Workshop Solutions to Chapter 4

1) If $f(x)$ is a differentiable function, then $f'(x) =$

<u>Solution:</u>

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

2) If $f(x) = 4x^2$, then $f'(x) =$

<u>Solution:</u>

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{4(x+h)^2-4x^2}{h}$$

3) If $f(x) = x^2 - 3$, then $f'(x) =$

<u>Solution:</u>

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$
$$= \lim_{h\to 0}\frac{[(x+h)^2-3]-[x^2-3]}{h}$$

4) If $f(x) = \sqrt{x}$, $x \geq 0$, then $f'(x) =$

<u>Solution:</u>

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$$

5) If f is a differentiable function at a, then f is a continuous function at a.

6) If f is a continuous function at a, then f is a differentiable function at a.

<u>Solution:</u>

False

7) If $y = x^4 + 5x^2 + 3$, then $y' =$

<u>Solution:</u>

$$y' = 4x^3 + 10x$$

8) If $y = x^4 - 5x^2 + 3$, then $y' =$

<u>Solution:</u>

$$y' = 4x^3 - 10x$$

9) If $y = x^{-5/2}$, then $y' =$

<u>Solution:</u>

$$y' = -\frac{5}{2}x^{-\frac{5}{2}-1} = -\frac{5}{2}x^{-7/2}$$

10) If $y = \frac{1}{3x^3} + 2\sqrt{x} = \frac{1}{3}x^{-3} + 2x^{1/2}$, then $y' =$

<u>Solution:</u>

$$y' = (-3)\left(\frac{1}{3}\right)x^{-3-1} + \left(\frac{1}{2}\right)(2)x^{\frac{1}{2}-1}$$
$$= -x^{-4} + x^{-1/2} = -\frac{1}{x^4} + \frac{1}{x^{1/2}} = -\frac{1}{x^4} + \frac{1}{\sqrt{x}}$$

11) If $y = (x-3)(x-2)$, then $y' =$

<u>Solution:</u>

$$y = (x-3)(x-2) = x^2 - 5x + 6$$
$$y' = 2x - 5$$

12) If $y = (x^3+3)(x^2-1)$, then $y' =$

<u>Solution:</u>

$$y = (x^3+3)(x^2-1) = x^5 - x^3 + 3x^2 - 3$$
$$y' = 5x^4 - 3x^2 + 6x$$

13) If $y = \sqrt{x}(2x+1)$, then $y' =$

<u>Solution:</u>

$$y = \sqrt{x}(2x+1) = 2x\sqrt{x} + \sqrt{x} = 2x^{\frac{3}{2}} + x^{\frac{1}{2}}$$
$$y' = \left(\frac{3}{2}\right)(2)x^{\frac{3}{2}-1} + \left(\frac{1}{2}\right)x^{\frac{1}{2}-1} = 3x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}}$$
$$= 3\sqrt{x} + \frac{1}{2\sqrt{x}}$$

OR

Use the rule $(f.g)' = f'g + fg'$

$$y' = (2)(\sqrt{x}) + \left(\frac{1}{2\sqrt{x}}\right)(2x+1) = 2\sqrt{x} + \frac{2x+1}{2\sqrt{x}}$$

14) If $y = \frac{x+3}{x-2}$, then $y' =$

<u>Solution:</u>

Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$

$$y' = \frac{(1)(x-2)-(x+3)(1)}{(x-2)^2} = \frac{x-2-x-3}{(x-2)^2} = \frac{-5}{(x-2)^2}$$
$$= -\frac{5}{(x-2)^2}$$

15) If $y = \frac{x+3}{x-2}$, then $y'|_{x=4} =$

<u>Solution:</u>

$$y' = \frac{(1)(x-2)-(x+3)(1)}{(x-2)^2} = \frac{x-2-x-3}{(x-2)^2}$$
$$= \frac{-5}{(x-2)^2} = -\frac{5}{(x-2)^2}$$
$$y'|_{x=4} = -\frac{5}{(4-2)^2} = -\frac{5}{4}$$

16) If $y = \frac{x-1}{x+2}$, then $y' =$

<u>Solution:</u>

Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$

$$y' = \frac{(1)(x+2)-(x-1)(1)}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$$

