}{3\left(\sqrt{3}\right)}$$

$$33-if\ \lim_{x\to1}f(x)=3\ ,\lim_{x\to1}g(x)=-4\ ,\lim_{x\to1}h(x)=-1$$

$$a)\ \lim_{x\to1}[f(x)+h(x)]=\lim_{x\to1}f(x)+\lim_{x\to1}h(x)=3-1=2$$

$$b)\ \lim_{x\to1}\left[\frac{5f(x)}{2g(x)}+h(x)\right]=\frac{5\lim_{x\to1}f(x)}{2\lim_{x\to1}g(x)}+\lim_{x\to1}h(x)=\frac{5(3)}{2(-4)}-1$$

$$=\frac{15}{-8}-1=-\frac{15}{8}-\frac{8}{8}=\frac{-23}{8}$$

$$c)\ \lim_{x\to1}[2f(x)\ g(x)h(x)]=2\left(\lim_{x\to1}f(x)\right)\left(\lim_{x\to1}g(x)\right)\left(\lim_{x\to1}h(x)\right)$$

$$=2(3)(-4)(-1)=24$$

$$d)\ \lim_{x\to1}\sqrt{g(x)h(x)}=\sqrt{\lim_{x\to1}g(x)\ \lim_{x\to1}h(x)}=\sqrt{(-4)(-1)}=\sqrt{4}=2$$

$$e)\ \lim_{x\to1}\left(f(x)\right)^6=(\lim_{x\to1}f(x))^6=3^6=729$$

$$f)\ \lim_{x\to1}\sqrt[3]{h(x)}=\sqrt[3]{\lim_{x\to1}h(x)}=\sqrt[3]{-1}=-\sqrt[3]{1}=-1$$

$$g)\ \lim_{x\to1}\sqrt{h(x)}=\sqrt{\lim_{x\to1}h(x)}=\sqrt{-1}=\textit{does not exist}$$

$$h)\ \lim_{x\to1}\frac{f(x)}{g(x)+6h(x)}=\frac{\lim_{x\to1}f(x)}{\lim_{x\to1}g(x)+6\lim_{x\to1}h(x)}=\frac{3}{-4+6(-1)}$$

$$=\frac{3}{-4-6}=\frac{3}{-10}=-\frac{3}{10}$$

$$i)\ \lim_{x\to1}[f(x)+x^2-3x]=\lim_{x\to1}f(x)+\lim_{x\to1}x^2-3\lim_{x\to1}x$$

$$=3+1^2-3(1)=3+1-3=1$$

$$j)\ \lim_{x\to1}\sqrt{3+h(x)}=\sqrt{\lim_{x\to1}3+\lim_{x\to1}h(x)}=\sqrt{3-1}=\sqrt{2}$$

$$k)\ \lim_{x\to1}[x^2f(x)]=\lim_{x\to1}x^2\ \lim_{x\to1}f(x)=(1^2)(3)=(1)(3)=3$$

$$l)\ \lim_{x\to1}[g(x)]^{1/4}=\left[\lim_{x\to1}g(x)\right]^{1/4}=\sqrt[4]{\lim_{x\to1}g(x)}=\sqrt[4]{-4}$$

$$=\textit{does not exsist}$$

$$35-\lim_{x\to3}\frac{f(x)+4}{x-1}=3$$

$$\frac{\lim_{x\to3} f(x) + \lim_{x\to3} 4}{\lim_{x\to3}(x-1)} = 3$$

$$\frac{\lim_{x\to3} f(x) + 4}{3-1} = 3$$

$$\frac{\lim_{x\to3} f(x) + 4}{2} = 3$$

$$\lim_{x\to3} f(x) + 4 = (2)(3) \Rightarrow \lim_{x\to3} f(x) + 4 = 6 \Rightarrow \lim_{x\to3} f(x) = 6-4$$
$$\Rightarrow \lim_{x\to3} f(x) = 2$$

$37 - \dfrac{x^2+1}{x-1} \le f(x) \le -x-1$

By Sandwich Theorem:

$$\lim_{x\to0} \frac{x^2+1}{x-1} = \frac{0+1}{0-1} = \frac{1}{-1} = -1$$

$$\lim_{x\to0} -x-1 = -0-1 = -1$$

$$\Rightarrow \lim_{x\to0} f(x) = -1$$

$41 - find\ \lim_{x\to3} f(x)$

Let $\lim_{x\to3} f(x) = z$

$$\lim_{x\to3} \frac{[f(x)]^2 - 2f(x)}{x-2} = -1$$

$$\frac{\left[\lim_{x\to3} f(x)\right]^2 - 2\lim_{x\to3} f(x)}{\lim_{x\to3}(x-2)} = -1$$

$$\frac{[z]^2 - 2z}{3-2} = -1$$

$$[z]^2 - 2z = -1 \Rightarrow [z]^2 - 2z + 1 = 0 \Rightarrow (z-1)(z-1) = 0$$

$$z - 1 = 0 \Rightarrow z = 1 \Rightarrow \lim_{x \to 3} f(x) = 1$$

ملحوظه : إذا كان للنهايه ناتجين محتلفين نقول انها D.N.E.