

OPTICS

PHYS 311

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INTERFERENCE

INTRODUCTION

The effect of enhancement or diminution of light waves due to superposition are called *interference*.

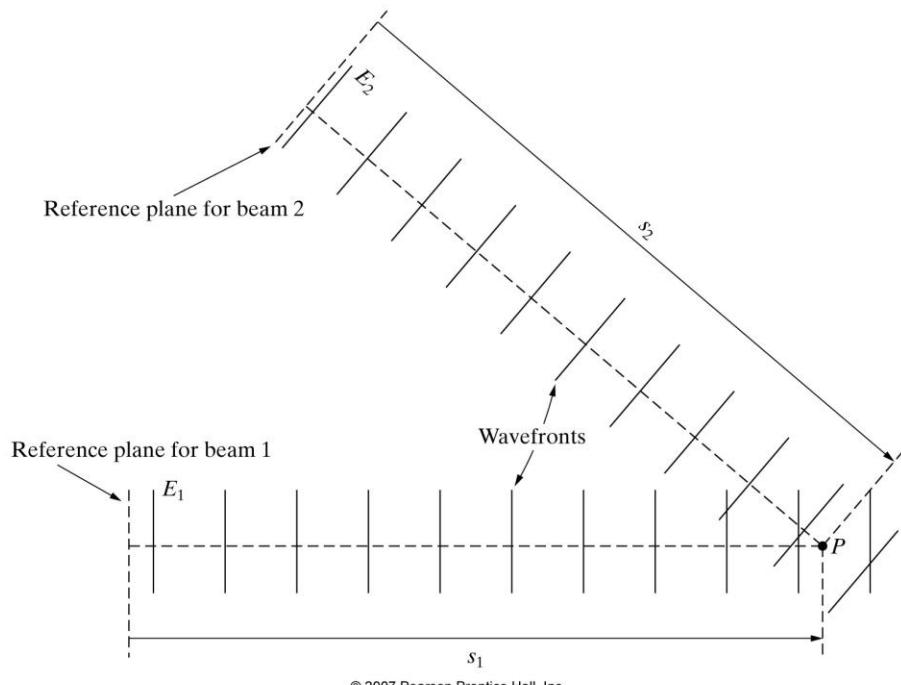
When conditions of *constructive interference* and *destructive interference* alternate in a spatial display, the interference is said to produce a pattern of *fringes*.

Same condition may lead to enhancement of one visible wavelength interval or color at the expenses of the others, in which case interference colors are produced ← oil slicks and soap films

TWO-BEAM INTERFERENCE

Lets us consider the interference of two plane waves of the same frequency

$$E_1 = E_{01} \cos(k s_1 - \omega t + \varphi_1) \quad E_2 = E_{02} \cos(k s_2 - \omega t + \varphi_2)$$



$$E_p = E_1 + E_2$$

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TWO-BEAMS INTERFERENCE

$$\mathbf{E}_p = \mathbf{E}_1 + \mathbf{E}_2$$

The irradiance at P measures the time average of the square of the wave amplitude and is given by

$$I = \varepsilon_0 c \langle \vec{E}_p^2 \rangle = \varepsilon_0 c \langle \vec{E}_p \cdot \vec{E}_p \rangle$$

$$\begin{aligned} I &= \varepsilon_0 c \langle (\vec{E}_1 + \vec{E}_2)^2 \rangle \\ &= \varepsilon_0 c \langle (\vec{E}_1 + \vec{E}_2) \cdot (\vec{E}_1 + \vec{E}_2) \rangle \\ &= \varepsilon_0 c \langle (\vec{E}_1 \cdot \vec{E}_1) + (\vec{E}_2 \cdot \vec{E}_2) + 2(\vec{E}_1 \cdot \vec{E}_2) \rangle \\ &= \varepsilon_0 c \langle \vec{E}_1^2 \rangle + \varepsilon_0 c \langle \vec{E}_2^2 \rangle + 2\varepsilon_0 c \langle \vec{E}_1 \cdot \vec{E}_2 \rangle \\ &= I_1 + I_2 + I_{12} \end{aligned}$$



TWO-BEAM INTERFERENCE

$$I = I_1 + I_2 + I_{12}$$

If light behaved like classical particles, then...

$$I = I_1 + I_2$$

If light behaved like waves, then...

$$I_{12}$$

How can we evaluate the interference term?

$$I_{12} = 2\epsilon_0 c \langle \vec{E}_1 \cdot \vec{E}_2 \rangle$$

$$\vec{E}_1 \cdot \vec{E}_2 = \vec{E}_{01} \cdot \vec{E}_{02} \cos(\mathbf{k}\mathbf{s}_1 - \omega t + \phi_1) \cos(\mathbf{k}\mathbf{s}_2 - \omega t + \phi_2)$$

Let's take...

$$\alpha \equiv \mathbf{k}\mathbf{s}_1 + \phi_1 \quad , \quad \beta \equiv \mathbf{k}\mathbf{s}_2 + \phi_2$$

TWO-BEAM INTERFERENCE

$$2\vec{E}_1 \cdot \vec{E}_2 = 2\vec{E}_{01} \cdot \vec{E}_{02} \cos(\alpha - \omega t) \cos(\beta - \omega t)$$

Since..

$$2\cos(A)\cos(B) = \cos(A+B) + \cos(B-A)$$

Then...

$$2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle = \vec{E}_{01} \cdot \vec{E}_{02} [\langle \cos(\alpha + \beta - 2\omega t) \rangle + \langle \cos(\beta - \alpha) \rangle]$$

The time average taken over a rapidly oscillating cosine function is zero

$$\begin{aligned} 2\langle \vec{E}_1 \cdot \vec{E}_2 \rangle &= \vec{E}_{01} \cdot \vec{E}_{02} \langle \cos(\beta - \alpha) \rangle \\ &= \vec{E}_{01} \cdot \vec{E}_{02} \langle \cos(k(s_2 - s_1) + \phi_2 - \phi_1) \rangle \\ &\equiv \vec{E}_{01} \cdot \vec{E}_{02} \langle \cos \delta \rangle \end{aligned}$$

TWO-BEAM INTERFERENCE

Where...

$$\delta = k(s_2 - s_1) + \phi_2 - \phi_1$$

Then...