17) If $y = \sqrt{3x^2 + 6x}$, then $y' =$

Solution:

Use the rule $\left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$

$$y' = \frac{6x + 6}{2\sqrt{3x^2 + 6x}} = \frac{6(x + 1)}{2\sqrt{3x^2 + 6x}} = \frac{3(x + 1)}{\sqrt{3x^2 + 6x}}$$

18) If $y = \sqrt{3x^2 + 6x}$, then $y'|_{x=1} =$

Solution:

$$y' = \frac{6x + 6}{2\sqrt{3x^2 + 6x}} = \frac{6(x + 1)}{2\sqrt{3x^2 + 6x}} = \frac{3(x + 1)}{\sqrt{3x^2 + 6x}}$$

$$y'|_{x=1} = \frac{3((1) + 1)}{\sqrt{3(1)^2 + 6(1)}} = \frac{6}{\sqrt{9}} = \frac{6}{3} = 2$$

19) The tangent line equation to the curve $y = x^2 + 2$ at the point $(1,3)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = 2x$$

Thus, the slope at $x = 1$ is

$$y'|_{x=1} = 2(1) = 2$$

Hence, the tangent line equation passing through the point $(1,3)$ with slope $m = 2$ is

$$y - 3 = 2(x - 1)$$
$$y - 3 = 2x - 2$$
$$y = 2x - 2 + 3$$
$$y = 2x + 1$$

20) The tangent line equation to the curve $y = \frac{2x}{x+1}$ at the point $(0,0)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = \frac{(2)(x + 1) - (2x)(1)}{(x + 1)^2} = \frac{2x + 2 - 2x}{(x + 1)^2} = \frac{2}{(x + 1)^2}$$

Thus, the slope at $x = 0$ is

$$y'|_{x=0} = \frac{2}{(0 + 1)^2} = 2$$

Hence, the tangent line equation passing through the point $(0,0)$ with slope $m = 2$ is

$$y - 0 = (2)(x - 0)$$
$$y = 2x$$

21) The tangent line equation to the curve $y = 3x^2 - 13$ at the point $(2, -1)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = 6x$$

Thus, the slope at $x = 2$ is

$$y'|_{x=2} = 6(2) = 12$$

Hence, the tangent line equation passing through the point $(2, -1)$ with slope $m = 12$ is

$$y - (-1) = 12(x - 2)$$
$$y + 1 = 12x - 24$$
$$y = 12x - 24 - 1$$
$$y = 12x - 25$$

22) The tangent line equation to the curve $y = 3x^2 + 2x + 5$ at the point $(0,5)$ is

Solution:

First, we have to find the slope of the curve which is

$$y' = 6x + 2$$

Thus, the slope at $x = 2$ is

$$y'|_{x=0} = 6(0) + 2 = 2$$

Hence, the tangent line equation passing through the point $(0,5)$ with slope $m = 2$ is

$$y - 5 = 2(x - 0)$$
$$y - 5 = 2x$$
$$y = 2x + 5$$

23) If $y = xe^x$, then $y' =$

Solution:

Use the rules $(f.g)' = f'g + fg'$ and $(e^u) = e^u.u'$

$$y' = (1)(e^x) + (x)(e^x) = e^x + xe^x = e^x(1 + x)$$

24) If $y = x - e^x$, then $y'' =$

Solution:

Use the rules $(f - g)' = f' - g'$ and $(e^u) = e^u.u'$

$$y' = 1 - e^x$$
$$y'' = -e^x$$

25) If $x^2 - y^2 = 4$, then $y' =$

Solution:

$$2x - 2yy' = 0$$
$$-2yy' = -2x$$
$$y' = \frac{-2x}{-2y}$$
$$y' = \frac{x}{y}$$

26) If $x^2 + y^2 = 4$, then $y' =$

Solution:

$$2x + 2yy' = 0$$
$$2yy' = -2x$$
$$y' = \frac{-2x}{2y}$$
$$y' = -\frac{x}{y}$$

27) If $y = \frac{x+1}{x+2}$, then $y' =$

Solution:

Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$

$$y' = \frac{(1)(x + 2) - (x + 1)(1)}{(x + 2)^2} = \frac{x + 2 - x - 1}{(x + 2)^2}$$
$$= \frac{1}{(x + 2)^2}$$

