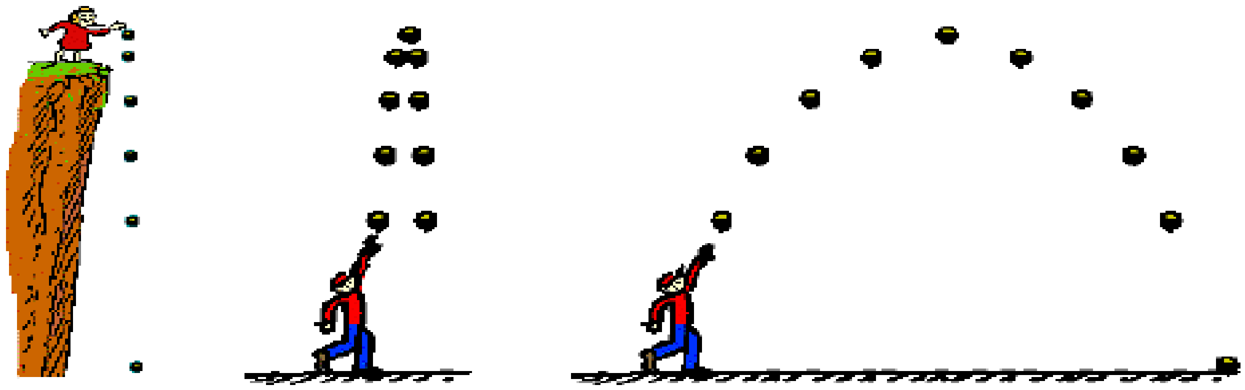
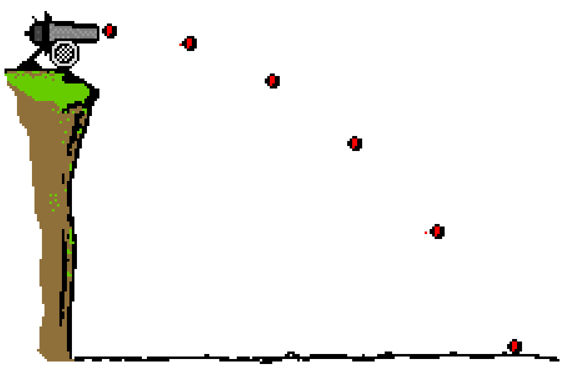
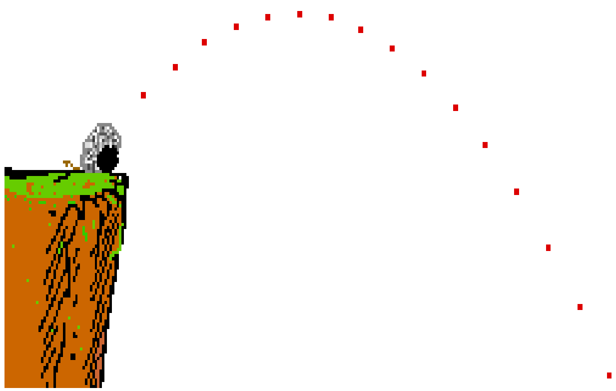


