math 312 CAR | Quiz 2 | Computer Number:
Date: 5-6-1434 | | Serial Number

(QI) Prove that the function $f(x) = \begin{cases} x+5 & \text{if } x \in Q \\ 0 & \text{if } x \notin Q \end{cases}$ is not Riemann integrable on $[1,2]$. 3

Let p be any partition of $[1,2]$

$M_i > 6 \ \forall i$, $m_i = 0 \ \forall i \Rightarrow u(p,f) > 6 \sum(x_i - x_{i-1}) = 6$ .75

$L(p,f) = 0(2-1) = 0$ .5 $\Rightarrow \overline{\int_a^b} f(x)dx \geq 6$, .25 $\underline{\int_a^b} f(x)dx = 0$ .25

$\Rightarrow \overline{\int_a^b} f(x)dx \neq \underline{\int_a^b} f(x)dx \Rightarrow f$ is not R.i. on $[1,2]$ 1

(QII) Prove that every continuous function on $[a, b]$ is Riemann integrable. 4

f is cont. on $[a,b] \Rightarrow f$ is uniformly cont. Let $\epsilon > 0$, then $\exists\ \delta > 0 \ni$

if $x, y \in [a,b]$ and $|x-y| < \delta \Rightarrow |f(x) - f(y)| < \frac{\epsilon}{b-a}$

Consider any partition p with $\|p\| < \delta \Rightarrow x_i - x_{i-1} < \delta \ \forall i$

$\forall x, y \in [x_{i-1}, x_i]$, $|f(x) - f(y)| < \frac{\epsilon}{b-a}$

$\Rightarrow M_i - m_i < \frac{\epsilon}{b-a}$

$\Rightarrow u(p,f) - L(p,f) = \sum(M_i - m_i)(x_i - x_{i-1}) < \frac{\epsilon}{b-a} \sum(x_i - x_{i-1}) =$

$\Rightarrow f$ is R.i.

(QIII) Let $f(x) = x^2$ for $x \in [0,4]$. Calculate the upper and lower Riemann sums for the partition $P = \{0,1,2,3,4\}$. 3

$U(p,f) = \sum_{i=1}^{4} M_i(x_i - x_{i-1}) = f(1)(1-0) + f(2)(2-1) + f(3)(3-2) + f(4)($

$= 1 + 4 + 9 + 16 = 30$ 1.5

$L(p,f) = \sum_{i=1}^{4} m_i(x_i - x_{i-1}) = 0 + 1(2-1) + 4(3-2) + 9(4-3)$ 1.5

$= 1 + 4 + 9 = 14$