28) If $y = \frac{1}{\sqrt[2]{x^5}} + \sec x$, then $y' =$

Solution:

Use the rules

$$(f + g)' = f' + g' \quad \text{and} \quad (\sec u)' = \sec u \tan u \,.\, u'$$

$$y = \frac{1}{\sqrt[2]{x^5}} + \sec x = x^{-\frac{5}{2}} + \sec x$$

$$y' = \left(-\frac{5}{2}\right)x^{-\frac{5}{2}-1} + \sec x \tan x = -\frac{5}{2}x^{-7/2} + \sec x \tan x$$

29) If $y = \tan^{-1}(x^3)$, then $y' =$
Solution:
Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$
$$y' = \frac{1}{1+(x^3)^2} \, . (3x^2) = \frac{3x^2}{1+x^6}$$

30) If $y = \tan x - x$, then $y' =$
Solution:
Use the rules
$(f-g)' = f' - g'$ and $(\tan u)' = \sec^2 u \, . u'$
$$y' = \sec^2 x - 1$$

31) If $y = \sec^2 x - 1$, then $y' =$
Solution:
Use the rules $(f-g)' = f' - g'$, $(u)^n = n(u)^{n-1} . u'$ and $(\sec u)' = \sec u \tan u \, . u'$
$$y' = 2\sec x \, . \, \sec x \tan x = 2\sec^2 x \tan x$$

32) If $y = x^{\sin x}$, then $y' =$
Solution:
Use the rule $(\sin u)' = \cos u \, . u'$
$$y = x^{\sin x}$$
$$\ln y = \ln x^{\sin x}$$
$$\ln y = \sin x \, . \ln x$$
$$\frac{y'}{y} = \cos x \, . \, \ln x + \sin x \, . \frac{1}{x} = \cos x \, . \, \ln x + \frac{\sin x}{x}$$
$$y' = y\left(\cos x \, . \, \ln x + \frac{\sin x}{x}\right) = x^{\sin x}\left(\cos x \, . \, \ln x + \frac{\sin x}{x}\right)$$

33) If $y = x^{\cos x}$, then $y' =$
Solution:
Use the rule $(\cos u)' = -\sin u \, . u'$
$$y = x^{\cos x}$$
$$\ln y = \ln x^{\cos x}$$
$$\ln y = \cos x \, . \ln x$$
$$\frac{y'}{y} = -\sin x \, . \, \ln x + \cos x \, . \frac{1}{x} = -\sin x \, . \, \ln x + \frac{\cos x}{x}$$
$$y' = y\left(-\sin x \, . \, \ln x + \frac{\cos x}{x}\right)$$
$$= x^{\cos x}\left(\frac{\cos x}{x} - \sin x \, . \, \ln x\right)$$

34) If $y = (2x^2 + \csc x)^9$, then $y' =$
Solution:
Use the rules
$(u)^n = n(u)^{n-1} . u'$ and $(\csc u)' = -\csc u \cot u \, . u'$
$$y' = 9(2x^2 + \csc x)^8 . (4x - \csc x \cot x)$$

35) If $y = \frac{5^x}{\cot x}$, then $y' =$
Solution:
Use the rules
$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$, $(a^u)' = a^u . \ln a \, . u'$
and $(\csc u)' = -\csc u \cot u \, . u'$
$$y' = \frac{(5^x \ln 5)(\cot x) - (5^x)(-\csc^2 x)}{(\cot x)^2}$$
$$= \frac{5^x(\ln 5 \cot x + \csc^2 x)}{\cot^2 x}$$

36) If $y = e^{2x}$, then $y^{(6)} =$
Solution:
Use the rule $(e^u)' = e^u . u'$
$$y' = 2e^{2x}$$
$$y'' = 4e^{2x}$$
$$y''' = 8e^{2x}$$
$$y^{(4)} = 16e^{2x}$$
$$y^{(5)} = 32e^{2x}$$
$$y^{(6)} = 64e^{2x}$$