4 Motion in Two and Three Dimensions

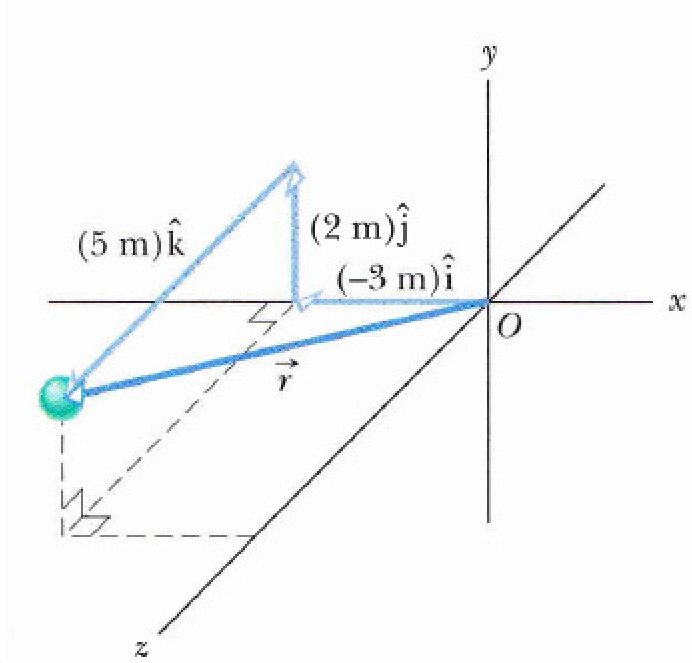




4-2 | Position and Displacement

Position

$$\vec{r} = x\hat{\mathrm{i}} + y\hat{\mathrm{j}} + z\hat{\mathrm{k}}$$



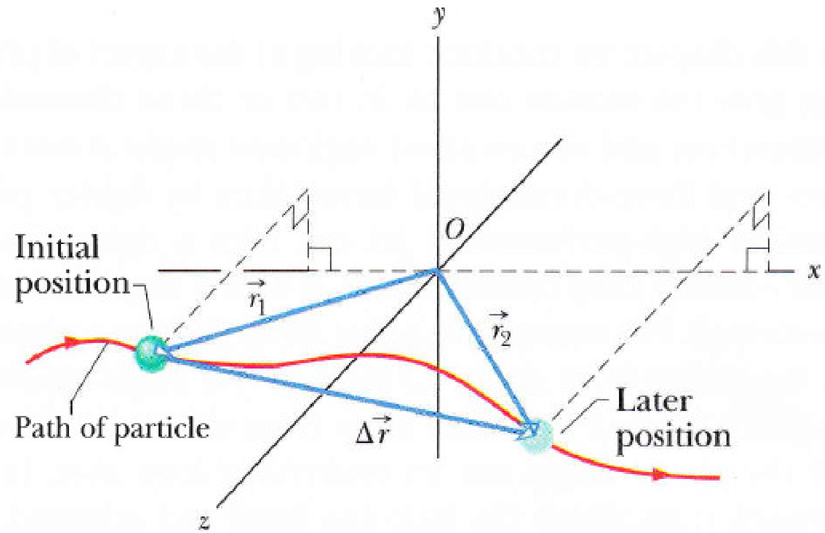
Displacmen

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\vec{r}_1 = x_1\hat{\mathrm{i}} + y_1\hat{\mathrm{j}} + z_1\hat{\mathrm{k}} \ \text{ and } \vec{r}_2 = x_2\hat{\mathrm{i}} + y_2\hat{\mathrm{j}} + z_2\hat{\mathrm{k}}$$

$$\Delta\vec{r} = (x_2\hat{\mathrm{i}} + y_2\hat{\mathrm{j}} + z_2\hat{\mathrm{k}}) - (x_1\hat{\mathrm{i}} + y_1\hat{\mathrm{j}} + z_1\hat{\mathrm{k}})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\hat{\mathrm{k}}$$



$$\Delta\vec{r} = \Delta x\,\hat{\mathrm{i}} + \Delta y\,\hat{\mathrm{j}} + \Delta z\,\hat{\mathrm{k}}$$

Sample Problem 4-1

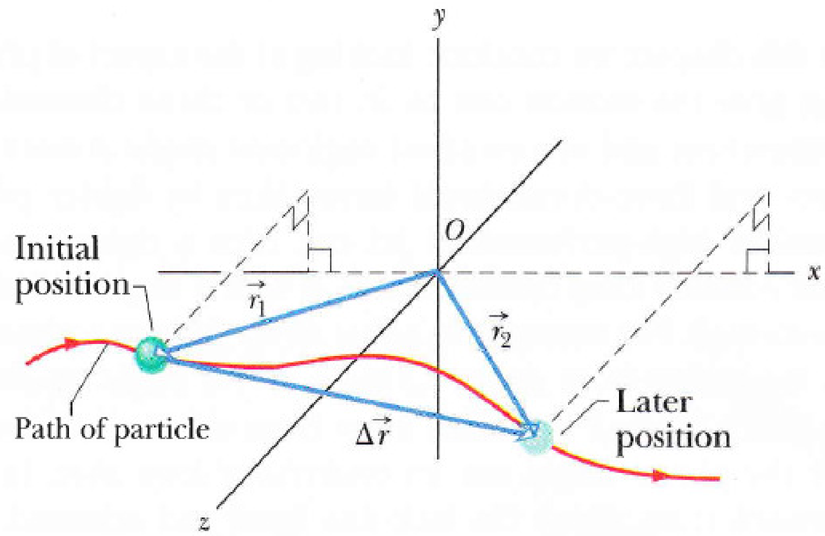
In Fig. 4-2, the position vector for a particle initially is

$$\vec{r}_1 = (-3.0\text{ m})\hat{\mathrm{i}} + (2.0\text{ m})\hat{\mathrm{j}} + (5.0\text{ m})\hat{\mathrm{k}}$$

and then later is

$$\vec{r}_2 = (9.0\text{ m})\hat{\mathrm{i}} + (2.0\text{ m})\hat{\mathrm{j}} + (8.0\text{ m})\hat{\mathrm{k}}.$$

What is the particle's displacement $\Delta\vec{r}$ from $\vec{r}_1$ to $\vec{r}_2$?



