Critical Path Method (C.P.M.)

Acknowledgments: - <u>An Introduction to Management Science</u>
David R. Anderson
Dennis J. Sweeney
Thomas A. Williams

-<u>User's Guide to Microsoft Project 2000</u>
Microsoft Corporation

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1.1   What is a C.P.M.?

   C.P.M. or **Critical Path Method** was developed in the 1950s as a way of organizing and tracking the numerous activities associated with the Polaris missile defense program.

   Using a C.P.M. will help to identify and coordinate various tasks associated with the completion of a project. It allows for various information about a specific project to be found:

1. The total time to complete the project
2. The scheduled start and finish dates foe each task pertaining to the projects completion.
3. The tasks that are "**critical**" to the project and must be completed exactly as scheduled.
4. The "**slack**" time available in non-critical tasks, as well as how long they can be delayed before they affect the project finish date.

1.2   Steps to creating a C.P.M.

1.   Develop a list of tasks that are required to complete the project.
ie. The actual work to be carried out.
To name a task so that its purpose is clear. It is usually best to use two words; a *verb* followed by a *noun*. For example; build walls, install windows.
Define your tasks so that they are the right size. If the task you assign is too large in scope it may hide smaller tasks that need to be completed. For example the task "construct building" would be too large, as there are many steps that go into the construction of a building, such as Excavate the site, Pour foundation...
On the other hand, turning on the cement mixer is too finely detailed to be valuable to programming your C.P.M. This level of detail would quickly lead to a cluttered and confusing diagram.
In short the tasks should be well defined, precise and detailed but they should also be significant to furthering the project along.

2.   Order your tasks and determine the immediate predecessors for each. Simply put "What tasks need to be completed before the next task can be carried out. ie. The architectural drawings must be completed before the building can be built. Hence the task "**Build house**" is dependant on "**Prepare drawings**".

3. Determine completion time for each task. How long will the individual task take to complete?
Sources for determining these times are;

                  -Your own judgement and experience.
                  -Draw on the experience of others.
                  -Obtain estimates from professionals.

The more information you have the more accurate your prediction for the project completion will be.

I.D.	Task Name	Duration	Predecessor
1.	prepare drawings	5w	-
2.	identify tenants	6w	-
3.	develop prospectus	4w	1
4.	select G.C.	3w	1
5.	prepare permits	1w	1
6.	obtain permits	4w	5
7.	build mall	14w	4,6
8.	finalize contracts	12w	2,3
9.	tenants move in	2w	7,8

