MATH 110

قسم الرياضيات – كلية العلوم

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Chapter 1

Basics of Sets

- ✓ **A set is a collection of objects. These objects are called members, or elements.**

- ✓ **Sets are denoted by uppercase letters such as X, Y, A, B, and their elements by lowercase letters such as x, y, a, b. The object x is a member of a set X is symbolized by $x \in X$. If x is not a member of a set X, then we denote that by $x \notin X$.**

Example 1

The set of days of the week can be represented by listing

- ✓ **For two sets X and Y, we define:**

- $X \setminus Y = \{x: x \in X \text{ and } x \notin Y\}$, **called the complement of Y with respect to X.**
- $X \cap Y = \{x: x \in X \text{ and } x \in Y\}$, **called the intersection of X and Y.**
- $X \cup Y = \{x: x \in X \text{ or } x \in Y\}$, **called the union of X and Y.**

- ✓ **Main sets of numbers**

The Natural numbers N= $\{1, 2, 3, 4, \ldots\}$.

The Integers numbers Z= $\{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$.

The Rational numbers Q= $\left\{\frac{a}{b} : a,b \in \mathbb{Z} \text{ and } b \neq 0\right\}$.

- ✓ **For** $a, b \in \mathbb{R}$ **such that** $a < b$**:**

1. $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
2. $(a, \infty) = \{x \in \mathbb{R} : a < x\}$
3. $(-\infty, a) = \{x \in \mathbb{R} : x < a\}$
4. $(-\infty, \infty) = \{x \in \mathbb{R} : -\infty < x < \infty\} = \mathbb{R}$
5. $[a, b] = \{x \in \mathbb{R} : a \leq x \leq b\}$
6. $[a, \infty) = \{x \in \mathbb{R} : a \leq x\}$
7. $(-\infty, a] = \{x \in \mathbb{R} : x \leq a\}$
8. $[a, b) = \{x \in \mathbb{R} : a \leq x < b\}$
9. $(a, b] = \{x \in \mathbb{R} : a < x \leq b\}$.

Example

a) $(2,6] \setminus (4,8) = (2,4]$.

b) $(1,3] \cup (2,7) = (1,7)$.

c) $(0,6] \cap (2,8) = (2,6]$.

Equations and Inequalities

✓ **To solve a linear equation we follow these steps:**

- **Eliminate each bracket.**
- **Put the terms of variable on one side and the constants on the other side.**
- **Divide both sides by the coefficient of the variable.**

Example

Solve the equation $4x - 1 = 2\left(3 - \frac{x}{2}\right)$.

Solution

$$4x - 1 = 2\left(3 - \frac{x}{2}\right) \Rightarrow 4x - 4 = 6 - x$$

$$\Rightarrow 4x + x = 6 + 4 \Rightarrow 5x = 10 \Rightarrow x = \frac{10}{5} \Rightarrow x = 2.$$

- ✓ **For** $ax^2 + bx + c = 0$, **where** $a, b, c \in \mathbb{R}$ **and** $a \neq 0$. **The following formula will solve any equation of this kind**

$$x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

To apply this formula we have to determine $a, b,$ **and** c **correctly. The expression** $b^2 - 4ac$**, called the distinguisher, will distinguish the roots as follows:**

- **If** $b^2 - 4ac < 0$**, then there is no real root.**
- **If** $b^2 - 4ac = 0$**, then there is only one repeat root.**
- **If** $b^2 - 4ac > 0$**, then there are two distinct roots.**

Example 2

Solve the equation $x^2 - 3x - 10 = 0$.

Solution

$\because a = 1, b = -3, c = -10$

$$\therefore x_1, x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 + 40}}{2} = \frac{3 \pm 7}{2}$$

Thus the first root is $x_1 = \frac{3+7}{2} = 5$**, and the second root is** $x_2 = \frac{3-7}{2} = -2$.