Sample Problem 4-2

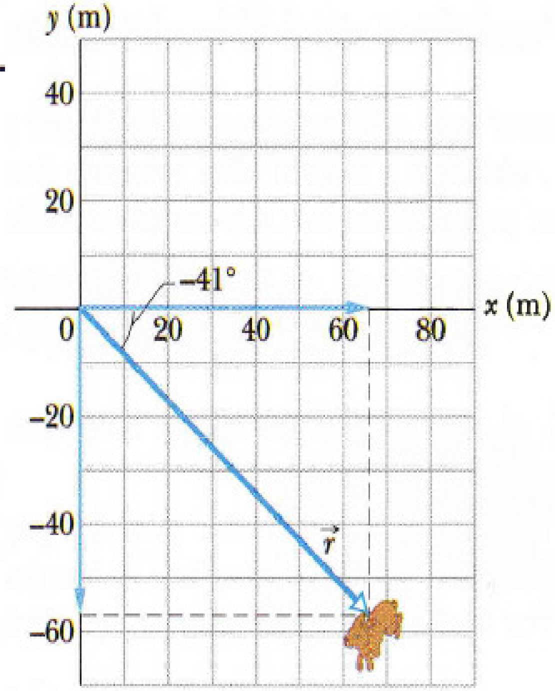
A rabbit runs across a parking lot on which a set of coordinate axes has, strangely enough, been drawn. The coordinates (meters) of the rabbit's position as functions of time t (seconds) are given by

$$x = -0.31t^2 + 7.2t + 28 \tag{4-5}$$

and

$$y = 0.22t^2 - 9.1t + 30. \tag{4-6}$$

(a) At $t = 15$ s, what is the rabbit's position vector $\vec{r}$ in unit-vector notation and in magnitude-angle notation?



4-3 | Average Velocity and Instantaneous Velocity

Average velocity

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v}_{avg} = \frac{\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

Example:

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(12\text{ m})\hat{i} + (3.0\text{ m})\hat{k}}{2.0\text{ s}}$$

$$= (6.0\text{ m/s})\hat{i} + (1.5\text{ m/s})\hat{k}.$$

Instantaneous velocity (Or velocity)

$$\vec{v} = \frac{d\vec{r}}{dt}$$

but $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}.$

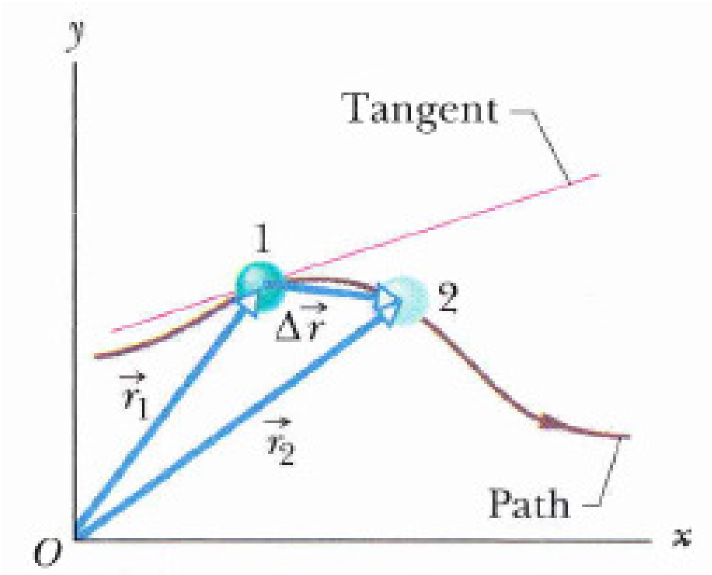
$$\vec{v} = \frac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$$

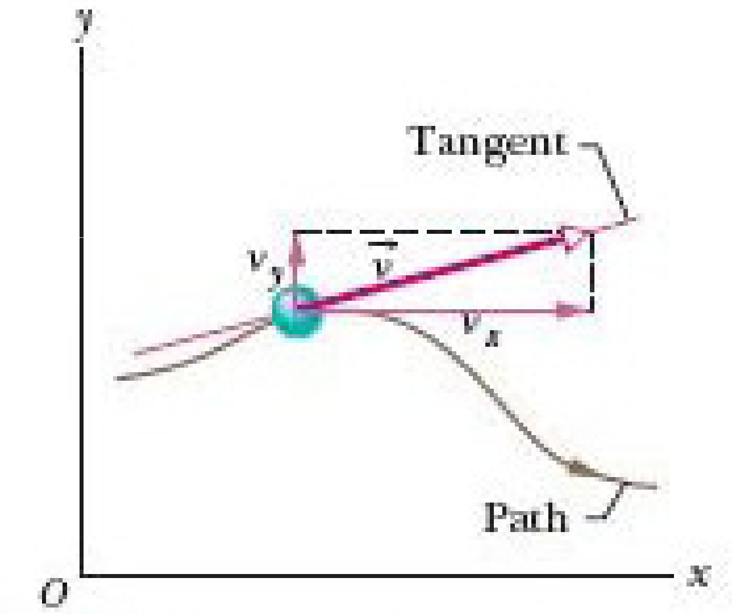
$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}.$$