$$I_{12} = \varepsilon_o c \vec{E}_{01} \cdot \vec{E}_{02} \langle \cos \delta \rangle$$

What are the irradiance terms I_1 and I_2 ??

$$I_1 = \varepsilon_o c \langle E_1 \cdot E_1 \rangle = \varepsilon_o c E_{01}^2 \langle \cos^2(\alpha - \omega t) \rangle = \frac{1}{2} \varepsilon_o c E_{01}^2$$

$$I_2 = \varepsilon_o c \langle E_2 \cdot E_2 \rangle = \varepsilon_o c E_{02}^2 \langle \cos^2(\beta - \omega t) \rangle = \frac{1}{2} \varepsilon_o c E_{02}^2$$

$$I_{12} = 2\sqrt{I_1 I_2} \langle \cos \delta \rangle$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \langle \cos \delta \rangle$$

TWO-BEAM INTERFERENCE

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \langle \cos \delta \rangle$$



$$2\sqrt{I_1 I_2} \langle \cos(k(s_2 - s_1) + \phi_2(t) - \phi_1(t)) \rangle$$

Interference of mutually incoherent beams..

$$\langle \cos \delta \rangle_T = 0$$

$$I = I_1 + I_2$$

Interference of mutually coherent beams..

$$2\sqrt{I_1 I_2} \langle \cos(k(s_2 - s_1)) \rangle$$

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

TWO-BEAM INTERFERENCE

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$

$$\begin{aligned} I_{\max} &= I_1 + I_2 + 2\sqrt{I_1 I_2} \\ &= (\sqrt{I_1} + \sqrt{I_2})^2 \end{aligned}$$

$$\begin{aligned} I_{\min} &= I_1 + I_2 - 2\sqrt{I_1 I_2} \\ &= (\sqrt{I_1} - \sqrt{I_2})^2 \end{aligned}$$

$$\delta = 2m\pi \quad m = 0, \pm 1, \pm 2, \dots$$

$$\delta = (2m+1)\pi \quad m = 0, \pm 1, \pm 2, \dots$$

in phase

180° out of phase

**TOTAL CONSTRUCTIVE
INTERFERENCE**

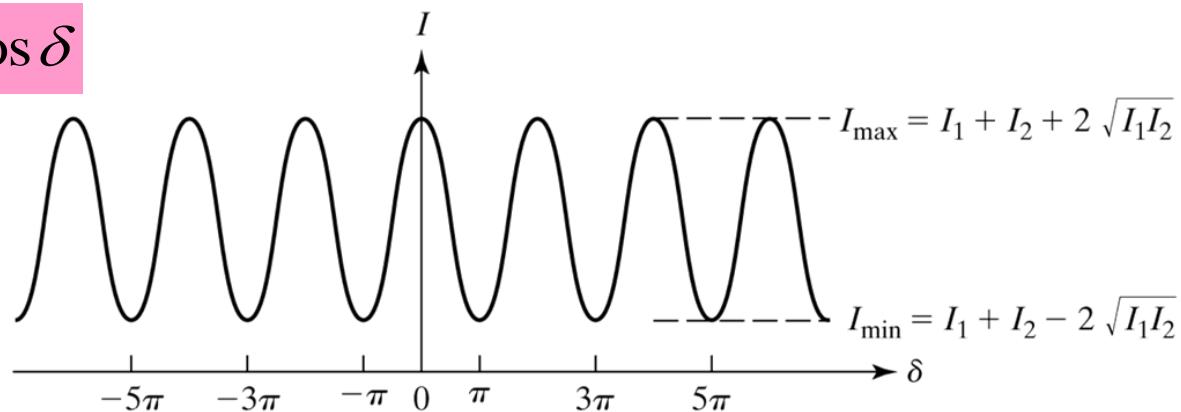
**TOTAL DESTRUCTIVE
INTERFERENCE**

TWO-BEAM INTERFERENCE

$\delta = 1$	$0 < \cos \delta < 1$	$\delta = \pi/2$ $\cos \delta = 0$	$0 > \cos \delta > -1$	$\delta = -1$
$I = I_{\max}$	$I_1 + I_2 < I < I_{\max}$	$I = I_1 + I_2$	$I_1 + I_2 > I > I_{\min}$	$I = I_{\min}$
<i>in phase</i>	<i>out of phase</i>	<i>90° out of phase</i>	<i>out of phase</i>	<i>180° out of phase</i>
<i>TOTAL CONSTRUCTIVE INTERFERENCE</i>	<i>CONSTRUCTIVE INTERFERENCE</i>		<i>DESTRUCTIVE INTERFERENCE</i>	<i>TOTAL DESTRUCTIVE INTERFERENCE</i>

TWO-BEAM INTERREFERENCE

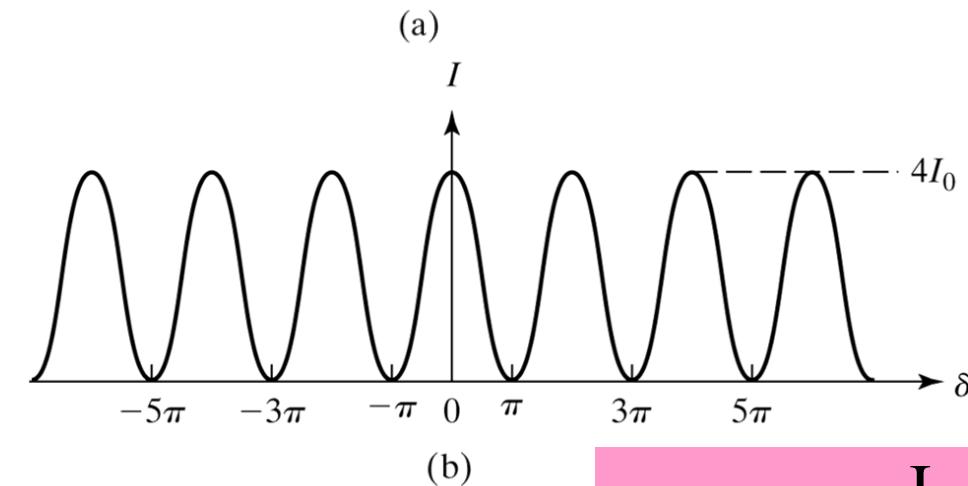
$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \delta$$