4. Using the chart above we are able to develop a graphical version of the network.
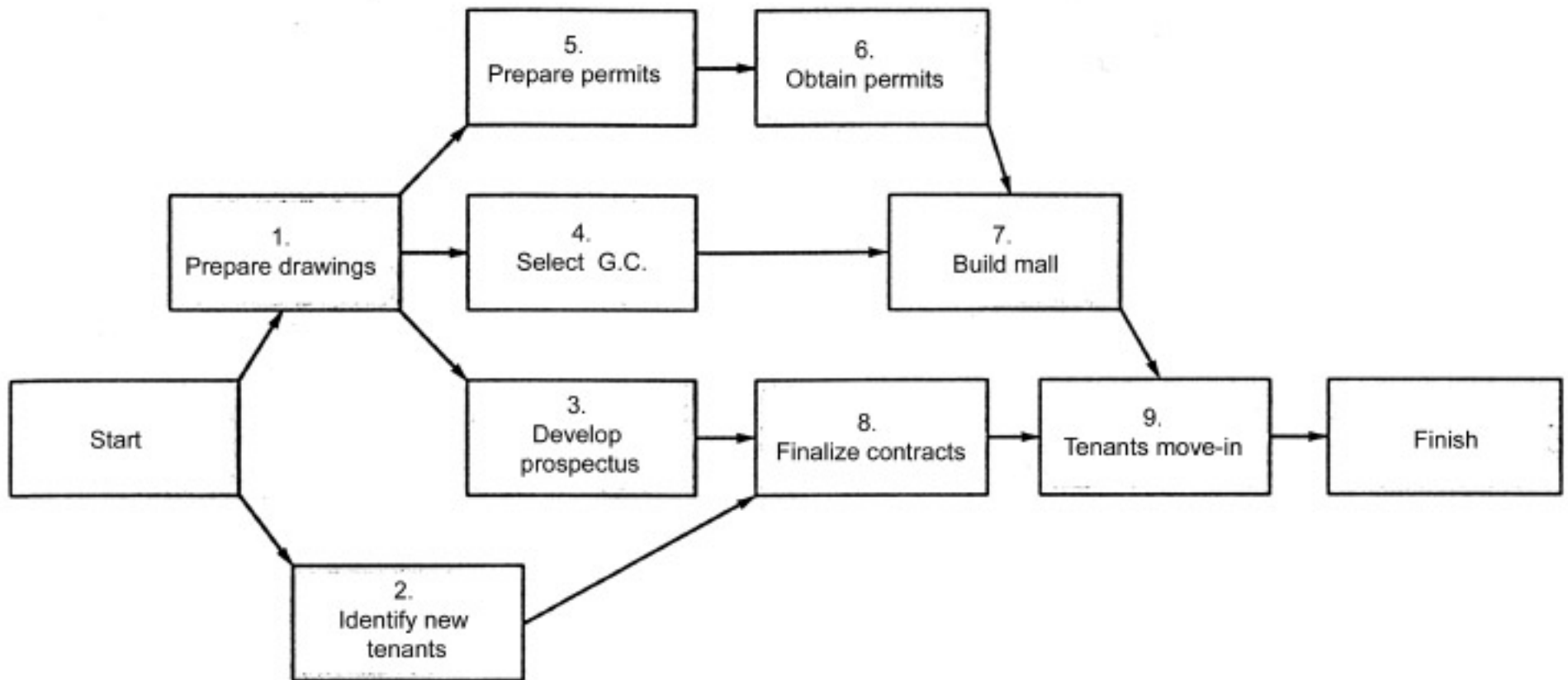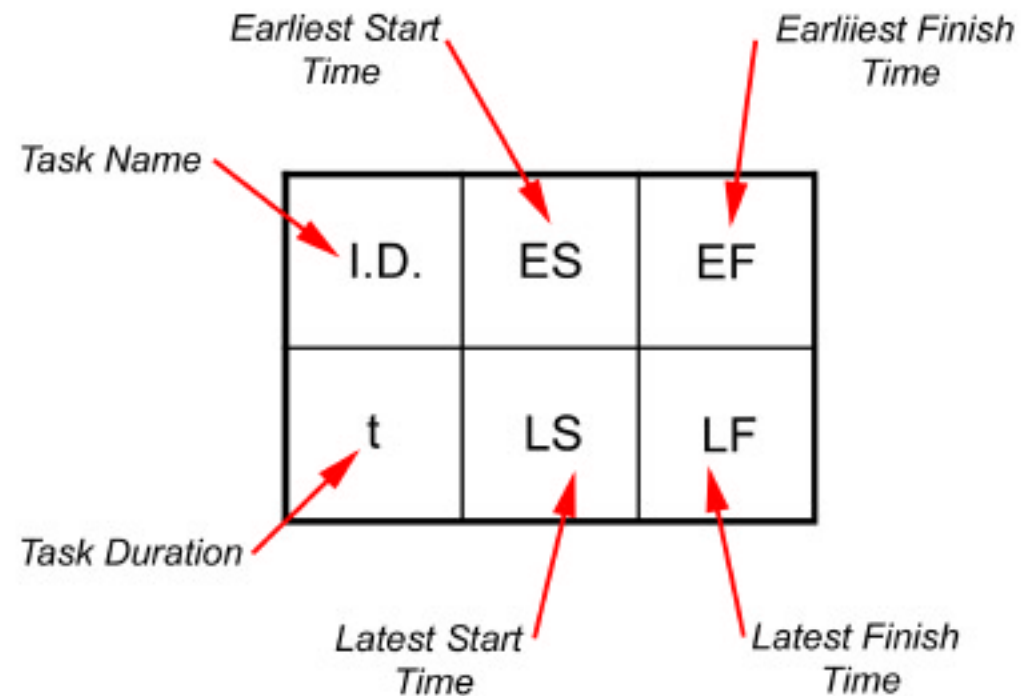
In order to determine the length of time needed to complete the project we must identify, in the network the critical path. Typically the longest path, time wise, will determine the completion time for a project.
If any of the tasks on the critical path are delayed then the entire project will be delayed. Hence the task is "critical" to the completion of the project on time.