37) If $y = x^{-2}e^{\sin x}$, then $y' =$
Solution:
Use the rules $(f.g)' = f'g + fg'$, $(e^u) = e^u . u'$ and $(\sin u)' = \cos u \, . u'$
$$y' = (-2x^{-3})(e^{\sin x}) + (x^{-2})(e^{\sin x} . \cos x)$$
$$= -2x^{-3}e^{\sin x} + x^{-2}\cos x \, e^{\sin x}$$
$$= x^{-3}e^{\sin x}(-2 + x\cos x)$$
$$= x^{-3}e^{\sin x}(x\cos x - 2)$$

38) If $y = 5^{\tan x}$, then $y' =$
Solution:
Use the rules
$(a^u)' = a^u . \ln a \, . u'$ and $(\tan u)' = \sec^2 u \, . u'$
$$y' = 5^{\tan x} . \, \ln 5 \, . \sec^2 x$$

39) If $x^2 + y^2 = 3xy + 7$, then $y' =$
Solution:
$$2x + 2yy' = 3y + 3xy'$$
$$2yy' - 3xy' = 3y - 2x$$
$$y'(2y - 3x) = 3y - 2x$$
$$y' = \frac{3y - 2x}{2y - 3x}$$

40) If $y = \sin^3(4x)$, then $y^{(6)} =$
Solution:
Use the rules
$(u)^n = n(u)^{n-1} . u'$ and $(\sin u)' = \cos u \, . u'$
$$y' = 3\sin^2(4x) . \cos(4x) . (4)$$
$$= 12\sin^2(4x) . \cos(4x)$$

41) If $y = 3^x \cot x$, then $y' =$

Solution:

Use the rules $(f.g)' = f'g + fg'$, $(a^u)' = a^u . \ln a . u'$ and $(\cot u)' = -\csc^2 u \; .u'$

$$y' = (3^x . \ln 3)(\cot x) + (3^x)(-\csc^2 x)$$
$$= 3^x \ln 3 \;\cot x - 3^x \csc^2 x$$
$$= 3^x(\ln 3 \;\cot x - \csc^2 x)$$

42) If $y = (2x^2 + \sec x)^7$, then $y' =$

Solution:

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\sec u)' = \sec u \tan u . u'$

$$y' = 7(2x^2 + \sec x)^6.(4x + \sec x \tan x)$$

43) If $f(x) = \cos x$, then $f^{(45)}(x) =$

Solution:

$$f'(x) = -\sin x$$
$$f''(x) = -\cos x$$
$$f'''(x) = \sin x$$
$$f^{(4)}(x) = \cos x$$

Note: $f^{(n)}(x) = \cos x$ whenever n is a multiple of 4.

Hence,

$$f^{(44)}(x) = \cos x$$
$$f^{(45)}(x) = -\sin x$$

44) If $D^{47}(\sin x) =$

Solution:

$$D(\sin x) = \cos x$$
$$D^2(\sin x) = -\sin x$$
$$D^3(\sin x) = -\cos x$$
$$D^4(\sin x) = \sin x$$

Note: $D^n(\sin x) = \sin x$ whenever n is a multiple of 4.

Hence,

$$D^{44}(\sin x) = \sin x$$
$$D^{45}(\sin x) = \cos x$$
$$D^{46}(\sin x) = -\sin x$$
$$D^{47}(\sin x) = -\cos x$$

45) If $y = x^x$, then $y' =$

Solution:

Use the rule $(\ln u)' = \frac{u'}{u}$

$$y = x^x$$
$$\ln y = \ln x^x$$
$$\ln y = x \ln x$$
$$\frac{y'}{y} = (1)(\ln x) + (x)\left(\frac{1}{x}\right)$$
$$\frac{y'}{y} = \ln x + 1$$
$$y' = y(1 + \ln x) = x^x(1 + \ln x)$$

46) If $f(x) = \frac{\ln x}{x^2}$, then $f'(1) =$

Solution:

Use the rules $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ and $(\ln u)' = \frac{u'}{u}$

$$f'(x) = \frac{\left(\frac{1}{x}\right)(x^2) - (\ln x)(2x)}{(x^2)^2} = \frac{x - 2x \ln x}{x^4}$$
$$= \frac{x(1 - 2\ln x)}{x^4} = \frac{1 - 2\ln x}{x^3}$$

$$\therefore \quad f'(1) = \frac{1 - 2\ln(1)}{(1)^3} = \frac{1 - 2(0)}{1} = 1$$

47) If $y = \cot^{-1}(e^x)$, then $y' =$

Solution:

Use the rules $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ and $(e^u) = e^u.u'$

$$y' = -\frac{1}{1 + (e^x)^2} . e^x = -\frac{e^x}{1 + e^{2x}}$$

48) If $y = \tan^{-1}(e^x)$, then $y' =$

Solution:

Use the rules $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ and $(e^u) = e^u.u'$

$$y' = \frac{1}{1 + (e^x)^2} . e^x = \frac{e^x}{1 + e^{2x}}$$

49) If $y = \sin^{-1}(e^x)$, then $y' =$

Solution:

Use the rules $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ and $(e^u) = e^u.u'$

$$y' = \frac{1}{\sqrt{1 - (e^x)^2}} . e^x = \frac{e^x}{\sqrt{1 - e^{2x}}}$$

50) If $y = \cos^{-1}(e^x)$, then $y' =$

Solution:

Use the rules $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ and $(e^u) = e^u.u'$

$$y' = -\frac{1}{\sqrt{1 - (e^x)^2}} . e^x = -\frac{e^x}{\sqrt{1 - e^{2x}}}$$

51) If $y = \cos(2x^3)$, then $y' =$

Solution:

Use the rule $(\cos u)' = -\sin u \; .u'$

$$y' = -\sin(2x^3).(6x^2) = -6x^2 \sin(2x^3)$$

52) If $y = \csc x \cot x$, then $y' =$

Solution:

Use the rules $(f.g)' = f'g + fg'$,

$(\csc u)' = -\csc u \cot u . u'$ and $(\cot u)' = -\csc^2 u \; .u'$

$$y' = (-\csc x \cot x)(\cot x) + (\csc x)(-\csc^2 x)$$
$$= -\csc x \cot^2 x - \csc^3 x = -\csc x(\cot^2 x + \csc^2 x)$$

53) If $y=\sqrt{x^2-2\sec x}$, then $y'=$
Solution:
Use the rules
$$\left(\sqrt{u}\right)'=\frac{u'}{2\sqrt{u}} \quad \text{and} \quad (\sec u)'=\sec u\tan u\,.\,u'$$
$$y'=\frac{2x-2\sec x\tan x}{2\sqrt{x^2-2\sec x}}=\frac{2(x-\sec x\tan x)}{2\sqrt{x^2-2\sec x}}$$
$$=\frac{x-\sec x\tan x}{\sqrt{x^2-2\sec x}}$$

54) If $y=(3x^2+1)^6$, then $y'=$
Solution:
Use the rule $\quad (u)^n=n(u)^{n-1}.u'$
$$y'=6(3x^2+1)^5.(6x)=36x(3x^2+1)^5$$

55) If $xy+\tan x=2x^3+\sin y$, then $y'=$
Solution:
$$[(1)(y)+(x)(y')]+\sec^2 x=6x^2+\cos y\,.\,y'$$
$$y+xy'+\sec^2 x=6x^2+y'\cos y$$
$$xy'-y'\cos y=6x^2-y-\sec^2 x$$
$$y'(x-\cos y)=6x^2-y-\sec^2 x$$
$$y'=\frac{6x^2-y-\sec^2 x}{x-\cos y}$$

56) If $y=x^{-1}\sec x$, then $y'=$
Solution:
Use the rules
$$(f.g)'=f'g+fg' \quad \text{and} \quad (\sec u)'=\sec u\tan u\,.\,u'$$
$$y'=(-x^{-2})(\sec x)+(x^{-1})(\sec x\tan x)$$
$$=x^{-1}\sec x\tan x-x^{-2}\sec x$$
$$=x^{-2}\sec x\,(x\tan x-1)$$

57) If $y=\sin^{-1}(x^3)$, then $y'=$
Solution:
Use the rule $\quad (\sin^{-1}u)'=\frac{u'}{\sqrt{1-u^2}}$
$$y'=\frac{1}{\sqrt{1-(x^3)^2}}\,.3x^2=\frac{3x^2}{\sqrt{1-x^6}}$$