The direction of the Instantaneous velocity

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k},$$





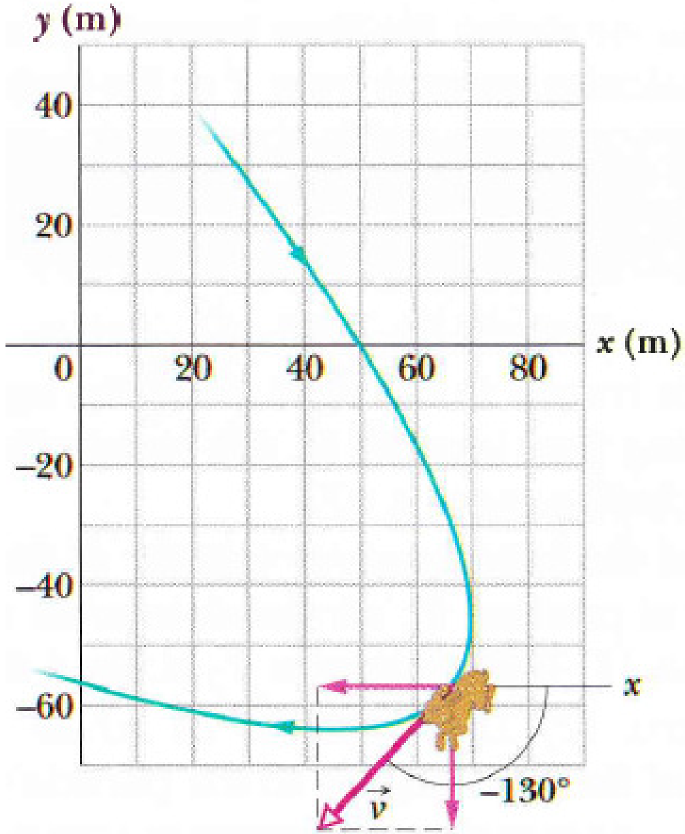
The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle's position.

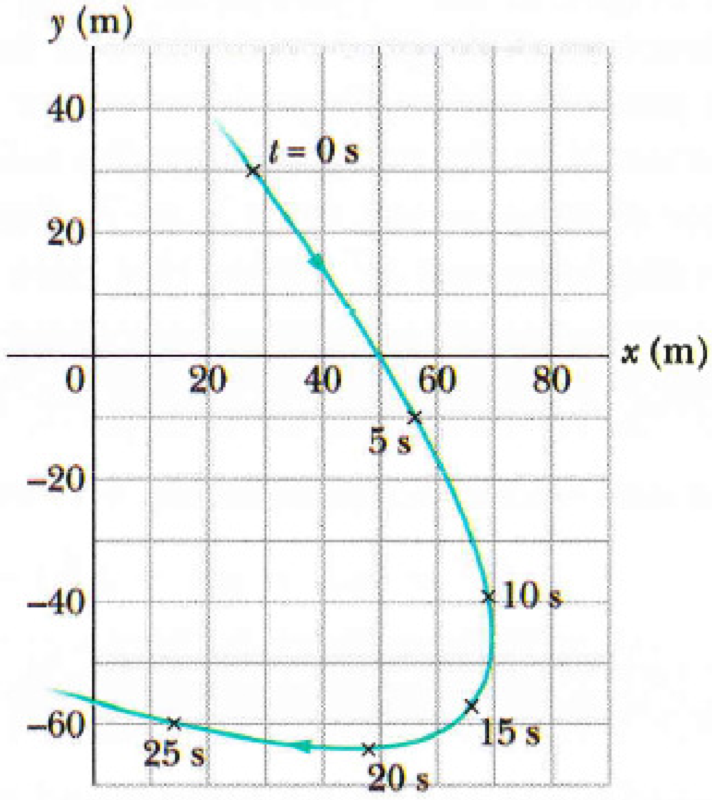
Sample Problem 4-3

For the rabbit in Sample Problem 4-2 find the velocity $\vec{v}$ at time $t = 15$ s.

