

Workshop Solutions to Sections 2.1 and 2.2

<p>1) Find the domain of the function $f(x) = 9 - x^2$.</p> <p><u>Solution:</u> Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p> <p>Note: The domain of any polynomial is \mathbb{R}.</p>	<p>2) Find the range of the function $f(x) = 9 - x^2$.</p> <p><u>Solution:</u> $R_f = (-\infty, 9]$</p>
<p>3) Find the domain of the function $f(x) = 6 - 2x$.</p> <p><u>Solution:</u> Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>	<p>4) Find the range of the function $f(x) = 6 - 2x$.</p> <p><u>Solution:</u> Since $f(x)$ is a polynomial of degree one (<i>i. e.</i> is of an odd degree), then $R_f = \mathbb{R} = (-\infty, \infty)$</p>
<p>5) Find the domain of the function $f(x) = x^2 - 2x - 3$.</p> <p><u>Solution:</u> Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>	<p>6) Find the domain of the function $f(x) = 1 + 2x^3 - x^5$.</p> <p><u>Solution:</u> Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>
<p>7) Find the domain of the function $f(x) = 5$.</p> <p><u>Solution:</u> Since $f(x)$ is a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>	<p>8) Find the range of the function $f(x) = 5$.</p> <p><u>Solution:</u> $R_f = \{5\}$</p>
<p>9) Find the domain of the function $f(x) = x - 1$.</p> <p><u>Solution:</u> Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p> <p>Note: The domain of an absolute value of any polynomial is \mathbb{R}.</p>	<p>10) Find the domain of the function $f(x) = x + 5$.</p> <p><u>Solution:</u> Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>
<p>11) Find the domain of the function $f(x) = x$.</p> <p><u>Solution:</u> Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>	<p>12) Find the range of the function $f(x) = x$.</p> <p><u>Solution:</u> $R_f = [0, \infty)$</p> <p>Note: The range of an absolute value of any polynomial is always $[0, \infty)$.</p>
<p>13) Find the domain of the function $f(x) = 3x - 6$.</p> <p><u>Solution:</u> Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>	<p>14) Find the domain of the function $f(x) = 9 - 3x$.</p> <p><u>Solution:</u> Since $f(x)$ is an absolute value of a polynomial, then $D_f = \mathbb{R} = (-\infty, \infty)$</p>
<p>15) Find the domain of the function</p> $f(x) = \frac{x + 2}{x - 3}$ <p><u>Solution:</u> $f(x)$ is defined when $x - 3 \neq 0 \Rightarrow x \neq 3$. So, $D_f = \mathbb{R} \setminus \{3\} = (-\infty, 3) \cup (3, \infty)$</p>	<p>16) Find the domain of the function</p> $f(x) = \frac{x - 2}{x + 3}$ <p><u>Solution:</u> $f(x)$ is defined when $x + 3 \neq 0 \Rightarrow x \neq -3$. So, $D_f = \mathbb{R} \setminus \{-3\} = (-\infty, -3) \cup (-3, \infty)$</p>