When $I_1 = I_2 = I_0$

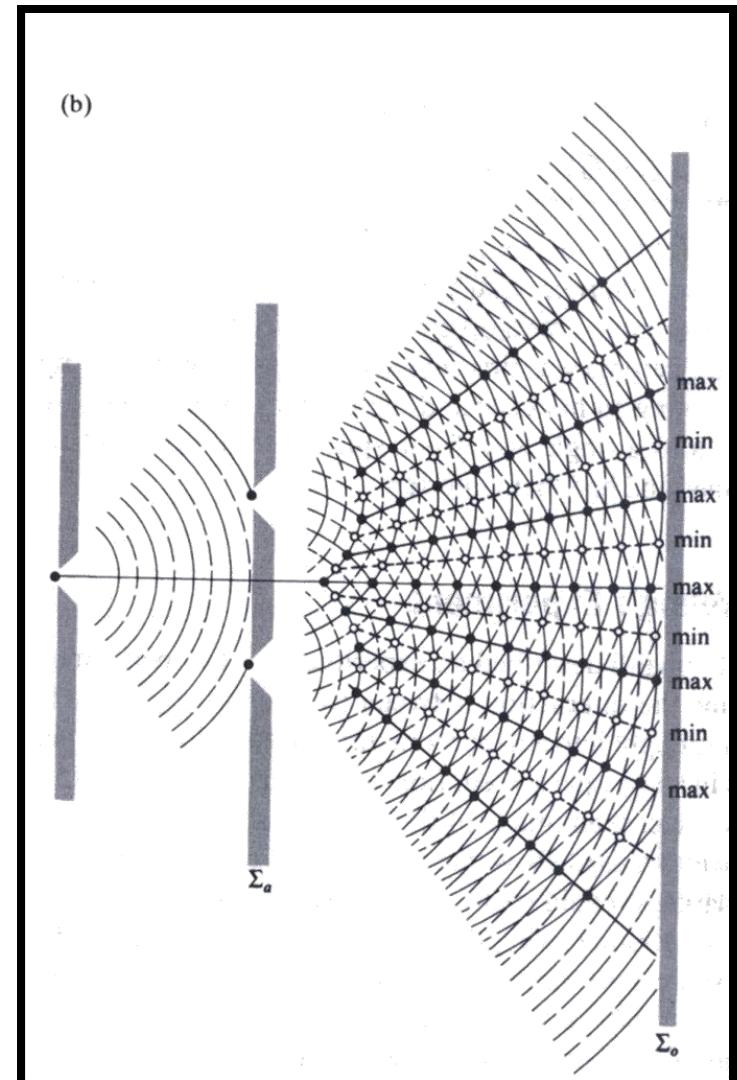
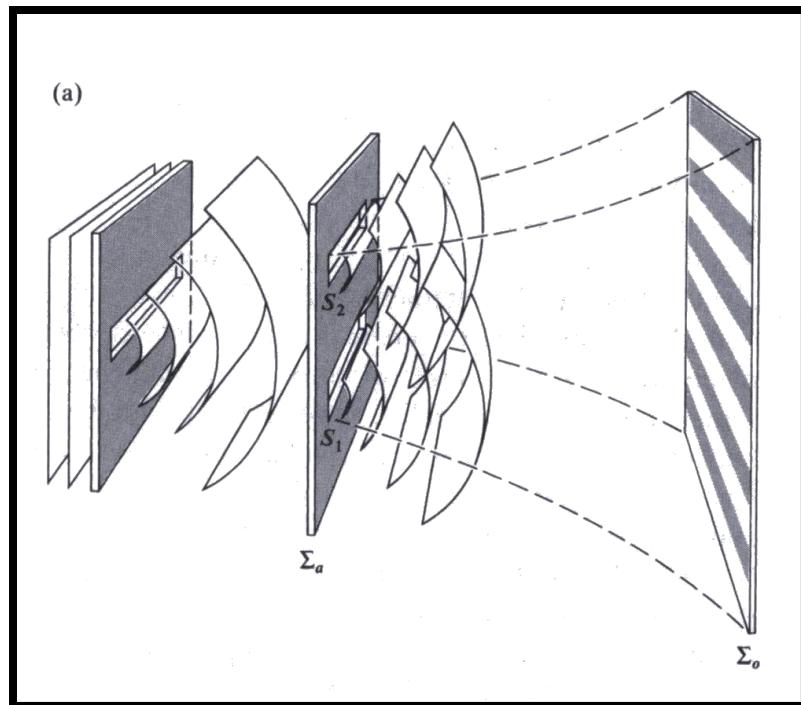
$$I = 4I_0 \cos^2 \frac{\delta}{2}$$

