Sample Problem 5-1

Figures 5-3*a* to *c* show three situations in which one or two forces act on a puck that moves over frictionless ice along an x axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces $\vec{F}_1$ and $\vec{F}_2$ are directed along the axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\vec{F}_3$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?

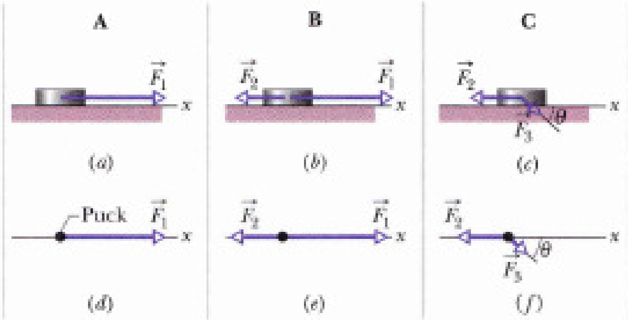


FIG. 5-3 (*a*)–(*c*) In three situations, forces act on a puck that moves along an x axis. (*d*)–(*f*) Free-body diagrams.

KEY IDEA In each situation we can relate the acceleration $\vec{a}$ to the net force $\vec{F}_{net}$ acting on the puck with Newton's second law, $\vec{F}_{net} = m\vec{a}$. However, because the motion is along only the x axis, we can simplify each situation by writing the second law for x components only:

$$F_{net,x} = ma_x. \tag{5-4}$$

The free-body diagrams for the three situations are given in Figs. 5-3*d* to *f* with the puck represented by a dot.

Situation A: For Fig. 5-3*d*, where only one horizontal force acts, Eq. 5-4 gives us

$$F_1 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1}{m} = \frac{4.0\text{ N}}{0.20\text{ kg}} = 20\text{ m/s}^2. \quad \text{(Answer)}$$

The positive answer indicates that the acceleration is in the positive direction of the x axis.

Situation B: In Fig. 5-3*e*, two horizontal forces act on the puck, $\vec{F}_1$ in the positive direction of x and $\vec{F}_2$ in the negative direction. Now Eq. 5-4 gives us

$$F_1 - F_2 = ma_x,$$

which, with given data, yields

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\text{ N} - 2.0\text{ N}}{0.20\text{ kg}} = 10\text{ m/s}^2.$$

(Answer)

Thus, the net force accelerates the puck in the positive direction of the x axis.

Situation C: In Fig. 5-3*f*, force $\vec{F}_3$ is not directed along the direction of the puck's acceleration; only x component $F_{3,x}$ is. (Force $\vec{F}_3$ is two-dimensional but the motion is only one-dimensional.) Thus, we write Eq. 5-4 as

$$F_{3,x} - F_2 = ma_x. \tag{5-5}$$

From the figure, we see that $F_{3,x} = F_3 \cos\theta$. Solving for the acceleration and substituting for $F_{3,x}$ yield

$$a_x = \frac{F_{3,x} - F_2}{m} = \frac{F_3\cos\theta - F_2}{m}$$
$$= \frac{(1.0\text{ N})(\cos 30°) - 2.0\text{ N}}{0.20\text{ kg}} = -5.7\text{ m/s}^2.$$

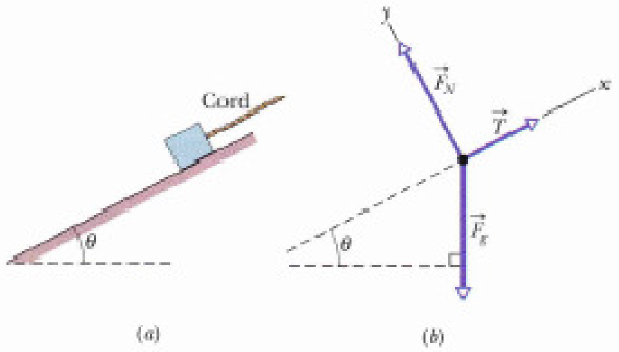
(Answer)

Thus, the net force accelerates the puck in the negative direction of the x axis.

Sample Problem 5-5

In Fig. 5-16*a*, a cord pulls on a box of sea biscuits up along a frictionless plane inclined at $\theta = 30°$. The box has mass $m = 5.00$ kg, and the force from the cord has magnitude $T = 25.0$ N. What is the box's acceleration component a along the inclined plane?

KEY IDEA The acceleration along the plane is set by the force components along the plane (not by force components perpendicular to the plane), as expressed by Newton's second law (Eq. 5-1).