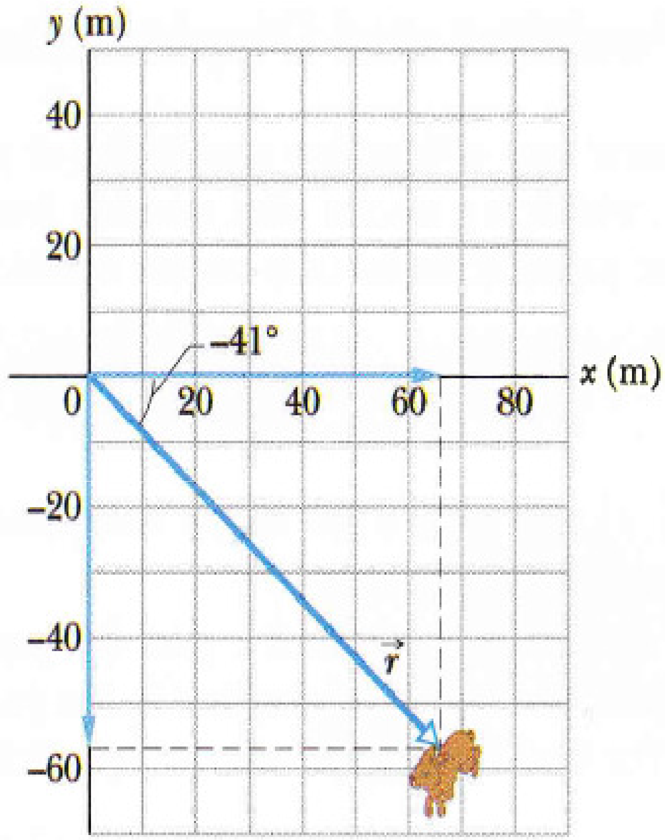
$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30.$$





y (m)
40
20
0
20
40
60
80
x (m)
−20
−40
−60
t = 0 s
5 s
10 s
15 s
20 s
25 s



y (m)
40
20
0
20
40
60
80
x (m)
−20
−40
−60
−41°
$\vec{r}$

4-4 Average Acceleration and Instantaneous Acceleration

Average Acceleration

$$\text{average acceleration} = \frac{\text{change in velocity}}{\text{time interval}}$$

$$\vec{a}_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

Instantaneous Acceleration (Or acceleration)

$$\vec{a} = \frac{d\vec{v}}{dt}$$

but

$$\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

$$\vec{a} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

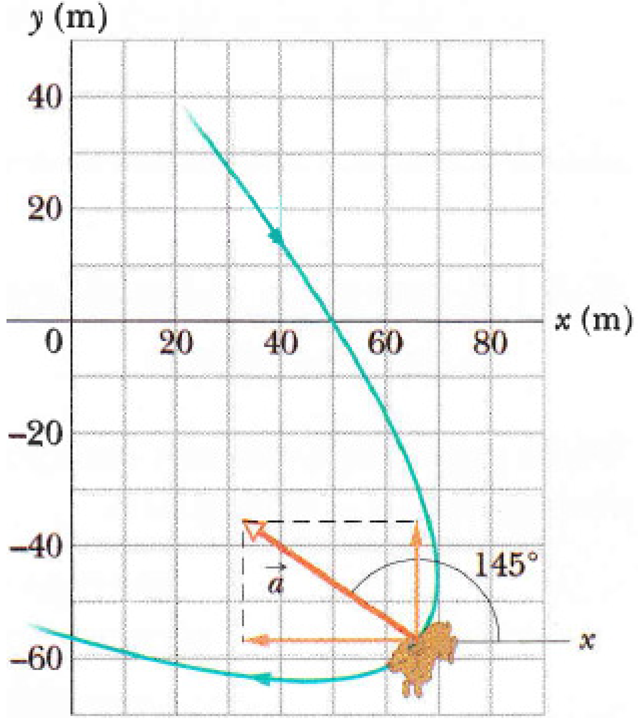
$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Sample Problem 4-4

For the rabbit in Sample Problems 4-2 and 4-3, find the acceleration $\vec{a}$ at time $t = 15$ s.