$$I_{\max} = 4I_0 \quad I_{\min} = 0$$

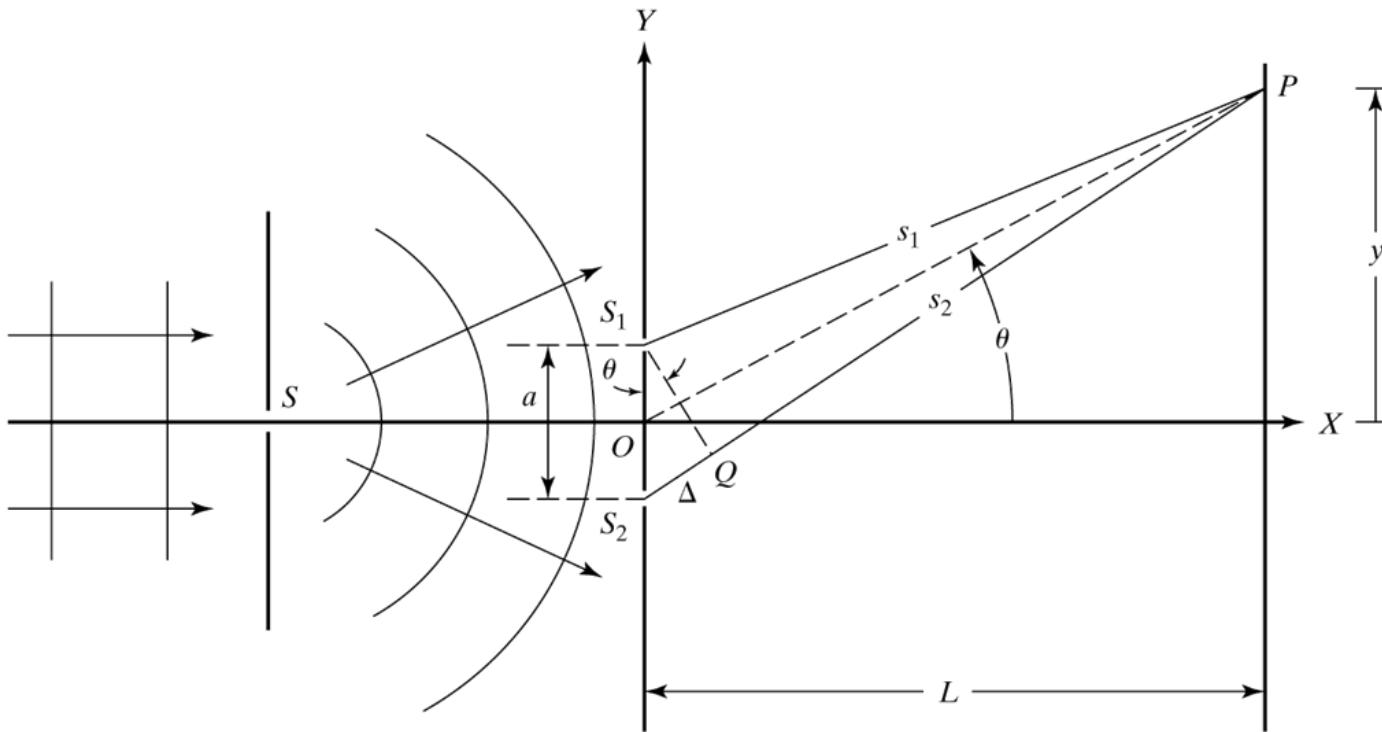


$$\text{visibility} = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

YOUNG'S DOUBLE-SLIT EXPERIMENT



YOUNG'S DOUBLE-SLIT EXPERIMENT



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constructive interference →

$$S_1 - S_2 = \Delta = m\lambda \cong a \sin \theta$$

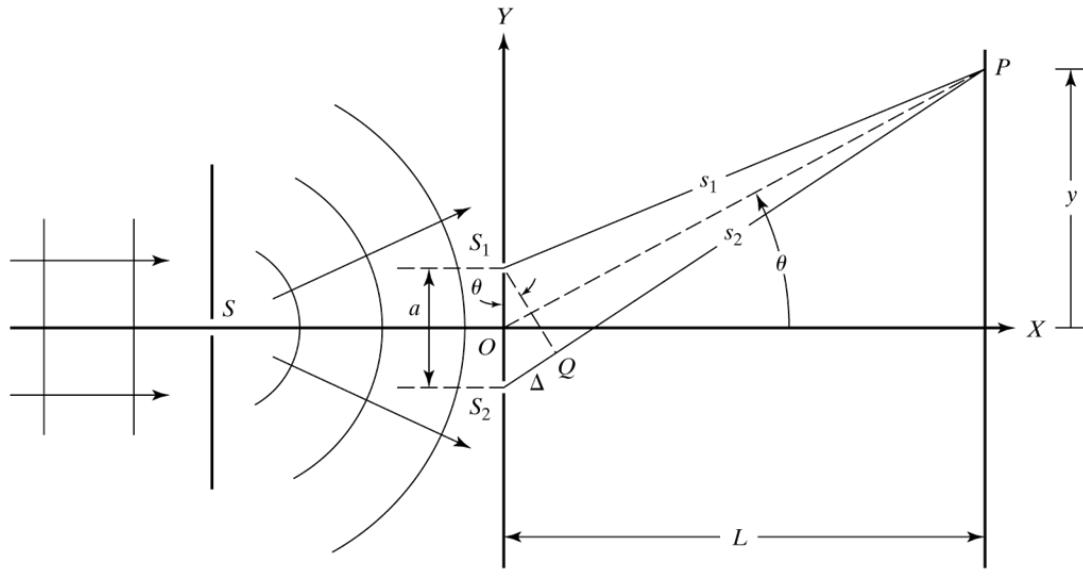
destructive interference →

$$S_1 - S_2 = \Delta = \left(m + \frac{1}{2} \right) \lambda \cong a \sin \theta$$

YOUNG'S DOUBLE-SLIT EXPERIMENT

The relation between path and phase difference..

$$\delta = k(s_2 - s_1) = \frac{2\pi}{\lambda} \Delta$$



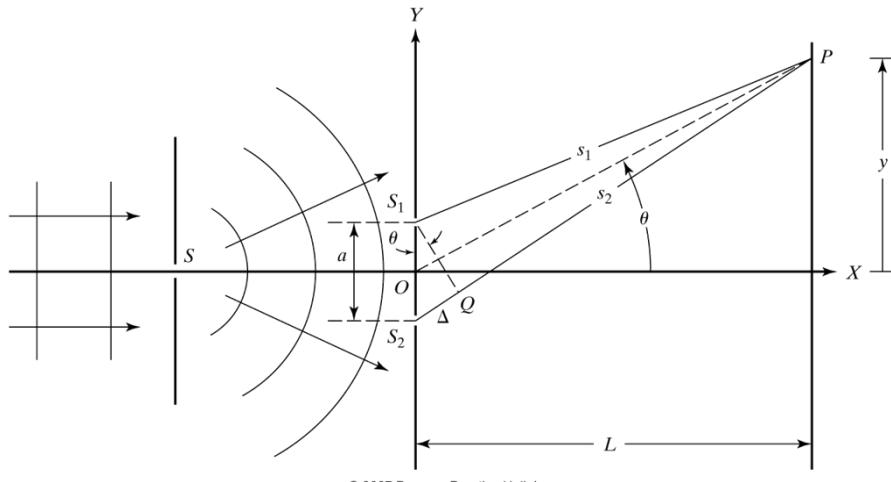
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$$I = 4I_0 \cos^2 \frac{\delta}{2} = 4I_0 \cos^2 \left(\frac{\pi \Delta}{\lambda} \right) = 4I_0 \cos^2 \left(\frac{\pi a \sin \theta}{\lambda} \right)$$

$$I = 4I_0 \cos^2 \left(\frac{\pi a y}{\lambda L} \right)$$

Small angle relation:
 $y \ll L \rightarrow \sin \theta \approx \tan \theta \approx y/L$

