


Capacitance

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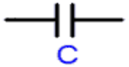
Capacitance - Capacitors

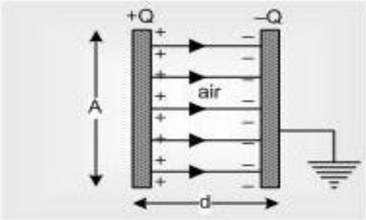
- **Capacitors** are electric devices used for saving electric charge (energy) and enable us to use later.
- Any capacitor consists of two plates with equal and opposite charges on each one.
- The magnitude of **charge** kept in a capacitor is proportional to the **voltage** applied across the capacitor. That is

$$q = CV$$

where C is the **capacitance** . The SI unit of capacitance is Farad (F) which is (from the above equation) is Coulomb/Volt (C/V). Its symbol is 

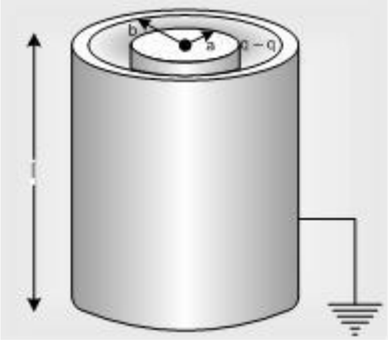
Types of Capacitors

- **A parallel-plate capacitor:** two parallel plates having equal and opposite charges with plate are A and separation by a distance d. Its capacitance is



$$C = \frac{A\varepsilon_0}{d}$$

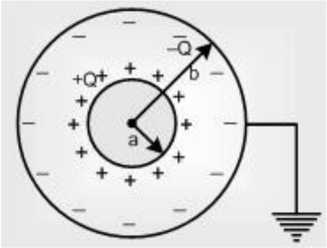
- **A cylindrical capacitor:** two long coaxial cylinders of length L and inner radius a and outer radius b. Its capacitance is



$$C = \frac{2\pi\varepsilon_0 L}{\ln\left[\frac{b}{a}\right]}$$

TYPES OF CAPACITORS

- **A spherical capacitor:** two concentric spheres inner radius a and outer radius b. Its capacitance is



$$C = \frac{4\pi\varepsilon_0\, ab}{b-a}$$

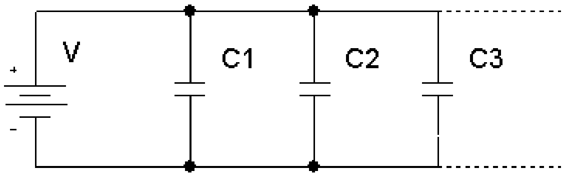
- **An isolated spherical capacitor:** for the previous capacitor, if $b \to \infty$ and $a = R$, we get an isolated spherical capacitor with capacitance

$$C = 4\pi\varepsilon_0\, R$$

COMBINATION OF CAPACITORS

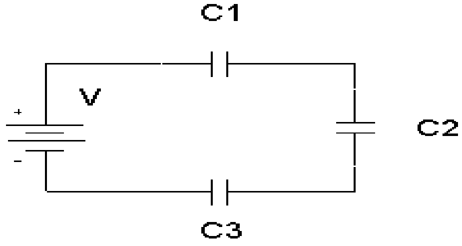
- If capacitors be connected in **parallel**, the voltage across each capacitor is the same as the total voltage, but the charge on each is different. The equivalent capacitance of a group of capacitors connected in parallel is

$$C_{eq} = \sum_{i=1}^{n} C_i$$



- If capacitors be connected in **series**, the charge on each capacitor is the same as the total charge, but the voltage across each is different. The equivalent capacitance of a group of capacitors connected in series is

$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$



Energy of Capacitors

- The electric potential energy of a charged capacitor is the work needed to charge it.

$$U = \frac{1}{2}CV^2 = \frac{1}{2}qV = \frac{q^2}{2C}$$

- The energy density is the potential energy per unit volume

$$u = \frac{1}{2}\varepsilon_0 E^2$$

where its unit is J/m^3.

Capacitors with Dielectric

- The capacitance of any capacitor can be increased if the space between the its plates is entirely filled by a dielectric material. The new capacitance is

$$C = \kappa C_{air}$$

where C_{air} is the capacitance with air between the plates and κ is the dielectric constant. The permittivity of the dielectric ε is

$$\varepsilon = \varepsilon_0 \kappa$$

Therefore all the equations containing ε_0 must be replaced by those of ε if the case is dielectric filled region.

