


Current, Resistance, and Circuits

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CURRENT

- Moving charges create an **electric current** which is defined as **the rate of change of electric charges**.

$$i = \frac{dq}{dt}$$

The SI unit of current is the Ampere (A) which is 1 A = 1 C/s. By convention, the direction of the current is the direction of the flow of *positive* charges. The actual charge carriers are electrons; hence they move in the opposite direction to current.

- **Current density** is the amount of current per unit area. Mathematically it is

$$J = \frac{i}{A}$$

It is worthy to note that the current density is a vector quantity whereas current is a scalar quantity.

RESISTANCE

- When electrons move under the influence of an electric field, their randomly speed (drift speed) is defined as.

$$\vec{v}_d = \frac{1}{Ne}\vec{J}$$

where *(Ne)* is the electronic charge density (electronic charge per unit volume). It is quite important to note that the drift velocity has the same direction of electric field if the carrier charge is positive. Otherwise they have opposite directions.

- **Resistance** is the ability to resist the flow of current through a resistor (any wire). For a wire of length L, cross-sectional area of A, and resistivity ρ, the resistance R is

$$R = \frac{\rho L}{A}$$

The SI unit of resistance is Ohm (Ω). Its symbol in circuits is 

Ohm's Law

- When a potential V is applied across a resistor R, the electric current will be

$$i = \frac{V}{R} \quad \rightarrow \quad R = \frac{V}{i}$$

Here we note that larger resistance leads to smaller current. Also we find a new unit for resistance which is 1 Ω = 1 Volt (V)/Ampere (A).

- **Resistivity ρ(***its inverse* **conductivity σ)** can be now defined as

$$\rho = \frac{RA}{L} = \frac{\left(V/i\right)A}{L} = \frac{V}{L}\cdot\frac{A}{i} = E\cdot\frac{1}{J} = \frac{E}{J} \quad \rightarrow \quad \sigma = \frac{1}{\rho} = \frac{J}{E}$$

The unit of resistivity is **Ω.m**

POWER

- When a potential V is applied across a resistor R, the electric power transfer to the resistor (device) is

$$P = iV$$

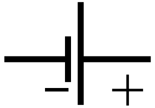
- The amount of power dissipated through the resistor (as heat) is

$$P = i^2 R = \frac{V^2}{R}$$

Electric Circuits

- An **electric circuit** is a loop comprised of elements like resistors and capacitors around which current flows.

- For current to continue to flow in a circuit, there must be an energy source such as a **battery**. There are two types of batteries; ideal and real. The former has zero internal resistance while the latter has non-zero resistance denoted by r.

- For each battery, it has an electromotive force (emf) which denoted as ε. The symbol of batteries in circuits is 

- **Simple circuit** consists of a battery and a resistor.

Kirchhoff's Rules

➢**Loop Rule:** The sum of the potential difference across all the elements around any closed circuit loop must be zero.

$$\sum V = 0$$

➢**Junction Rule:** The sum of currents entering a junction equals the sum of currents leaving the junction.

$$\sum i = 0$$

➢For a simple-loop circuit, the current is

$$i = \frac{\varepsilon}{R + r}$$

When the battery is ideal, the current is

$$i = \frac{\varepsilon}{R}$$

RESISTORS IN CIRCUITS

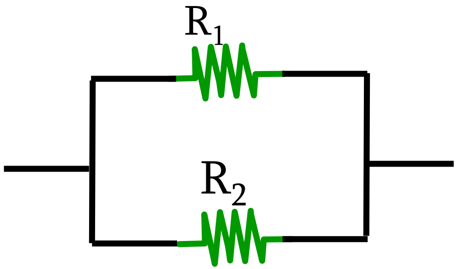
- Resistors can be joined in **series** and **parallel**.