YOUNG'S DOUBLE-SLIT EXPERIMENT



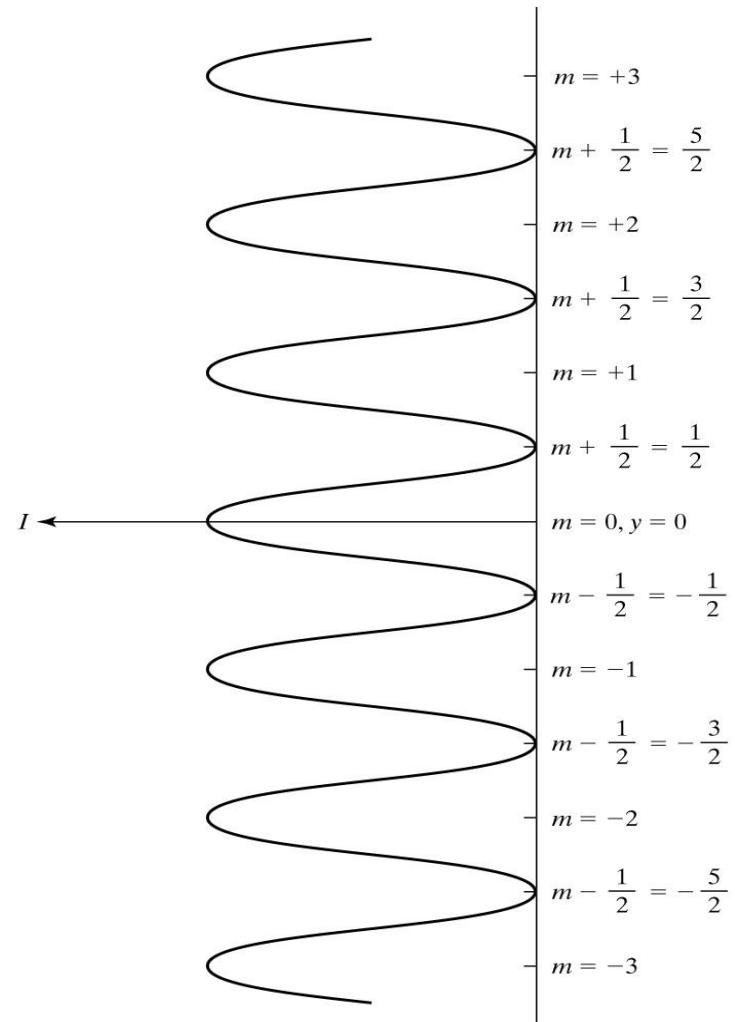
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$$S_1 - S_2 = \Delta = m\lambda \cong a \sin \theta$$

$$y \ll L \rightarrow \sin \theta \approx \tan \theta \approx y/L$$

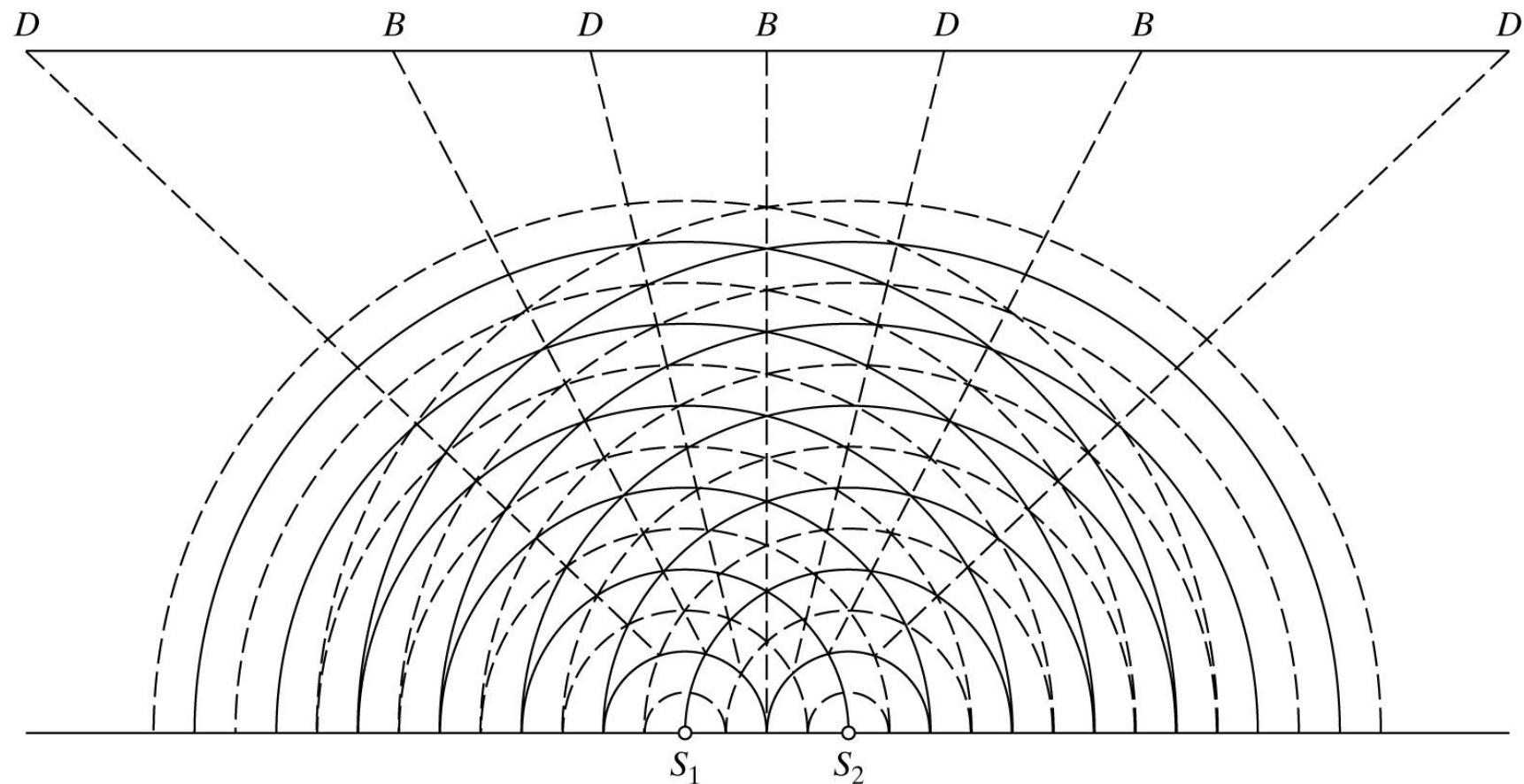
$$y_m \approx \frac{m\lambda L}{a}, \quad m = 0, \pm 1, \pm 2, \dots$$