5. Determining the critical path.
When laying out a c.p.m. network, each task will be represented by a box with the following information.

Earliest Start Time

Earliiest Finish Time

Task Name

I.D.	ES	EF
t	LS	LF

Task Duration

Latest Start Time

Latest Finish Time

To begin determining the critical path, we must first determine the earliest start time and a latest finish time for all tasks.

ES= earliest start time
EF= earliest finish time
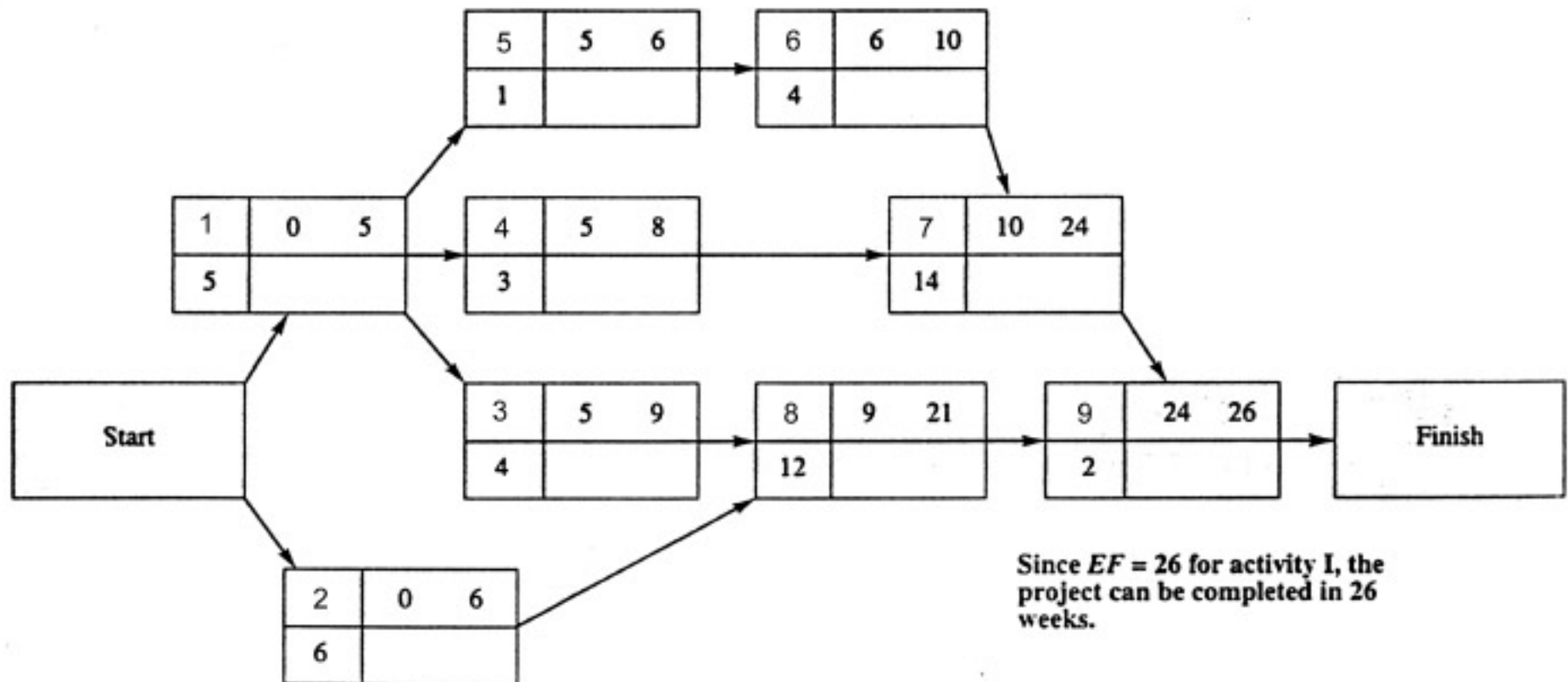
The earliest finish time for a task is;