$$x = -0.31t^2 + 7.2t + 28$$

$$y = 0.22t^2 - 9.1t + 30.$$

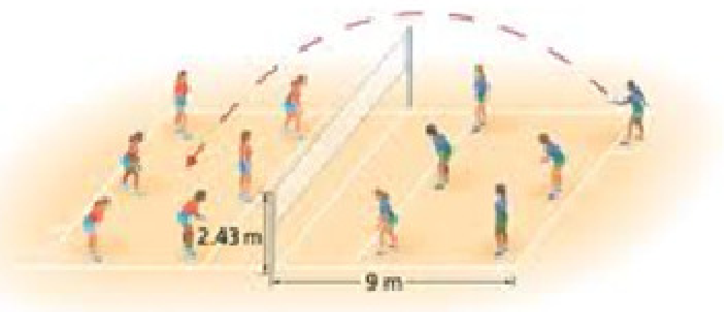


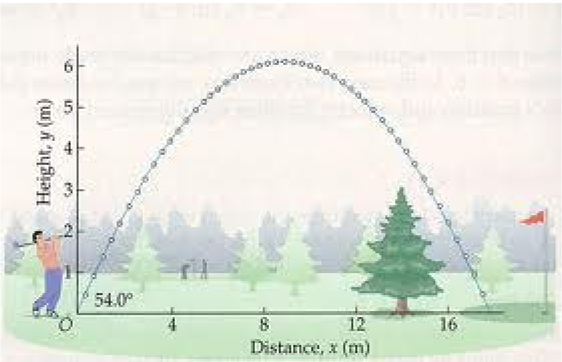
Sample Problem 4-5

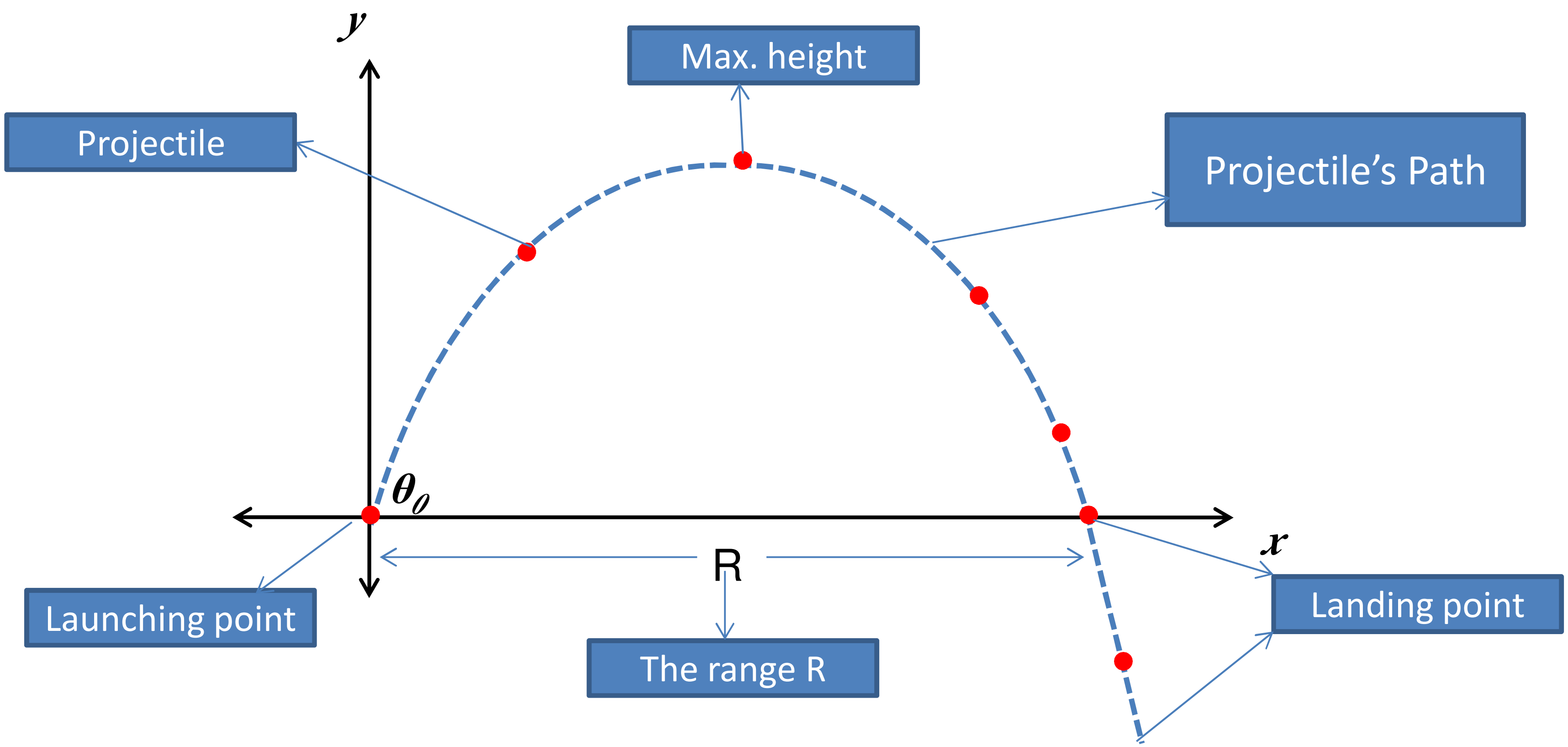
A particle with velocity $\vec{v}_0 = -2.0\hat{i} + 4.0\hat{j}$ (in meters per second) at $t = 0$ undergoes a constant acceleration $\vec{a}$ of magnitude $a = 3.0$ m/s^2 at an angle $\theta = 130°$ from the positive direction of the x axis. What is the particle's velocity $\vec{v}$ at $t = 5.0$ s?