<p>17) Find the domain of the function</p> $f(x) = \frac{x+2}{x^2-9}$ <p><u>Solution:</u> $f(x)$ is defined when $x^2 - 9 \neq 0 \Rightarrow x^2 \neq 9 \Rightarrow x \neq \pm 3$. So, $D_f = \mathbb{R} \setminus \{-3, 3\} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty)$</p>	<p>18) Find the domain of the function</p> $f(x) = \frac{x+2}{x^2-5x+6}$ <p><u>Solution:</u> $f(x)$ is defined when $x^2 - 5x + 6 \neq 0$ $\Rightarrow (x-2)(x-3) \neq 0 \Rightarrow x \neq 2$ or $x \neq 3$. So, $D_f = \mathbb{R} \setminus \{2, 3\} = (-\infty, 2) \cup (2, 3) \cup (3, \infty)$</p>
<p>19) Find the domain of the function</p> $f(x) = \frac{x+2}{x^2-x-6}$ <p><u>Solution:</u> $f(x)$ is defined when $x^2 - x - 6 \neq 0$ $\Rightarrow (x+2)(x-3) \neq 0 \Rightarrow x \neq -2$ or $x \neq 3$. So, $D_f = \mathbb{R} \setminus \{-2, 3\} = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$</p>	<p>20) Find the domain of the function</p> $f(x) = \frac{x+2}{x^2+9}$ <p><u>Solution:</u> $f(x)$ is defined when $x^2 + 9 \neq 0$ but for any value x the denominator $x^2 + 9$ cannot be 0. So, $D_f = \mathbb{R} = (-\infty, \infty)$</p>
<p>21) Find the domain of the function</p> $f(x) = \sqrt[3]{x-3}$ <p><u>Solution:</u> $D_f = \mathbb{R} = (-\infty, \infty)$</p> <p>Note: The domain of an odd root of any polynomial is \mathbb{R}.</p>	<p>22) Find the domain of the function</p> $f(x) = \sqrt{x-3}$ <p><u>Solution:</u> $f(x)$ is defined when $x - 3 \geq 0 \Rightarrow x \geq 3$ because $f(x)$ is an even root. So, $D_f = [3, \infty)$</p>
<p>23) Find the domain of the function</p> $f(x) = \sqrt{3-x}$ <p><u>Solution:</u> $f(x)$ is defined when $3 - x \geq 0 \Rightarrow -x \geq -3 \Rightarrow x \leq 3$ because $f(x)$ is an even root. So, $D_f = (-\infty, 3]$</p>	<p>24) Find the domain of the function</p> $f(x) = \sqrt{x+3}$ <p><u>Solution:</u> $f(x)$ is defined when $x + 3 \geq 0 \Rightarrow x \geq -3$ because $f(x)$ is an even root. So, $D_f = [-3, \infty)$</p>
<p>25) Find the domain of the function</p> $f(x) = \sqrt{-x}$ <p><u>Solution:</u> $f(x)$ is defined when $-x \geq 0 \Rightarrow x \leq 0$ because $f(x)$ is an even root. So, $D_f = (-\infty, 0]$</p>	<p>26) Find the range of the function</p> $f(x) = \sqrt{-x}$ <p><u>Solution:</u> $R_f = [0, \infty)$</p> <p>Note: The range of an even root is always ≥ 0.</p>
<p>27) Find the domain of the function</p> $f(x) = \sqrt{9-x^2}$ <p><u>Solution:</u> $f(x)$ is defined when $9 - x^2 \geq 0 \Rightarrow -x^2 \geq -9 \Rightarrow x^2 \leq 9 \Rightarrow \sqrt{x^2} \leq \sqrt{9} \Rightarrow x \leq 3 \Rightarrow -3 \leq x \leq 3$. So, $D_f = [-3, 3]$</p>	<p>28) Find the domain of the function</p> $f(x) = \frac{x+2}{\sqrt{x-3}}$ <p><u>Solution:</u> $f(x)$ is defined when $x - 3 > 0 \Rightarrow x > 3$. So, $D_f = (3, \infty)$</p>