$$\Delta y = y_{m+1} - y_m = \frac{\lambda L}{a}$$



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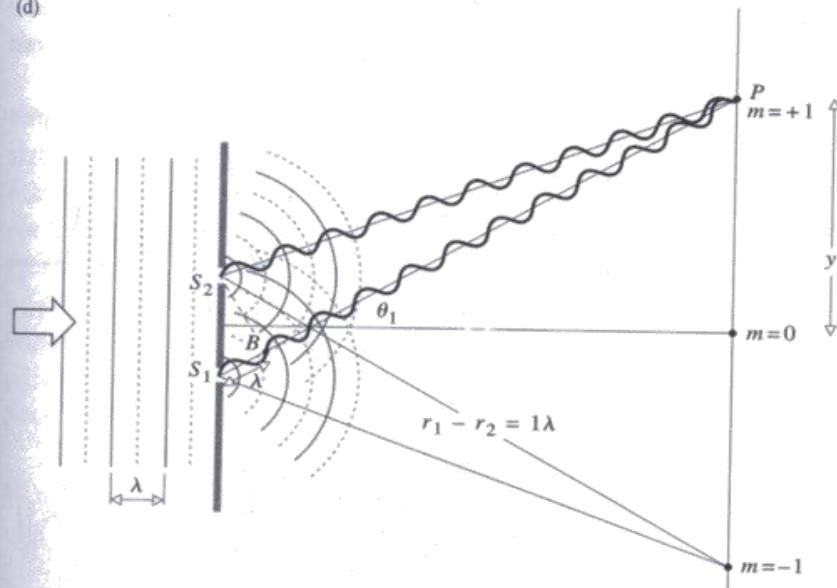
YOUNG'S DOUBLE-SLIT EXPERIMENT



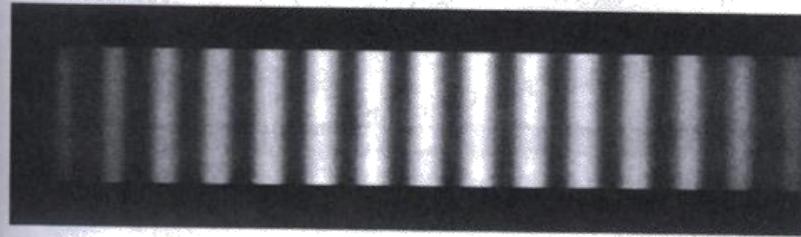
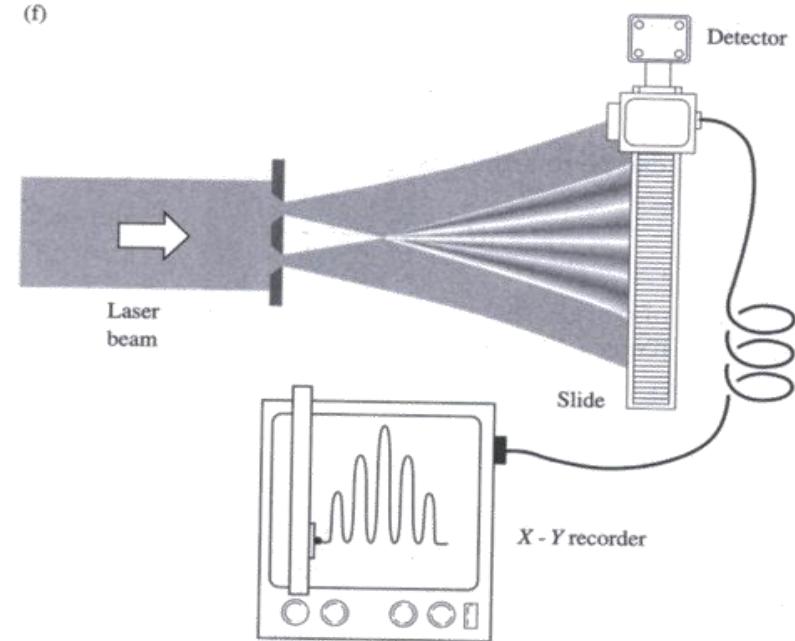
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YOUNG'S DOUBLE-SLIT EXPERIMENT

(d)