4-5 | Projectile Motion

Projectile Motion *Projectile motion* is the motion of a particle that is launched with an initial velocity $\vec{v}_0$. During its flight, the particle's horizontal acceleration is zero and its vertical acceleration is the free-fall acceleration $-g$.

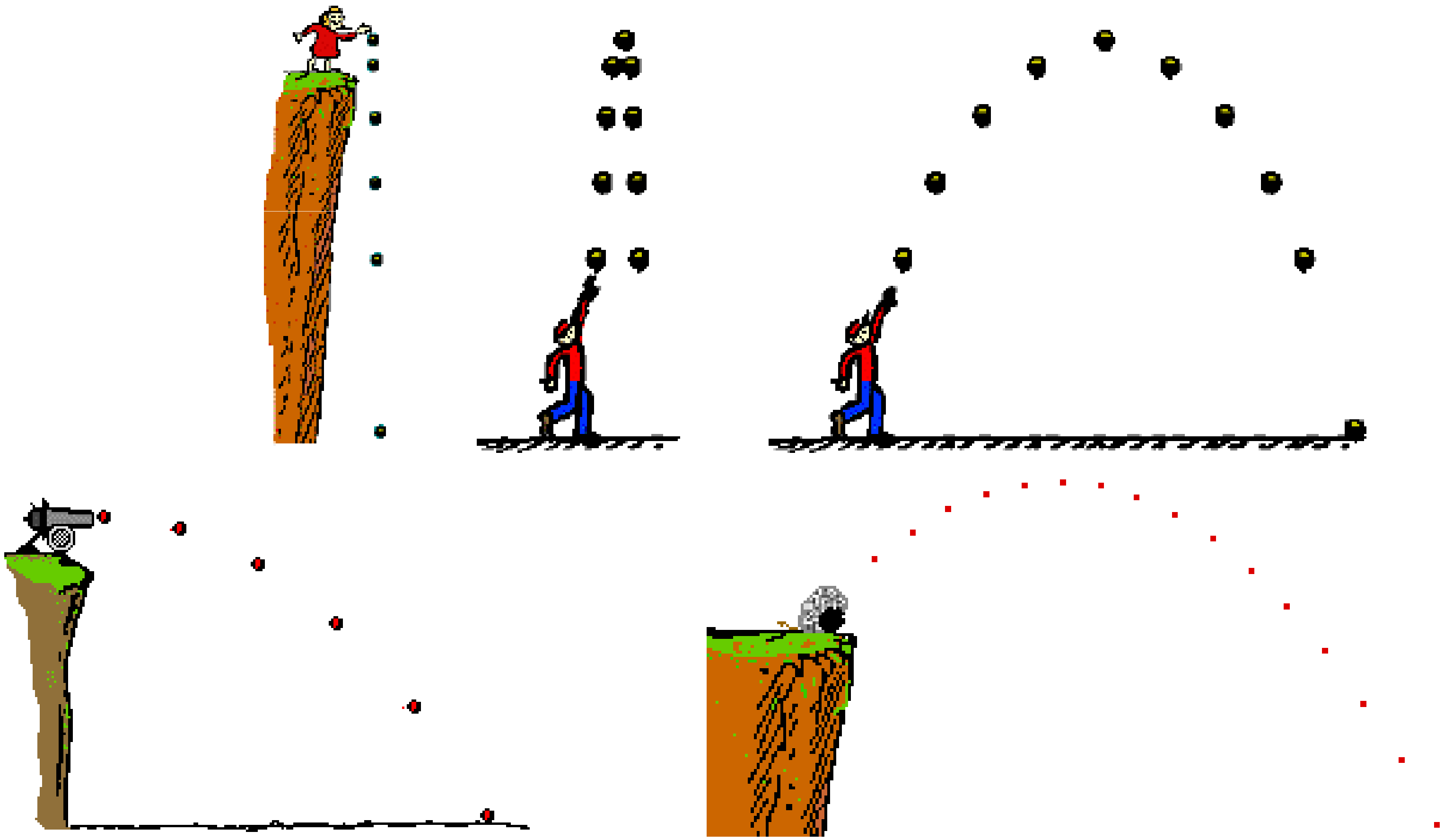






y
Max. height
Projectile
Projectile's Path
θ_0
x
R
Launching point
The range R
Landing point