58) If $y=\cos^{-1}(x^3)$, then $y'=$
Solution:
Use the rule $\quad (\cos^{-1}u)'=-\frac{u'}{\sqrt{1-u^2}}$
$$y'=-\frac{1}{\sqrt{1-(x^3)^2}}\,.3x^2=-\frac{3x^2}{\sqrt{1-x^6}}$$

59) If $y=\sec^{-1}(x^3)$, then $y'=$
Solution:
Use the rule $\quad (\sec^{-1}u)'=\frac{u'}{|u|\sqrt{u^2-1}}$
$$y'=\frac{1}{x^3\sqrt{(x^3)^2-1}}\,.3x^2=\frac{3x^2}{x^3\sqrt{x^6-1}}=\frac{3}{x\sqrt{x^6-1}}$$

60) If $y=\csc^{-1}(x^3)$, then $y'=$
Solution:
Use the rule $\quad (\csc^{-1}u)'=-\frac{u'}{|u|\sqrt{u^2-1}}$
$$y'=-\frac{1}{x^3\sqrt{(x^3)^2-1}}\,.3x^2=-\frac{3x^2}{x^3\sqrt{x^6-1}}=-\frac{3}{x\sqrt{x^6-1}}$$

61) If $y=\ln(x^3-2\sec x)$, then $y'=$
Solution:
Use the rules
$$(\ln u)'=\frac{u'}{u} \quad \text{and} \quad (\sec u)'=\sec u\tan u\,.\,u'$$
$$y'=\frac{1}{x^3-2\sec x}\,.(3x^2-2\sec x\tan x)$$
$$=\frac{3x^2-2\sec x\tan x}{x^3-2\sec x}$$

62) If $y=\ln(\cos x)$, then $y'=$
Solution:
Use the rules
$$(\ln u)'=\frac{u'}{u} \quad \text{and} \quad (\cos u)'=-\sin u\,.\,u'$$
$$y'=\frac{1}{\cos x}\,.(-\sin x)=-\frac{\sin x}{\cos x}=-\tan x$$

63) If $y=\ln(\sin x)$, then $y'=$
Solution:
Use the rules
$$(\ln u)'=\frac{u'}{u} \quad \text{and} \quad (\sin u)'=\cos u\,.\,u'$$
$$y'=\frac{1}{\sin x}\,.(\cos x)=\frac{\cos x}{\sin x}=\cot x$$

64) If $y=\ln\sqrt{3x^2+5x}$, then $y'=$
Solution:
Use the rules $\quad (\ln u)'=\frac{u'}{u} \quad$ and $\quad \left(\sqrt{u}\right)'=\frac{u'}{2\sqrt{u}}$
$$y'=\frac{1}{\sqrt{3x^2+5x}}\cdot\left(\frac{6x+5}{2\sqrt{3x^2+5x}}\right)=\frac{6x+5}{2(3x^2+5x)}$$

65) If $y = \log_5(x^3 - 2\csc x)$, then $y' =$

<u>Solution:</u>

Use the rules

$$(\log_a u)' = \frac{u'}{u \ln a} \quad \text{and} \quad (\csc u)' = -\csc u \cot u \,.\, u'$$

$$y' = \frac{1}{(x^3 - 2\csc x)(\ln 5)} \,.\, [3x^2 - 2\,(-\csc x \cot x)]$$

$$= \frac{3x^2 + 2\csc x \cot x}{(x^3 - 2\csc x)(\ln 5)}$$

66) If $y = \ln \frac{x-1}{\sqrt{x+2}}$, then $y' =$

<u>Solution:</u>

Use the rules

$$(\ln u)' = \frac{u'}{u} \;,\; \left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2} \quad \text{and} \quad \left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$$

$$y' = \frac{1}{\frac{x-1}{\sqrt{x+2}}} \cdot \left(\frac{(1)\left(\sqrt{x+2}\right) - (x-1)\left(\frac{1}{2\sqrt{x+2}}\right)}{\left(\sqrt{x+2}\right)^2}\right)$$

$$= \frac{\sqrt{x+2}}{x-1} \cdot \left(\frac{\sqrt{x+2} - \frac{x-1}{2\sqrt{x+2}}}{x+2}\right)$$