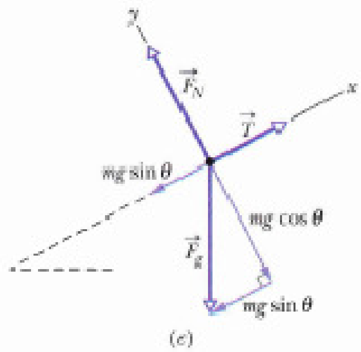


FIG. 5-16 (*a*) A box is pulled up a plane by a cord. (*b*) The three forces acting on the box: the cord's force $\vec{T}$, the gravittional force $\vec{F}_g$, and the normal force $\vec{F}_N$. (*c*) The components of $\vec{F}_g$ along the plane and pependicular to it.

Calculation: For convenience, we draw a coordinate system and a free-body diagram as shown in Fig. 5-16*b*. The positive direction of the x axis is up the plane. Force $\vec{T}$ from the cord is up the plane and has magnitude $T = 25.0$ N. The gravitational force $\vec{F}_g$ is downward and has magnitude $mg = (5.00\text{ kg})(9.8\text{ m/s}^2) = 49.0$ N. More important, its component along the plane is down the plane and has magnitude $mg \sin\theta$ as indicated in Fig. 5-16*c*. (To see why that trig function is involved, compare the right triangles in Figs. 5-16*b* and *c*.) To indicate the direction, we can write the component as $-mg\sin\theta$. The normal force $\vec{F}_N$ is perpendicular to the plane and thus does not determine acceleration along the plane.

We write Newton's second law ($\vec{F}_{net} = m\vec{a}$) for motion along the x axis as

$$T - mg\sin\theta = ma. \qquad (5\text{-}22)$$

Substituting data and solving for a, we find

$$a = 0.100\text{ m/s}^2, \qquad \text{(Answer)}$$

where the positive result indicates that the box accelerates up the plane.

Sample Problem 5-7 Build your skill

Figure 5-18*a* shows the general arrangement in which two forces are applied to a 4.00 kg block on a frictionless floor, but only force $\vec{F}_1$ is indicated. That force has a fixed magnitude but can be applied at angle θ to the positive direction of the x axis. Force $\vec{F}_2$ is horizontal and fixed in both magnitude and angle. Figure 5-18*b* gives the horizontal acceleration a_x of the block for any given value of θ from 0° to 90°. What is the value of a_x for $\theta = 180°$?

KEY IDEAS (1) The horizontal acceleration a_x depends on the net horizontal force $F_{net,x}$, as given by Newton's second law. (2) The net horizontal force is the sum of the horizontal components of forces $\vec{F}_1$ and $\vec{F}_2$.

Calculations: The x component of $\vec{F}_2$ is F_2 because the vector is horizontal. The x component of $\vec{F}_1$ is $F_1\cos\theta$. Using these expressions and a mass m of 4.00 kg, we can write Newton's second law ($\vec{F}_{net} = m\vec{a}$) for motion along the x axis as

$$F_1\cos\theta + F_2 = 4.00a_x. \qquad (5\text{-}25)$$

From this equation we see that when $\theta = 90°$, $F_1\cos\theta$ is zero and $F_2 = 4.00a_x$. From the graph we see that the corresponding acceleration is 0.50 m/s². Thus, $F_2 = 2.00$ N and $\vec{F}_2$ must be in the positive direction of the x axis.

From Eq. 5-25, we find that when $\theta = 0°$,

$$F_1\cos 0° + 2.00 = 4.00a_x. \qquad (5\text{-}26)$$

From the graph we see that the corresponding acceleration is 3.0 m/s². From Eq. 5-26, we then find that $F_1 = 10$ N.

Substituting $F_1 = 10$ N, $F_2 = 2.00$ N, and $\theta = 180°$ into Eq. 5-25 leads to

$$a_x = -2.00\text{ m/s}^2. \qquad \text{(Answer)}$$

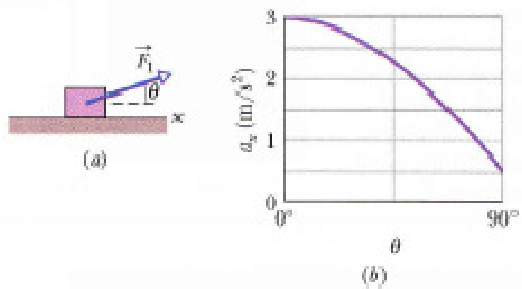


FIG. 5-18 (*a*) One of the two forces applied to a block is shown. Its angle θ can be varied. (*b*) The block's acceleration component a_x versus θ.