- **Series** combinations leads to **constant current** passing through each element. The equivalent resistance is

$$R_{eq} = R_1 + R_2 + \cdots + R_n$$

- **Parallel** combinations leads to **constant voltage** across each element. The equivalent resistance is

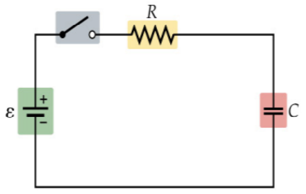
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}$$



RC Circuits

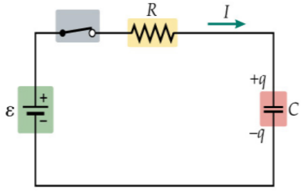
➢Circuits consisting of a resistor and a capacitor is said to be **RC circuits**. It is used to **charge/discharge** capacitors.

➢The use of the **resistor** in RC circuits is to **control the current** and hence the time of charging/discharging the capacitor.



(a) $t < 0$

➢ Before the switch is closed ($t = 0$) there is no current in the circuit and no charge on the capacitor.



(b) $t > 0$

➢ After the switch is closed ($t > 0$) current flows and the charge on the capacitor builds up over a finite time.

Charging a Capacitor

- During charging, the **charge** on the capacitor is

$$q = q_0\left(1 - e^{-t/RC}\right)$$

The **capacitive time constant** (RC) is denoted by τ where τ=RC.

- The **maximum charge** is

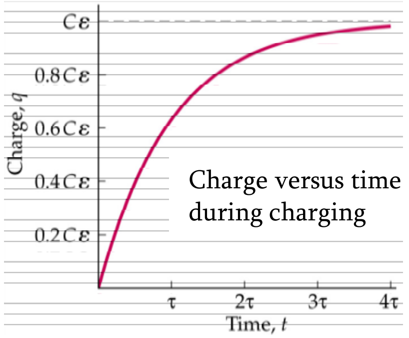
$$q_0 = CV$$

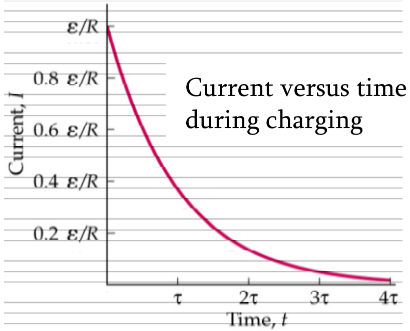
- The **current** is the rate of change of charge, which is

$$i = i_0 e^{-t/RC}$$

- The **maximum current** is

$$i_0 = \frac{V}{R}$$





Discharging a Capacitor

- During discharging, the charge on the capacitor is

$$q = q_0 e^{-t/RC}$$

- The current is the rate of change of charge, which is

$$i = -i_0 e^{-t/RC}$$

WORKED EXERCISES

1. A 4 Ω resistor is connected to a potential of 12 V. Calculate the current passing through the resistor.

Solution

It is well known from Ohm's law that the current is defined as

$$i = \frac{V}{R} = \frac{12}{4} = 3\,A$$

Worked Exercises

2. A cylindrical wire of radius 10 mm has a current of 2 A. Calculate he current density in the wire.

Solution

It is well known that the current density is

$$J = \frac{i}{A} = \frac{i}{\pi r^2} = \frac{2}{\pi (0.010)^2} = 6.4 \times 10^3 \, A/m^2$$

Worked Exercises

3. A wire of length 5 cm and cross-sectional area 2 mm^2 is connected to a potential of 12 V. If the current passing through the wire is 2 A, determine the **resistivity** of the wire.

Solution

It is well known that the resistivity is

$$\rho = \frac{RA}{L}$$

From Ohm's law, we find the resistance as

$$R = \frac{V}{i} = \frac{12}{2} = 6\Omega$$

Therefore the resistivity is

$$\rho = \frac{6 \times 2 \times 10^{-6}}{0.05} = 2.4 \times 10^{-4}\ \Omega.m$$

Worked Exercises

4. The power dissipation rate through a 5 Ω resistor is 3.2 W. Determine the potential difference across the resistor.

Solution

It is well known that the power dissipated through a resistor is

$$P = \frac{V^2}{R}$$

Therefore the potential across the resistor is

$$V = \sqrt{PR} = \sqrt{3.2 \times 5} = 4V$$

Worked Exercises

5. The electric field inside a cylindrical wire of radius 1.2 mm is 0.1 V/m. If the current in the wire is measured to be 16 A, calculate the conductivity of the wire.