$$= \frac{\sqrt{x+2}}{x-1} \cdot \left(\frac{\frac{2(x+2)-(x-1)}{2\sqrt{x+2}}}{x+2}\right)$$

$$= \frac{\sqrt{x+2}}{x-1} \cdot \left(\frac{\frac{x+5}{2\sqrt{x+2}}}{x+2}\right)$$

$$= \frac{\sqrt{x+2}}{x-1}\left(\frac{x+5}{2(x+2)\sqrt{x+2}}\right)$$

$$= \frac{x+5}{2(x-1)(x+2)}$$

67) If $y = 2x^3 - \sin x$, then $y' =$

<u>Solution:</u>

Use the rule $\quad (\sin u)' = \cos u \,.\, u'$

$$y' = 6x^2 - \cos x$$

68) If $y = x^3 \cos x$, then $y' =$

<u>Solution:</u>

Use the rules

$$(f.g)' = f'g + fg' \quad \text{and} \quad (\cos u)' = -\sin u \,.\, u'$$

$$y' = (3x^2)(\cos x) + (x^3)(-\sin x)$$

$$= 3x^2 \cos x - x^3 \sin x$$

69) If $y = x^{\sqrt{x}}$, then $y' =$

<u>Solution:</u>

Use the rule $\quad \left(\sqrt{u}\right)' = \frac{u'}{2\sqrt{u}}$

$$y = x^{\sqrt{x}}$$

$$\ln y = \ln x^{\sqrt{x}}$$

$$\ln y = \sqrt{x} \ln x$$

$$\frac{y'}{y} = \left(\frac{1}{2\sqrt{x}}\right)(\ln x) + \left(\sqrt{x}\right)\left(\frac{1}{x}\right)$$

$$\frac{y'}{y} = \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} = \frac{x \ln x + 2x}{2x\sqrt{x}} = \frac{x(\ln x + 2)}{2x\sqrt{x}}$$

$$= \frac{\ln x + 2}{2\sqrt{x}}$$

$$y' = y\left(\frac{\ln x + 2}{2\sqrt{x}}\right) = x^{\sqrt{x}}\left(\frac{\ln x + 2}{2\sqrt{x}}\right)$$

70) If $y = (\sin x)^x$, then $y' =$

<u>Solution:</u>

Use the rule $\quad (\sin u)' = \cos u \,.\, u'$

$$y = (\sin x)^x$$

$$\ln y = \ln(\sin x)^x$$

$$\ln y = x \ln(\sin x)$$

$$\frac{y'}{y} = (1)(\ln(\sin x)) + (x)\left(\frac{\cos x}{\sin x}\right)$$

$$\frac{y'}{y} = \ln(\sin x) + \frac{x \cos x}{\sin x} = \ln(\sin x) + x \cot x$$

$$y' = y(\ln(\sin x) + x \cot x)$$

$$= (\sin x)^x(\ln(\sin x) + x \cot x)$$

71) If $y = \log_7(x^3 - 2)$, then $y' =$

<u>Solution:</u>

Use the rule $\quad (\log_a u)' = \frac{u'}{u \ln a}$

$$y' = \frac{1}{(x^3 - 2)(\ln 7)} \,.\, (3x^2) = \frac{3x^2}{(x^3 - 2)(\ln 7)}$$

72) If $y = \cos(x^5)$, then $y' =$

<u>Solution:</u>

Use the rule $\quad (\cos u)' = -\sin u \,.\, u'$

$$y' = -\sin(x^5) \,.\, (5x^4) = -5x^4 \sin(x^5)$$

73) If $y = \sec x \tan x$, then $y' =$

<u>Solution:</u>

$(f.g)' = f'g + fg'$, $(\sec u)' = \sec u \tan u \,.\, u'$ and

$$(\tan u)' = \sec^2 u \,.\, u'$$

$$y' = (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)$$
$$= \sec x \tan^2 x + \sec^3 x = \sec x(\tan^2 x + \sec^2 x)$$

74) If $D^{99}(\cos x) =$

<u>Solution:</u>

$$D(\cos x) = -\sin x$$
$$D^2(\cos x) = -\cos x$$
$$D^3(\cos x) = \sin x$$
$$D^4(\cos x) = \cos x$$

Note: $D^n(\cos x) = \cos x$ whenever n is a multiple of 4.