Sample Problem 5-8 Build your skill

In Fig. 5-19*a*, a passenger of mass $m = 72.2$ kg stands on a platform scale in an elevator cab. We are concerned with the scale readings when the cab is stationary and when it is moving up or down.

(a) Find a general solution for the scale reading, whatever the vertical motion of the cab.

KEY IDEAS (1) The reading is equal to the magnitude of the normal force $\vec{F}_N$ on the passenger from the scale. The only other force acting on the passenger is the gravitational force $\vec{F}_g$, as shown in the free-body diagram of Fig. 5-19*b*. (2) We can relate the forces on the passenger to his acceleration $\vec{a}$ by using Newton's second law ($\vec{F}_{net} = m\vec{a}$). However, recall that we can use this law only in an inertial frame. If the cab accelerates, then it is *not* an inertial frame. So we choose the ground to be our inertial frame and make any measure of the passenger's acceleration relative to it.

Calculations: Because the two forces on the passenger and his acceleration are all directed vertically, along the y axis in Fig. 5-19*b*, we can use Newton's second law written for y components ($F_{net,y} = ma_y$) to get

$$F_N - F_g = ma$$

or

$$F_N = F_g + ma. \qquad (5\text{-}27)$$

This tells us that the scale reading, which is equal to F_N, depends on the vertical acceleration. Substituting mg for F_g gives us

$$F_N = m(g + a) \qquad \text{(Answer)} \qquad (5\text{-}28)$$

for any choice of acceleration a.

(b) What does the scale read if the cab is stationary or moving upward at a constant 0.50 m/s?

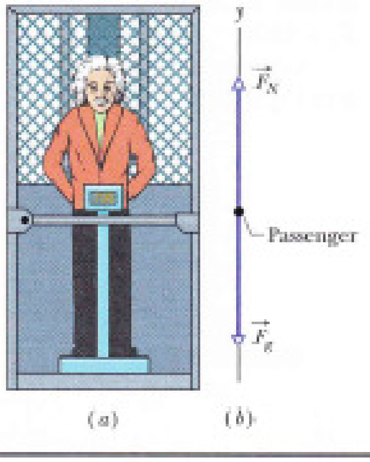


FIG. 5-19 (*a*) A passenger stands on a platform scale that indicates either his weight or his apparent weight. (*b*) The free-body diagram for the passenger, showing the normal force $\vec{F}_N$ on him from the scale and the gravitational force $\vec{F}_g$.

KEY IDEA For any constant velocity (zero or otherwise), the acceleration a of the passenger is zero.

Calculation: Substituting this and other known values into Eq. 5-28, we find

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 0) = 708 \text{ N}. \qquad \text{(Answer)}$$

This is the weight of the passenger and is equal to the magnitude F_g of the gravitational force on him.

(c) What does the scale read if the cab accelerates upward at 3.20 m/s² and downward at 3.20 m/s²?

Calculations: For $a = 3.20$ m/s², Eq. 5-28 gives

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 + 3.20 \text{ m/s}^2) = 939 \text{ N}, \qquad \text{(Answer)}$$

and for $a = -3.20$ m/s², it gives

$$F_N = (72.2 \text{ kg})(9.8 \text{ m/s}^2 - 3.20 \text{ m/s}^2) = 477 \text{ N}. \qquad \text{(Answer)}$$

For an upward acceleration (either the cab's upward speed is increasing or its downward speed is decreasing), the scale reading is greater than the passenger's weight. That reading is a measurement of an apparent weight, because it is made in a noninertial frame. For a downward acceleration (either decreasing upward speed or increasing downward speed), the scale reading is less than the passenger's weight.

(d) During the upward acceleration in part (c), what is the magnitude F_{net} of the net force on the passenger, and what is the magnitude $a_{p,cab}$ of his acceleration as measured in the frame of the cab? Does $\vec{F}_{net} = m\vec{a}_{p,cab}$?

Calculation: The magnitude F_g of the gravitational force on the passenger does not depend on the motion of the passenger or the cab; so, from part (b), F_g is 708 N. From part (c), the magnitude F_N of the normal force on the passenger during the upward acceleration is the 939 N reading on the scale. Thus, the net force on the passenger is

$$F_{net} = F_N - F_g = 939 \text{ N} - 708 \text{ N} = 231 \text{ N}, \qquad \text{(Answer)}$$

during the upward acceleration. However, his acceleration $a_{p,cab}$ relative to the frame of the cab is zero. Thus, in the noninertial frame of the accelerating cab, F_{net} is not equal to $ma_{p,cab}$, and Newton's second law does not hold.