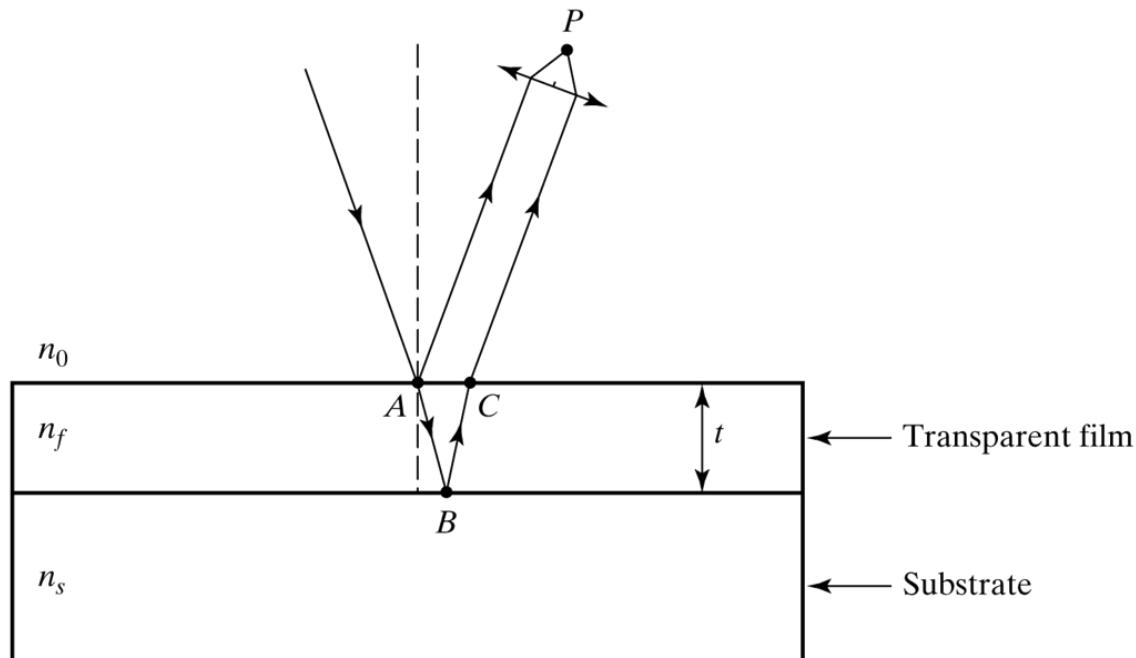


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(e)

INTERFERENCE IN DIELECTRIC FILMS



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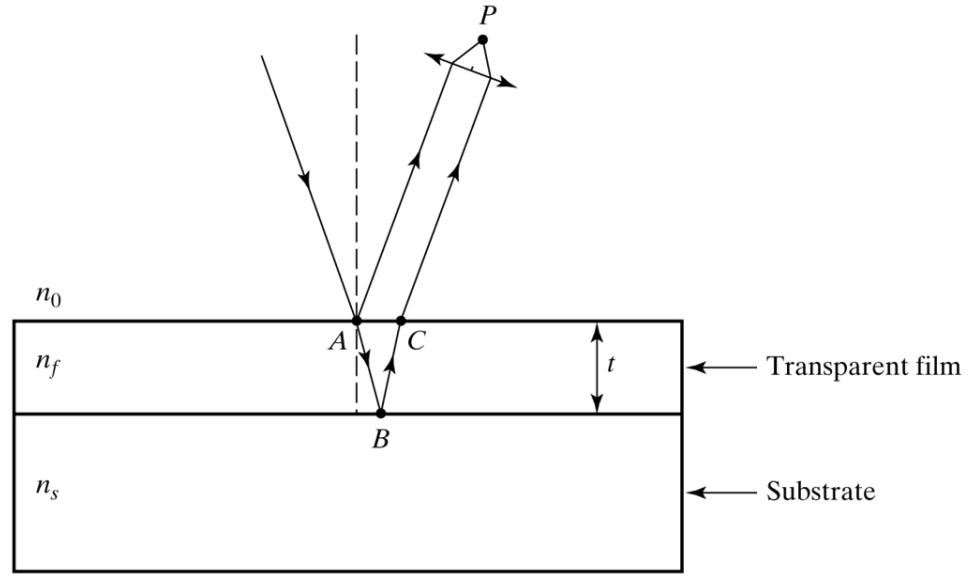
Interference takes place in a variety of situations depending on:

- Size and spectral width of the source.
- Shape and reflectance of the film.

Colors on the surface of oily water, soap films, mother-of-pearl, peacock feathers and butterfly wings are associated with the interference of light in single or multiple thin surface layers of transparent material.

INTERFERENCE IN DIELECTRIC FILMS

- At point A, part of the beam is reflected and part is refracted ← *amplitude division.*
- The refracted beam reflects again at B.
- The beam leaves the film at C.
- Part of the beam may reflect internally at C and experience multiple reflections with the film layer.
- Multiple parallel beams will emerge from the top.

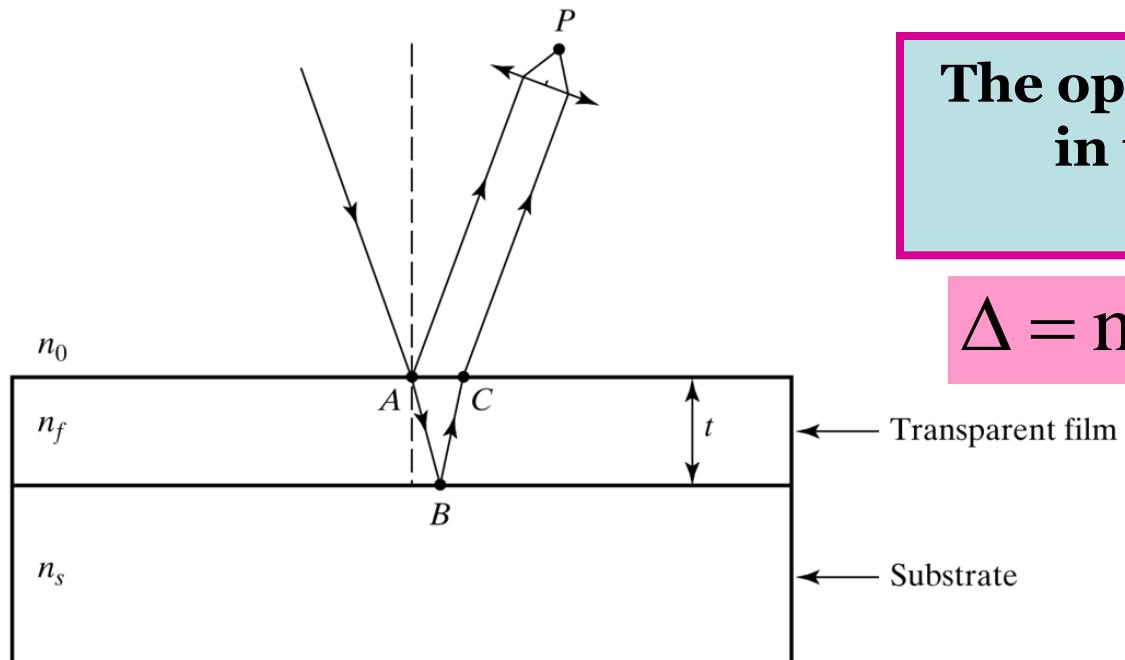


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The two parallel beams superpose and interfere at P after traveling different paths from point A.