<p>29) Find the domain of the function</p> $f(x) = \frac{x+2}{\sqrt{9-x^2}}$ <p><u>Solution:</u> $f(x)$ is defined when $9 - x^2 > 0 \Rightarrow -x^2 > -9$ $\Rightarrow x^2 < 9 \Rightarrow \sqrt{x^2} < \sqrt{9} \Rightarrow x < 3 \Rightarrow -3 < x < 3$. So, $D_f = (-3, 3)$</p>	<p>30) Find the domain of the function</p> $f(x) = \sqrt{x^2-9}$ <p><u>Solution:</u> $f(x)$ is defined when $x^2 - 9 \geq 0 \Rightarrow x^2 \geq 9$ $\Rightarrow \sqrt{x^2} \geq \sqrt{9} \Rightarrow x \geq 3 \Rightarrow x \geq 3$ or $x \leq -3$. So, $D_f = (-\infty, -3] \cup [3, \infty)$</p>

<p>31) Find the range of the function</p> $f(x) = \sqrt{x^2 - 9}$ <p><u>Solution:</u></p> $R_f = [0, \infty)$	<p>32) Find the domain of the function</p> $f(x) = \frac{x + 2}{\sqrt{x^2 - 9}}$ <p><u>Solution:</u></p> <p>$f(x)$ is defined when $x^2 - 9 > 0 \Rightarrow x^2 > 9$ $\Rightarrow \sqrt{x^2} > \sqrt{9} \Rightarrow x > 3 \Rightarrow x > 3$ or $x < -3$.</p> <p>So,</p> $D_f = (-\infty, -3) \cup (3, \infty)$
<p>33) Find the domain of the function</p> $f(x) = \sqrt{9 + x^2}$ <p><u>Solution:</u></p> <p>$f(x)$ is defined when $9 + x^2 \geq 0$ but it is always true for any value x. So,</p> $D_f = \mathbb{R}$	<p>34) Find the domain of the function</p> $f(x) = \sqrt[4]{x^2 - 25}$ <p><u>Solution:</u></p> <p>$f(x)$ is defined when $x^2 - 25 \geq 0 \Rightarrow x^2 \geq 25$ $\Rightarrow \sqrt{x^2} \geq \sqrt{25} \Rightarrow x \geq 5 \Rightarrow x \geq 5$ or $x \leq -5$.</p> <p>So,</p> $D_f = (-\infty, -5] \cup [5, \infty)$
<p>35) Find the domain of the function</p> $f(x) = \sqrt[6]{16 - x^2}$ <p><u>Solution:</u></p> <p>$f(x)$ is defined when $16 - x^2 \geq 0 \Rightarrow -x^2 \geq -16 \Rightarrow x^2 \leq 16 \Rightarrow \sqrt{x^2} \leq \sqrt{16} \Rightarrow x \leq 4 \Rightarrow -4 \leq x \leq 4$.</p> <p>So,</p> $D_f = [-4, 4]$	<p>36) Find the range of the function</p> $f(x) = \sqrt{16 - x^2}$ <p><u>Solution:</u></p> <p>We know that $f(x)$ is defined when $16 - x^2 \geq 0$ $\Rightarrow -x^2 \geq -16 \Rightarrow x^2 \leq 16 \Rightarrow \sqrt{x^2} \leq \sqrt{16}$ $\Rightarrow x \leq 4 \Rightarrow -4 \leq x \leq 4$. So,</p> $D_f = [-4, 4]$ <p>Using D_f we find the outputs vary from 0 to 4. Hence,</p> $R_f = [0, 4]$
<p>37) Find the domain of the function</p> $f(x) = \frac{x + x }{x}$ <p><u>Solution:</u></p> <p>$f(x)$ is defined when $x \neq 0$. So,</p> $D_f = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty)$	<p>38) Find the domain of the function</p> $f(x) = \begin{cases} -\frac{1}{x}, & x < 0 \\ x, & x \geq 0 \end{cases}$ <p><u>Solution:</u></p> <p>It is clear from the definition of the function $f(x)$ that</p> $D_f = \mathbb{R} = (-\infty, \infty)$