Example

Solve the equation $x^2 - 3x = 0$**.**

Solution

$x^2 - 3x = 0 \;\Rightarrow\; x(x-3) = 0 \;\Rightarrow$ **either** $x = 0$ **or** $x = 3$**.**

Example

Solve the equation $x^2 - 9 = 0$**.**

Solution

$x^2 - 9 = 0 \;\Rightarrow\; x^2 = 9 \;\Rightarrow\; x = \pm\sqrt{9} = \pm 3.$

Example

Solve the equation $x^2 - 3x - 10 = 0$**.**

Solution

$x^2 - 3x - 10 = 0 = 0 \;\Rightarrow\; (x-5)(x-2) = 0 \;\Rightarrow$

either $x - 5 = 0 \;\Rightarrow\; x = 5$ **or** $x - 2 = 0 \;\Rightarrow\; x = 2.$

Inequalities.

✓ **For** $a, b, c \in \mathbb{R}$**:**

- $a < b \Rightarrow a + c < b + c$.
- $a < b$ and $c > 0 \Rightarrow ac < bc$.
- $a < b$ and $c < 0 \Rightarrow ac > bc$.
- $0 < a < b \Rightarrow \frac{1}{a} > \frac{1}{b}$.

All of the properties above are true if we replace $<$ by $\leq$.

Example

Solve the inequality $x + 6 < 4(x - 1)$**.**

Solution

$x + 5 < 4(x - 1) \Rightarrow x + 5 < 4x - 4 \Rightarrow x - 4x < -4 - 5 \Rightarrow -3x < -9 \Rightarrow x > \frac{-9}{-3} \Rightarrow x > 3$.

So, the solution set is

$\{x \in \mathbb{R} : x \geq 3\} = [3, \infty)$.

Absolute value

For any $x \in \mathbb{R}$, we define the absolute value of x, denoted by $|x|$, to be the distance on the real line between x and 0. Thus

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}.$$

The absolute value has the following properties:

For any $x, a \in \mathbb{R}$, with $a > 0$:

- $|x| = a \iff x = a$ or $x = -a$.
- $|x| < a \iff -a < x < a$.
- $|x| > a \iff x > a$ or $x < -a$.

The last two properties are true if we replace $<$ by $\leq$.

Example

Solve the inequality $|x - 2| \leq 4$.

Solution

$|x - 2| \leq 4 \Rightarrow -4 \leq x - 2 \leq 4 \Rightarrow -2 \leq x \leq 6$. **The solution set is $[-2, 6]$.**

- ✓ **The distance between two real numbers x and y on the real line we can be measured by $|x - y|$.**
- ✓ **The distance between two points (x_1, y_1) and (x_2, y_2) in the plane can be measured by the formula $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.**

Example

Find the distance between $(3, 5)$ and $(2, 3)$.

Solution

$$\sqrt{(3-2)^2 + (5-3)^2} = \sqrt{1+4} = \sqrt{5}$$

Lines

- ✓ **The slope of a line passing through the points (x_1, y_1) and (x_2, y_2), denoted by m, is given by**

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}.$$

- ✓ **An equation of a line which has slope m and passing through the point (x_1, y_1) is given by**

$$y - y_1 = m(x - x_1).$$

Example

Find the slope of the line passing through the points $(3\ ,1)$ and $(1\ ,5)$.

Solution

The slope $m = \frac{1-5}{3-1} = \frac{-4}{2} = -2.$

Example

Find an equation of the line has slope -2 and passing through the point $(2\ ,1)$.

Solution

An equation is $y - 1 = 2(x - 2) \Rightarrow\ y - 1 = 2x - 4\ \Rightarrow\ y - 2x + 3 = 0.$

Example

Find an equation of the line passing through the points $(3\ ,1)$ and $(1\ ,5)$.

Solution

First, we have to find the slope. $m = \frac{1-5}{3-1} = -2$**. Then we choose any one of the two points. Let us take the first point, so, an equation is** $y - 1 = -2(x - 3) \Rightarrow\ y - 1 = -2x + 6\ \Rightarrow\ y + 2x - 7 = 0.$