

Chapter 10, Exercises 10-1, 10-2, Problem # 15

The director of an alumni association for a small college wants to determine whether there is any type of relationship between the amount of an alumnus' contribution (in dollars) and the years the alumnus has been out of school. The data follow. Also, find the equation of the regression line, and find \hat{y} value when $x=4$ years.

Years (x)	Contribution (y)	x^2	y^2	xy
1	500	1	250000	500
5	100	25	10000	500
3	300	9	90000	900
10	50	100	2500	500
7	75	49	5625	525
6	80	36	6400	480
$\Sigma x = 32$	$\Sigma y = 1105$	$\Sigma x^2 = 220$	$\Sigma y^2 = 364525$	$\Sigma xy = 3405$

Correlation Coefficient (r):

$$r = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{\sqrt{[n(\Sigma x^2) - (\Sigma x)^2][n(\Sigma y^2) - (\Sigma y)^2]}}$$
$$r = \frac{6(3405) - (32)(1105)}{\sqrt{[6(220) - (32)^2][6(364525) - (1105)^2]}}$$
$$r = \frac{-14930}{16910.736} = -0.883$$

Linear regression equation ($\hat{y} = a + b x$):

$$a = \frac{(\Sigma y)(\Sigma x^2) - (\Sigma x)(\Sigma xy)}{n(\Sigma x^2) - (\Sigma x)^2} = \frac{(1105)(220) - (32)(3405)}{6(220) - (32)^2} = 453.176$$
$$b = \frac{n(\Sigma xy) - (\Sigma x)(\Sigma y)}{n(\Sigma x^2) - (\Sigma x)^2} = \frac{6(3405) - (32)(1105)}{6(220) - (32)^2} = -50.439$$
$$\hat{y} = 453.176 - 50.439 x$$

when $x = 4$ years

$$\hat{y} = 453.176 - 50.439 (4) = 251.42 \text{ dollar}$$

Chapter 10, Exercises 10-1, 10-2, Problem # 21

A random sample of U.S. cities is selected to determine if there is a relationship between the population (in thousands) of people under 5 years of age and the population (in thousands) of those 65 years of age and older. The data for the sample are shown here. Also, find the equation of the regression line, and find \hat{y} value when $x= 200$ thousand.

Under 5 (x)	65 and over (y)	x^2	y^2	xy
178	361	31684	130321	64258
27	72	729	5184	1944
878	1496	770884	2238016	1313488
314	501	98596	251001	157314
322	585	103684	342225	188370
143	207	20449	42849	29601
$\sum x = 1862$	$\sum y = 3222$	$\sum x^2 = 1026026$	$\sum y^2 = 3009596$	$\sum xy = 1754975$

Correlation Coefficient (r):

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{[n(\sum x^2) - (\sum x)^2][n(\sum y^2) - (\sum y)^2]}}$$
$$r = \frac{6(1754975) - (1862)(3222)}{\sqrt{[6(1026026) - (1862)^2][6(3009596) - (3222)^2]}}$$
$$r = \frac{4530486}{4543391.787} = 0.997$$

Linear regression equation ($\hat{y} = a + b x$):

$$a = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2} = \frac{(3222)(1026026) - (1862)(1754975)}{6(1026026) - (1862)^2} = 14.165$$

$$b = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} = \frac{6(1754975) - (1862)(3222)}{6(1026026) - (1862)^2} = 1.685$$

$$\hat{y} = 14.165 + 1.685 x$$

when $x= 200$ thousand

$$\hat{y} = 14.165 + 1.685 (200) = 351.165 \approx 351 \text{ thousand}$$