Solution

It is known that the conductivity is

$$\sigma = \frac{J}{E}$$

But the current density is

$$J = \frac{i}{A} = \frac{16}{\pi\left(1.2\times10^{-3}\right)^2} = 3.54\times10^{6}\, A/m^2$$

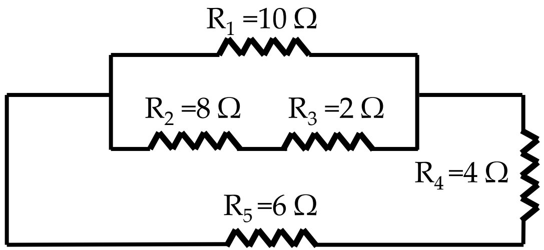
Therefore the conductivity is

$$\sigma = \frac{3.54\times10^{6}}{0.1} = 3.54\times10^{7}\,(\Omega.m)^{-1}$$

Worked Exercises

6. As shown in the figure, calculate the equivalent resistance.

Solution



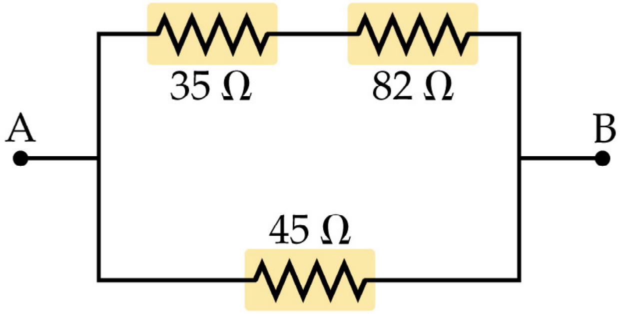
The resistors 2 and 3 are in series, their equivalent resistance R_{23} is 10 Ω. These two resistors are in parallel with resistor 1, so the equivalent resistance R_{123} is 5 Ω. These resistors are in series with the rest. Therefore the total resistance is

$$R_{12345} = 5+4+6=15\ \Omega$$

Worked Exercises

7. As shown in the figure, calculate the equivalent resistance between the point A and B.

Solution



The equivalent resistance of the upper side is

$$R_{up} = 35+82 = 117\ \Omega$$

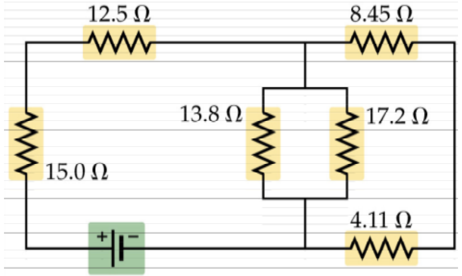
Thus the total resistance is

$$R_{eq} = 117\times45/(117+45) = 32.5\ \Omega$$

Worked Exercises

8. The current in the 13.8 Ω resistor is 0.750 A. Find the current passing through the 17.2 Ω resistor.

Solution



The 13.8 Ω and 17.2 Ω resistors are in parallel. Therefore the potential across each is similar. This is

$$V = iR = 0.75 \times 13.8 = 10.35 \text{ V}$$

Hence the current passing through the 17.2 Ω resistor is

$$i = V/R = 10.35/17.2 = 0.602 \text{ A}$$

Worked Exercises

9. A capacitor of 4 μC initially uncharged is connected with a battery of 12 V and a 10 MΩ resistor. Calculate the maximum charge on the capacitor and also the maximum current. What is the capacitive time constant?

Solution

The maximum charge is

$$q_0 = CV = 4 \times 12 = 48\ \mu C$$

The maximum current is

$$i_0 = \frac{V}{R} = \frac{12}{10} = 1.2\ \mu A$$

The maximum charge is

$$\tau = RC = 10 \times 10^6 \times 4 \times 10^{-6} = 40\ s$$