Worked Exercises

1. A parallel-plate capacitor with plate's area $25\ \text{cm}^2$ and separation of 17.7 mm is charged by applying a voltage of 12 V across its ends. Calculate the total charge of the capacitor.

Solution

The capacitance of a parallel-plate capacitor is

$$C = \frac{A\varepsilon_0}{d} = \frac{25 \times 10^{-4} \times 8.85 \times 10^{-12}}{17.7 \times 10^{-3}} = 1.25\ \text{pF}$$

Therefore the charge of the capacitor is

$$q = CV = 1.25 \times 10^{-12} \times 12 = 15\ \text{pC}$$

Worked Exercises

2. An isolated sphere of surface area 0.5 m^2 is completely charged to 111 pC. Calculate the applied voltage across the sphere.

Solution

The capacitance of an isolated sphere is

$$C = 4\pi\varepsilon_0\, R$$

with $Area = A = 4\pi R^2 \rightarrow \quad R = \left(\frac{A}{4\pi}\right)^{0.5} = \left(\frac{0.5}{4\pi}\right)^{0.5}$ =0.2 m

$$C = 4\pi\varepsilon_0\, R = 4\pi \times 8.85 \times 10^{-12} \times 0.2 = 22.2 \text{ pF}$$

It is known that

$$q = CV \quad \rightarrow \quad V = \frac{q}{C} = \frac{111 \times 10^{-12}}{22.2 \times 10^{-12}} = 5 \text{ V}$$

Worked Exercises

3. An isolated sphere is completely charged to 60 pC when a voltage of 12 V is applied. Calculate (i) the radius of the sphere and (ii) the energy stored within the sphere.

Solution

(i) The capacitance of any capacitor can be determined from

$$q = CV \quad \rightarrow \quad C = \frac{q}{V} = \frac{60 \times 10^{-12}}{12} = 5 \times 10^{-12}\ \text{F}$$

However we know that for an isolated sphere, the capacitance is

$$C = 4\pi\varepsilon_0 R \quad \rightarrow \quad R = \frac{C}{4\pi\varepsilon_0} = \frac{5 \times 10^{-12}}{4\pi \times 8.85 \times 10^{-12}} = 4.5\ \text{cm}$$

(ii) The energy stored is

$$U = \frac{1}{2}qV = \frac{1}{2} \times 12 \times 60 \times 10^{-12} = 360 \times 10^{-12}\text{J}$$

Worked Exercises

4. A parallel-plate capacitor has a capacitance of 8 μF. Calculate its capacitance if the (i) plate separation is doubled and (ii) plate area is doubled.

Solution

The capacitance of any parallel-plate capacitor is

$$C = \frac{A\varepsilon_0}{d}$$

(i) If the separation is doubled ($d' = 2d$), the new capacitance is

$$C' = \frac{A\varepsilon_0}{2d} = \frac{C}{2} = \frac{8}{2} = 4\mu\text{F}$$

(ii) If the area is doubled ($A' = 2A$), the new capacitance is

$$C' = \frac{2A\varepsilon_0}{d} = 2C = 2 \times 8 = 16\ \mu\text{F}$$

Worked Exercises

5. A coaxial cable of radii 5 mm and 3 mm is connected by a battery of 12 V. If the charge on each cable is 6 nC, find the length of the capacitor.

Solution

To calculate the capacitance, we use the equation

$$C = \frac{q}{V} = \frac{6}{12} = 0.5\ \text{nF}$$

The capacitance of any cylindrical capacitor is

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left[\frac{b}{a}\right]}$$

Therefore the length is

$$L = \frac{C\ln\left[\frac{b}{a}\right]}{2\pi\varepsilon_0} = \frac{0.5\times10^{-9}\times\ln\left[\frac{5}{3}\right]}{2\pi\times8.85\times10^{-12}} = 4.6\ \text{m}$$

Worked Exercises

6. Two concentric spherical shells of radii 4 cm and 3 cm has a charge of 5 nC. Find the potential difference across the capacitor.