<p>39) Find the domain of the function</p> $f(x) = \frac{2 - \sqrt{x}}{\sqrt{x^2 + 1}}$ <p><u>Solution:</u></p> <p>$f(x)$ is defined when</p> <ol style="list-style-type: none"> $x \geq 0 \Rightarrow D_{\sqrt{x}} = [0, \infty)$ $x^2 + 1 > 0$ but this is always true for all x $\Rightarrow D_{\sqrt{x^2 + 1}} = \mathbb{R}$. <p>Hence,</p> $D_f = D_{\sqrt{x}} \cap D_{\sqrt{x^2 + 1}} = [0, \infty) \cap \mathbb{R} = [0, \infty)$	<p>40) Find the domain of the function</p> $f(x) = \sqrt{x - 1} + \sqrt{x + 3}$ <p><u>Solution:</u></p> <p>$f(x)$ is defined when</p> <ol style="list-style-type: none"> $x - 1 \geq 0 \Rightarrow x \geq 1 \Rightarrow D_{\sqrt{x-1}} = [1, \infty)$ $x + 3 \geq 0 \Rightarrow x \geq -3 \Rightarrow D_{\sqrt{x+3}} = [-3, \infty)$ <p>Hence,</p> $D_f = D_{\sqrt{x-1}} \cap D_{\sqrt{x+3}} = [1, \infty) \cap [-3, \infty) = [1, \infty)$
<p>41) The function $f(x) = 3x^4 + x^2 + 1$ is a polynomial function.</p>	<p>42) The function $f(x) = 5x^3 + x^2 + 7$ is a cubic function.</p>
<p>43) The function $f(x) = -3x^2 + 7$ is a quadratic function.</p>	<p>44) The function $f(x) = 2x + 3$ is a linear function.</p>
<p>45) The function $f(x) = x^7$ is a power function.</p>	<p>46) The function $f(x) = \frac{2x+3}{x^2-1}$ is a rational function.</p>
<p>47) The function $f(x) = \frac{x-3}{x+2}$ is a rational function and we can say it is an algebraic function as well.</p>	<p>48) The function $f(x) = \sin x$ is a trigonometric function.</p>

49) The function $f(x) = e^x$ is a natural exponential function.	50) The function $f(x) = 3^x$ is a general exponential function.
51) The function $f(x) = x^2 + \sqrt{x-2}$ is an algebraic function.	52) The function $f(x) = -3$ is a constant function.
53) The function $f(x) = \log_3 x$ is a general logarithmic function.	54) The function $f(x) = \ln x$ is a natural logarithmic function.
55) The function $f(x) = 3x^4 + x^2 + 1$ is <u>Solution:</u> $f(-x) = 3(-x)^4 + (-x)^2 + 1 = 3x^4 + x^2 + 1 = f(x)$ Hence, $f(x)$ is an even function.	56) The function $f(x) = 9 - x^2$ is <u>Solution:</u> $f(-x) = 9 - (-x)^2 = 9 - x^2 = f(x)$ Hence, $f(x)$ is an even function.
57) The function $f(x) = x^5 - x$ is <u>Solution:</u> $f(-x) = (-x)^5 - (-x) = -x^5 + x$ $= -(x^5 - x) = -f(x)$ Hence, $f(x)$ is an odd function.	58) The function $f(x) = 2 - \sqrt[5]{x}$ is <u>Solution:</u> $f(-x) = 2 - \sqrt[5]{(-x)} = 2 - \sqrt[5]{-x} = 2 + \sqrt[5]{x}$ $= -(-2 - \sqrt[5]{x})$ Hence, $f(x)$ is neither even nor odd.
59) The function $f(x) = 3x + \frac{2}{\sqrt{x^2+9}}$ is <u>Solution:</u> $f(-x) = 3(-x) + \frac{2}{\sqrt{(-x)^2+9}} = -3x + \frac{2}{\sqrt{x^2+9}}$ $= -\left(3x - \frac{2}{\sqrt{x^2+9}}\right)$ Hence, $f(x)$ is neither even nor odd.	60) The function $f(x) = \frac{3}{\sqrt{x^2+9}}$ is <u>Solution:</u> $f(-x) = \frac{3}{\sqrt{(-x)^2+9}} = \frac{3}{\sqrt{x^2+9}} = f(x)$ Hence, $f(x)$ is an even function.
61) The function $f(x) = \sqrt{4+x^2}$ is <u>Solution:</u> $f(-x) = \sqrt{4+(-x)^2} = \sqrt{4+x^2} = f(x)$ Hence, $f(x)$ is an even function.	62) The function $f(x) = 3$ is <u>Solution:</u> Since the graph of the constant function 3 is symmetric about the y -axis, then $f(x)$ is an even function.
