

## Exercise Sheet 2

**Exercise 1** Classify the following PDE:

1.  $z^2 z_x + z_y = \cos z$ .
2.  $x^2 z_x + y^2 z_y + z = 2xy$ .
3.  $\frac{z_y}{z_x} = x^2$ .
4.  $(x - y)z_x^2 + 2z_y = 4y$ .
5.  $x^2 z_y - yz_x = \tan z$ .
6.  $z_x + z_y^2 - z_x = 4$ .
7.  $z_x - z_x z_y - z_y = 0$ .
8.  $z z_x + z_y = z^2$ .
9.  $-z_x^2 + z_y^2 - \sin(z_x) = 0$ .

**Exercise 2** Give two examples of linear, quasi-linear and non-linear PDEFO.

**Exercise 3** Find the general solution of the following PDE and verify the solution:

1.  $3z_x + 5z_y - xyz = 0$ .
2.  $z_x + 4z_y - xz = x$ .
3.  $xz_x - yz_y + z = x$ .
4.  $z_x - xz_y = 4$ .
5.  $z_x + z_y - z = y$ .
6.  $z_x + yz_y + xz = 0$ .

**Exercise 4** Solve the following:

1.  $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$ .
2.  $\frac{adx}{(b-c)yz} = \frac{bdy}{(c-a)zx} = \frac{cdz}{(a-b)xy}$ .
3.  $\frac{dx}{xz-y} = \frac{dy}{yz-x} = \frac{dz}{1-z^2}$ .
4.  $\frac{dx}{x^2(y^3-z^3)} = \frac{dy}{y^2(z^3-x^3)} = \frac{dz}{z^2(x^3-y^3)}$ .

**Exercise 5** Find the general solution of the following PDE and verify the solution:

1.  $(x+z)z_x + yz_y = z + y^2$ .
2.  $[y(x+y) + \alpha z]z_x + [x(x+y) - \alpha z]z_y = z(x+y)$ .
3.  $xz z_x + yz z_y = x^2 + y^2$ .
4.  $(y-x)z_x + (x+y)z_y = x^2 + y^2$ .
5.  $(z^2 - 2yz - y^2)z_x + (xy + xz)z_y = xy - xz$ .
6.  $x(y-z)z_x + y(z-x)z_y = z(x-y)$ .
7.  $(\frac{b-c}{a})yz z_x + (\frac{c-a}{b})xz z_y = (\frac{a-b}{c})xy$ .
8.  $(xz - y)z_x + (yz - x)z_y = 1 - z^2$ .

9.  $x^2(y^3 - z^3)z_x + y^2(z^3 - x^3)z_y = z^2(x^3 - y^3)$ .
10.  $x^2z_x + y^2z_y = (x + y)z$ .
11.  $x(z - 2y^2)z_x = (z - yz_y)(z - y^2 - 2x^3)$ .
12.  $z(xz_x - yz_y) = y^2 - x^2$ .
13.  $x(x + y)z_x = y(x + y)z_y - (x - y)(2x + 2y + z)$ .
14.  $(y + xz)z_x - (x + yz)z_y = x^2 - y^2$ .

**Exercise 6** Find the general solution of the following PDE and verify the solution:

1.  $(z_x^2 + z_y^2)y = zz_y$ .
  2.  $z_xz_y = 1$ .
  3.  $z_x = (z + yz_y)^2$ .
  4.  $z_x^2z^2 + z_y^2 = 1$ .
  5.  $z^2 = xyz_xz_y$ .
  6.  $z_x^2y(1 + x^2) = x^2z_y$ .
  7.  $z_x + z_y = z_xz_y$ .
  8.  $zz_xz_y = z_x + z_y$ .
  9.  $z_x^2z_y(x^2 + y^2) = z_x^2 + z_y$ .
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