Hence,

$$D^{96}(\cos x) = \cos x$$
$$D^{97}(\cos x) = -\sin x$$
$$D^{98}(\cos x) = -\cos x$$
$$D^{99}(\cos x) = \sin x$$

75) If $y = (x + \sec x)^3$, then $y' =$

<u>Solution:</u>

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\sec u)' = \sec u \tan u \,.\, u'$

$$y' = 3(x + \sec x)^2.(1 + \sec x \tan x)$$

76) If $x^2 = 5y^2 + \sin y$, then $y' =$

<u>Solution:</u>

$$2x = 10yy' + \cos y \,.\, y'$$
$$y'(10y + \cos y) = 2x$$
$$y' = \frac{2x}{10y + \cos y}$$

77) If $x^2 - 5y^2 + \sin y = 0$, then $y' =$

<u>Solution:</u>

$$2x - 10yy' + \cos y \,.\, y' = 0$$
$$y'(-10y + \cos y) = -2x$$
$$y' = \frac{-2x}{-10y + \cos y} = \frac{2x}{10y - \cos y}$$

78) If $y = \sin x \sec x$, then $y' =$

<u>Solution:</u>

$(f.g)' = f'g + fg'$, $(\sin u)' = \cos u \,.\, u'$ and

$$(\sec u)' = \sec u \tan u \,.\, u'$$

$$y' = (\cos x)(\sec x) + (\sin x)(\sec x \tan x)$$
$$= 1 + \sin x \,.\, \frac{1}{\cos x} \,.\, \frac{\sin x}{\cos x} = 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$$
$$= \sec^2 x$$

79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$

<u>Solution:</u>

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\sin u)' = \cos u \,.\, u'$

$$f'(x) = 2\sin(x^3 + 1) \,.\, (\cos(x^3 + 1)) \,.\, (3x^2)$$
$$= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$$

80) If $y = (x + \cot x)^3$, then $y' =$

<u>Solution:</u>

Use the rules

$(u)^n = n(u)^{n-1}.u'$ and $(\cot u)' = -\csc^2 u \,.\, u'$

$$y' = 3(x + \cot x)^2.(1 - \csc^2 x)$$

81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$

<u>Solution:</u>

Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$

$$y' = \frac{1}{1 + \left(\frac{x}{2}\right)^2} \,.\, \frac{1}{2} = \frac{1}{2\left(1 + \frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4 + x^2}{4}\right)} = \frac{2}{4 + x^2}$$

82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' =$

<u>Solution:</u>

Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$

$$y' = -\frac{1}{1 + \left(\frac{x}{2}\right)^2} \,.\, \frac{1}{2} = -\frac{1}{2\left(1 + \frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4 + x^2}{4}\right)}$$
$$= -\frac{2}{4 + x^2}$$

83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' =$

<u>Solution:</u>

Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \,.\, \frac{1}{3} = \frac{1}{3\sqrt{1 - \frac{x^2}{9}}} = \frac{1}{3\sqrt{\frac{9 - x^2}{9}}}$$
$$= \frac{1}{\sqrt{9 - x^2}}$$

84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' =$

<u>Solution:</u>

Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$

$$y' = -\frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \,.\, \frac{1}{3} = -\frac{1}{3\sqrt{1 - \frac{x^2}{9}}} = -\frac{1}{3\sqrt{\frac{9 - x^2}{9}}}$$
$$= -\frac{1}{\sqrt{9 - x^2}}$$

85) If $D^{99}(\sin x) =$

Solution:

$$D(\sin x) = \cos x$$
$$D^2(\sin x) = -\sin x$$
$$D^3(\sin x) = -\cos x$$
$$D^4(\sin x) = \sin x$$

Note: $D^n(\sin x) = \sin x$ whenever n is a multiple of 4.

Hence,

$$D^{96}(\sin x) = \sin x$$
$$D^{97}(\sin x) = \cos x$$
$$D^{98}(\sin x) = -\sin x$$
$$D^{99}(\sin x) = -\cos x$$