# Exercise Sheet 1

**Exercise 1** *Classify each of the following PDE (order, linear, homogenous and nonhomogenous):*

1. $u_x + u_y = \cos x$.
2. $x^2 u_x + y^2 u_y + u_{xy} = 2xy$.
3. $(x - y)u_x + u_{xy} = 1$.
4. $x^2 u_{yy} - y u_{xx} = 0$.
5. $u_x + u_y - u_{xx} = 4$.
6. $u_x + u_y = u$.
7. $u_{xy} - u_x + u_y - \sin(x + y)u = 0$.
8. $x^2 u_{xy} + u_y = 10u$.
9. $\cos x u_x + u_y = 0$.

**Exercise 2** *Classify each of the following PDE (hyperbolic, parabolic and elliptic):*

1. $u_{xx} + 2x u_{xy} + x^2 u_{yy} + u = 0$.
2. $x e^x u_{xx} + x^3 u_{yy} + \ln x u_y = 0$.
3. $y^2 u_{xx} + 5xy u_{xy} + x^2 u_{yy} + \sin x = 0$.

**Exercise 3** *Show that* $u(x,t) = f(x + ct) + g(x - ct)$ *is a solution of* $u_{tt} = c^2 u_{xx}$ *for any twice differentiable functions* $f$ *and* $g$ *of one variable.* $c$ *is a positive constant.*

**Exercise 4** *Show that* $u(x,y) = \ln((x - x_0)^2 + (y - y_0)^2)$ *satisfies Laplace's equation* $u_{xx} + u_{yy} = 0$ *for all pairs* $(x,y)$ *of real numbers except* $(x_0, y_0)$.

**Exercise 5** *Find the solution to* $u_{xy} = x^2 cosy$, *subject to the condition* $u_x(x,0) = e^x$ *and* $u(0,y) = 1$.

**Exercise 6** *Find the solution to* $u_{xx} + t^2 u = 0$, *subject to the condition* $u(0,t) = e^t$ *and* $u_x(0,t) = t^2$ *(*$t > 0$ *and* $u = u(x,t)$*).*

**Exercise 7** *Find the general solution to* $x u_y + y u = 0$.