63) The function $f(x) = \frac{9-x^2}{x-2}$ is <u>Solution:</u> $f(-x) = \frac{9-(-x)^2}{(-x)-2} = \frac{9-x^2}{-x-2}$ $= -\left(\frac{9-x^2}{x+2}\right)$ Hence, $f(x)$ is neither even nor odd.	64) The function $f(x) = \frac{x^2-4}{x^2+1}$ is <u>Solution:</u> $f(-x) = \frac{(-x)^2-4}{(-x)^2+1} = \frac{x^2-4}{x^2+1} = f(x)$ Hence, $f(x)$ is an even function.
65) The function $f(x) = 3 x $ is <u>Solution:</u> $f(-x) = 3 (-x) = 3 x = f(x)$ Hence, $f(x)$ is an even function.	66) The function $f(x) = x^{-2}$ is <u>Solution:</u> $f(x) = x^{-2} = \frac{1}{x^2}$ $f(-x) = \frac{1}{(-x)^2} = \frac{1}{x^2} = f(x)$ Hence, $f(x)$ is an even function.

<p>67) The function $f(x) = x^3 - 2x + 5$ is</p> <p><u>Solution:</u></p> $f(-x) = (-x)^3 - 2(-x) + 5 = -x^3 + 2x + 5$ $= -(x^3 - 2x - 5)$ <p>Hence, $f(x)$ is neither even nor odd.</p>	<p>68) The function $f(x) = \sqrt[3]{x^5} - x^3 + x$ is</p> <p><u>Solution:</u></p> $f(-x) = \sqrt[3]{(-x)^5} - (-x)^3 + (-x) = -\sqrt[3]{x^5} + x^3 - x$ $= -(\sqrt[3]{x^5} - x^3 + x) = -f(x)$ <p>Hence, $f(x)$ is an odd function.</p>
<p>69) The function $f(x) = 7$ is</p> <p><u>Solution:</u></p> <p>Since the graph of the constant function 7 is symmetric about the y-axis, then</p> <p>$f(x)$ is an even function.</p>	<p>70) The function $f(x) = \frac{x^3-4}{x^3+1}$ is</p> <p><u>Solution:</u></p> $f(-x) = \frac{(-x)^3-4}{(-x)^3+1} = \frac{-x^3-4}{-x^3+1} = -\frac{x^3+4}{-x^3+1}$ <p>Hence, $f(x)$ is neither even nor odd.</p>
<p>71) The function $f(x) = \frac{x^2-1}{x^3+3}$ is</p> <p><u>Solution:</u></p> $f(-x) = \frac{(-x)^2-1}{(-x)^3+3} = \frac{x^2-1}{-x^3+3} = -\frac{x^2-1}{x^3-3}$ <p>Hence, $f(x)$ is neither even nor odd.</p>	<p>72) The function $f(x) = x^6 - 4x^2 + 1$ is</p> <p><u>Solution:</u></p> $f(-x) = (-x)^6 - 4(-x)^2 + 1 = x^6 - 4x^2 + 1 = f(x)$ <p>Hence, $f(x)$ is an even function.</p>
<p>73) The function $f(x) = x^2$ is increasing on $(0, \infty)$.</p>	<p>74) The function $f(x) = x^2$ is decreasing on $(-\infty, 0)$.</p>
<p>75) The function $f(x) = x^3$ is increasing on $(-\infty, \infty)$.</p>	<p>76) The function $f(x) = x^3$ is not decreasing at all.</p>
<p>77) The function $f(x) = \sqrt{x}$ is increasing on $(0, \infty)$.</p>	<p>78) The function $f(x) = \sqrt{x}$ is not decreasing at all.</p>
<p>79) The function $f(x) = \frac{1}{x}$ is not increasing at all.</p>	<p>80) The function $f(x) = \frac{1}{x}$ is decreasing on $(-\infty, \infty)$.</p>