$$EF=ES+t$$

Using our network we begin from the start. Task 1. Can begin as soon as possible, so we set its t to 0 (t= 0). It has a duration time of 5 weeks so its earliest finish time will be:

    EF=0+5
      = 5

Since the task can not be started until all other preceding tasks have finished the earliest start time for a task is equal to the largest of all the earliest finish times for all its immediate predecessors.
We make a forward pass through the whole network determining all the earliest start and finish times for all tasks.
The earliest finish time for the last task will be how long the entire project will take to complete.

Network diagram (boxes shown as: Task | ES | EF and duration below task number):

- Task **5**: ES 5, EF 6; duration **1** → Task **6**: ES 6, EF 10; duration **4**
- Start → Task **1**: ES 0, EF 5; duration **5** → Task **4**: ES 5, EF 8; duration **3** → Task **7**: ES 10, EF 24; duration **14**
- Task **3**: ES 5, EF 9; duration **4** → Task **8**: ES 9, EF 21; duration **12** → Task **9**: ES 24, EF 26; duration **2** → Finish
- Task **2**: ES 0, EF 6; duration **6**

Since **EF** = 26 for activity I, the project can be completed in 26 weeks.

6. Determine critical tasks.

      We now make a backward pass through the network to determine the critical path.

      Since we know that the project will take 26 weeks to complete. This willbe our latest finish time.

LF= latest finish time

LS= latest start time

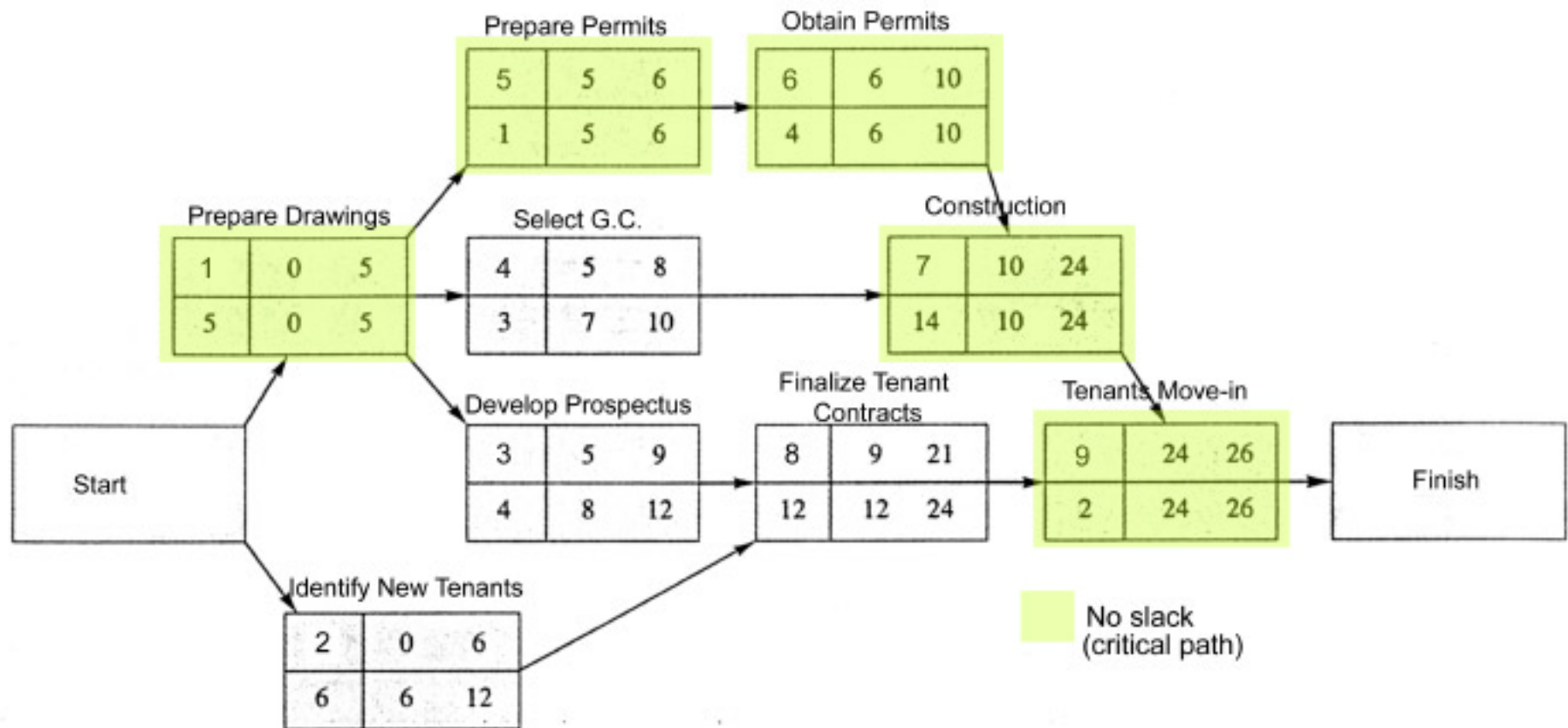The latest start time for the last task will be:

$$LS=LF\text{-}t$$

$$LS=26\text{-}2$$
$$=24$$

Moving backwards, the latest finish time for a task is the smallest of the latest start times for all tasks that immediately follow that task. The latest finish time is the earliest start time for the immediately following tasks.

Once we have completed the forward and backward passes through the network, we can determine how much "**slack**" is in each task that is the length of time the task can be delayed without it affecting the project completion date.

Slack is determined as follows: **Slack=LS-ES=LF-EF**

When a task has zero slack it is critical and lies on the critical path for that network configuration. Watch these tasks carefully for a delay here will delay the entire project.

Prepare Permits

5	5	6
1	5	6

Obtain Permits

6	6	10
4	6	10

Construction

7	10	24
14	10	24

Prepare Drawings

1	0	5
5	0	5

Select G.C.

4	5	8
3	7	10

Tenants Move-in

9	24	26
2	24	26

Develop Prospectus

3	5	9
4	8	12

Finalize Tenant Contracts

8	9	21
12	12	24

Start

Finish

Identify New Tenants

2	0	6
6	6	12

No slack (critical path)

1.3   C.P.M. Summary

1. Develop a list of tasks
2. Order and determine immediate predecessors for each task
3. Estimate completion time for tasks
4. Layout a project network using tasks and their predecessors
5. Determine earliest start and finishing times using a forward pass through the network.

Equations: forward pass   1. $ES_i = $ largest of (EF_{i-1})
                                              2. $EF_i = ES_i + t$
            backward pass   3. $LF_i = $ smallest of (LS_{i+1})
                                              4. $LS_i = LF_i - t$

6. Using the earliest finish time for the project make a backward pass through the network to determine the latest start and finish times for each task.
7. Determine slack for each task using difference between latest and earliest start times.
8. Find tasks with zero slack. These are critical tasks lying on the critical path.

1.4   Crashing the C.P.M.

   Once you have created a network and determined its critical path, you should reevaluate the network to see if the time for project completion can be shortened without considerable loss to the project. this shortening of the project network is called "**crashing**" the C.P.M..

There are several techniques to use when crashing a c.p.m.;

1. Shorten (if possible) the duration of tasks on the critical path by adding resources such as overtime hours or extra workers.
2. Overlap tasks on the critical path so that they may be worked on simultaneously. ie. Drywallers finish upstairs move to downstairs. Painting can begin on upstairs walls.
3. Break up larger tasks on the critical path into smaller more manageable ones
4. Does the plan include only tasks which are necessary to achieving project goals?