A phase difference develops that can interfere constructively or destructively at point P.

INTERFERENCE IN DIELECTRIC FILMS



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The optical path difference Δ ,
in the case of normal
incidence is:

$$\Delta = n_f(AB + BC) = n_f(2t)$$

Be careful of phase
change upon
reflection!!!

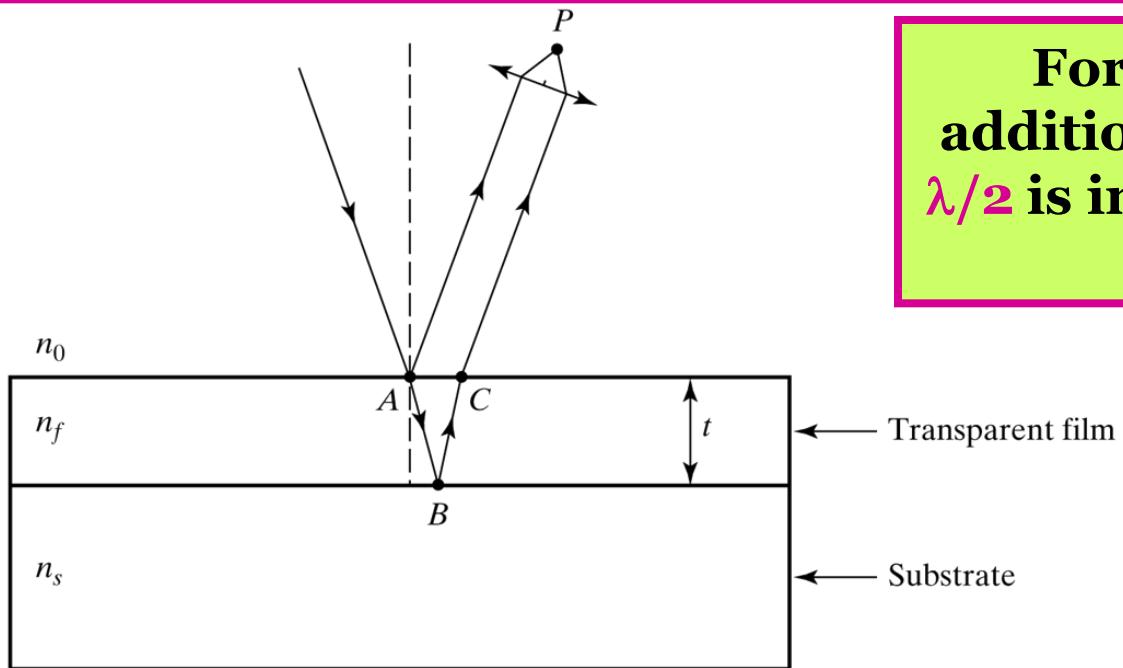
Lower index \rightarrow higher index : *external reflection*
Higher index \rightarrow lower index: *internal reflection*

A phase shift of π occurs between externally and internally
reflected beams!!

INTERFERENCE IN DIELECTRIC FILMS

A phase shift of π occurs between externally and internally reflected beams!!

$n_f > n_o$ & $n_f > n_s \leftarrow \text{Phase shift}$
 $n_o < n_f < n_s$ (both reflections are external) $\leftarrow \text{NO phase shift}$
 $n_o > n_f > n_s$ (both reflections are internal) $\leftarrow \text{NO phase shift}$



For a phase shift π , an additional path difference of $\lambda/2$ is introduced between the two beams.

How does antireflecting coating works??

INTERFERENCE IN DIELECTRIC FILMS

Let's generalize the conditions of constructive and destructive interference in the case incident rays are NOT normal.

$$\Delta = n_f(AB + BC) - n_o(AD)$$

$$\Delta = [n_f(AE + FC) - n_o AD] + n_f(EB + BF)$$

$$n_o \sin \theta_i = n_f \sin \theta_t$$

$$AE = AG \sin \theta_t = \left(\frac{AC}{2} \right) \sin \theta_t$$

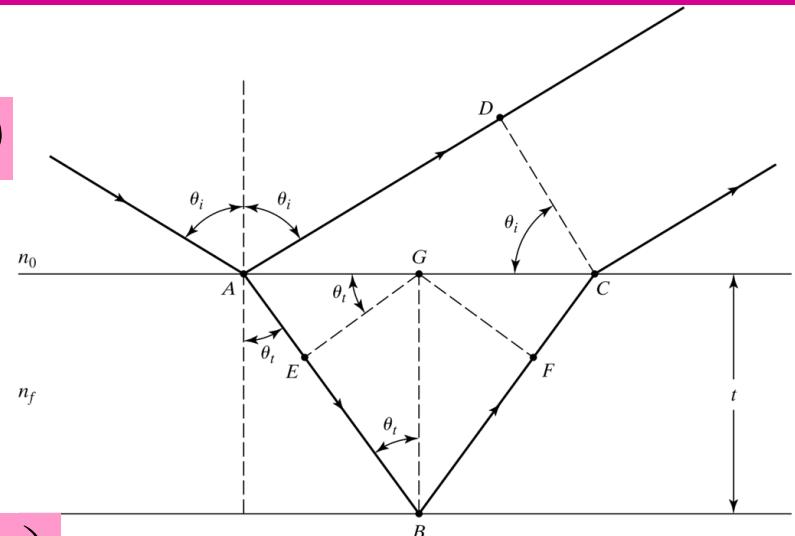
$$AD = AC \sin \theta_i$$

$$2AE = AC \sin \theta_t = AD \left(\frac{\sin \theta_t}{\sin \theta_i} \right) = AD \left(\frac{n_o}{n_f} \right)$$

$$n_o AD = 2n_f AE = n_f(AE + FC)$$

$$\Delta = n_f(EB + BF) = 2n_f EB$$

$$\Delta = 2n_f t \cos \theta_t$$



constructive interference

$$\Delta_p + \Delta_r = m\lambda$$

destructive interference

$$\Delta_p + \Delta_r = \left(m + \frac{1}{2} \right) \lambda$$