Solution

The capacitance of any concentric spherical capacitor is

$$C = \frac{4\pi\varepsilon_0 ab}{b-a} = \frac{4\pi \times 8.85 \times 10^{-12} \times 0.04 \times 0.02}{0.04 - 0.02} = 4.45 \times 10^{-12}\text{F} = 4.45\ \text{pF}$$

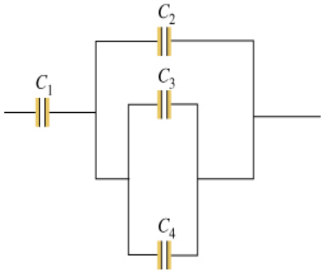
To calculate the potential difference, we use the equation

$$V = \frac{q}{C} = \frac{5.00 \times 10^{-9}}{4.45 \times 10^{-12}} = 1124\ \text{V} = 1.124\ \text{kV}$$

Worked Exercises

7. As shown in the figure, C_1=6μF and C_2= C_3=C_4=2μF. Calculate the equivalent capacitance.

Solution



From the figure, the three capacitors C_2 , C_3 and C_4 are in parallel. Their capacitance is

$$C_{234} = C_2 + C_3 + C_4 = 2 + 2 + 2 = 6\ \mu\text{F}$$

The capacitor C_1 is in series with the above three. The equivalent capacitance is

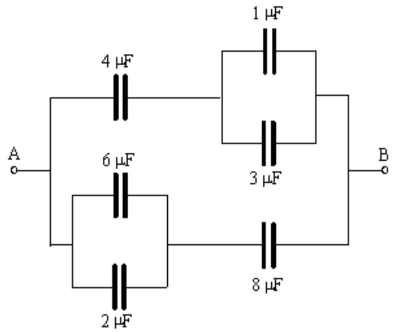
$$C_{eq} = \frac{C_1 C_{234}}{C_1 + C_{234}} = \frac{6 \times 6}{6 + 6} = 3\ \mu\text{F}$$

Worked Exercises

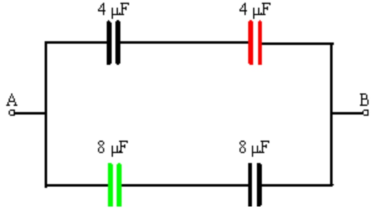
8. As shown in the figure, calculate the equivalent capacitance.

Solution

From the figure, the capacitors **1μF** and **3 μF** are in parallel and their equivalent capacitance is **4 μF** . The capacitors **6 μF** and **2 μF** are in parallel and their equivalent capacitance is **8 μF.**



$$C_{eq} = \frac{4 \times 4}{4 + 4} + \frac{8 \times 8}{8 + 8} = 6 \ \mu\text{F}$$



Worked Exercises

9. As shown in the figure, C_1=6μF , C_2= 2μF and V=12 V. calculate (i) their equivalent capacitance and (ii) the charge on the capacitor C_2.

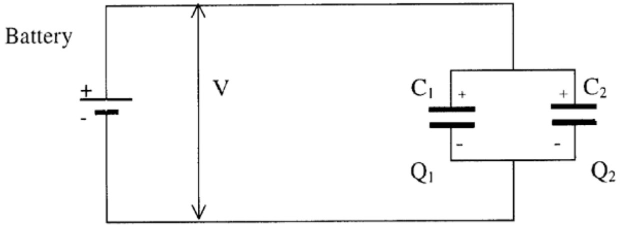
Solution

(i) From the figure, the capacitors are in parallel and their equivalent capacitance is

$$C_{eq} = C_1 + C_2 = 6 + 2 = 8\ \mu\text{F}$$

(ii) The voltage across each on is similar to the battery voltage. Hence the charge on the capacitor C_2 is

$$q_2 = C_2 V = 2.0 \times 10^{-6} \times 12 = 24 \times 10^{-6}\ \text{C}$$



Worked Exercises

10. A parallel-plate capacitor is fully charged to 8 μC when it is connected to 1 V battery. If the battery is removed and a dielectric of 2.5 is completely inserted between the plates, what is its capacitance and voltage across it.

Solution

To calculate the capacitance before inserting the material, we use the equation

$$C_{air} = \frac{q}{V} = \frac{8}{1} = 8\mu\text{F}$$

The capacitance of capacitor with a dielectric is

$$C = \kappa C_{air} = 2.5 \times 8 = 20\ \mu\text{F}$$

The voltage across the new capacitor is

$$V = \frac{q}{C_{air}} = \frac{8}{20} = 0.4\ \text{V}$$