Types of Projectiles

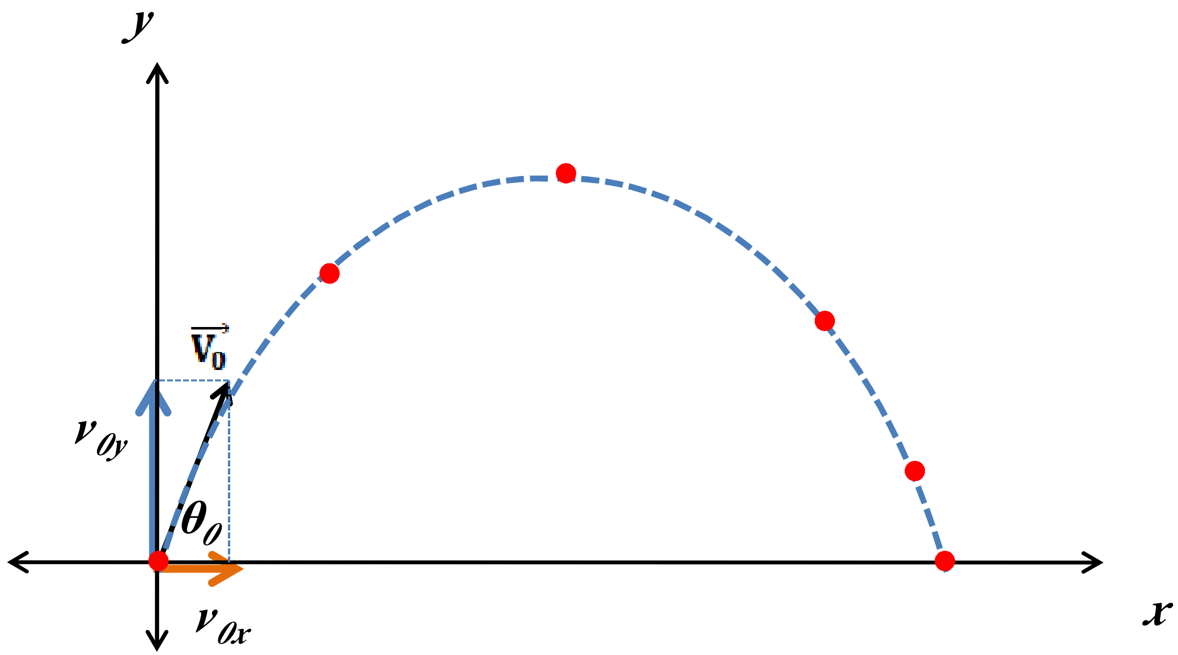


In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other.

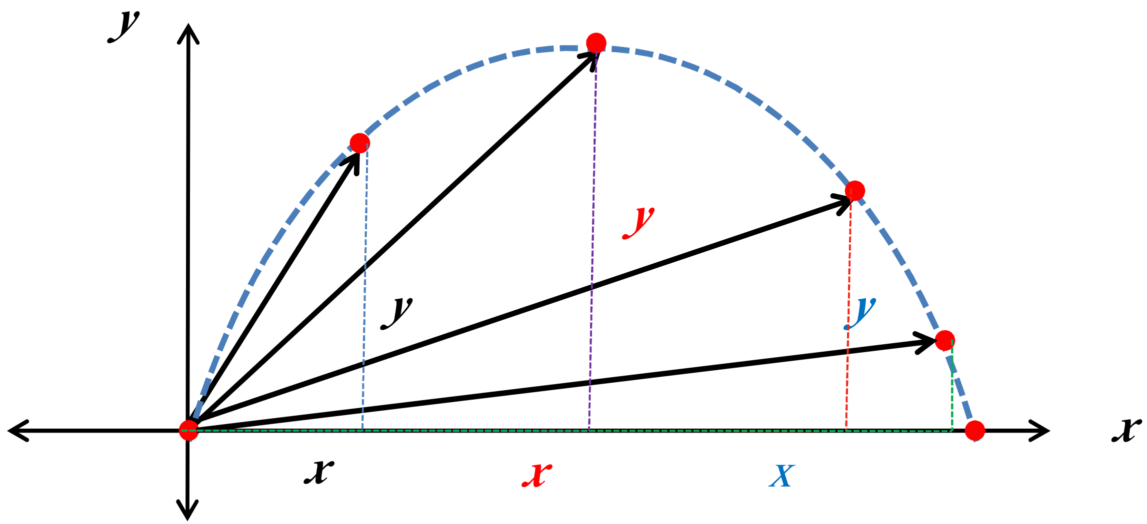
$$\vec{v}_0 = v_{0x}\hat{\mathrm{i}} + v_{0y}\hat{\mathrm{j}}.$$

$$v_{0x} = v_0 \cos\theta_0$$

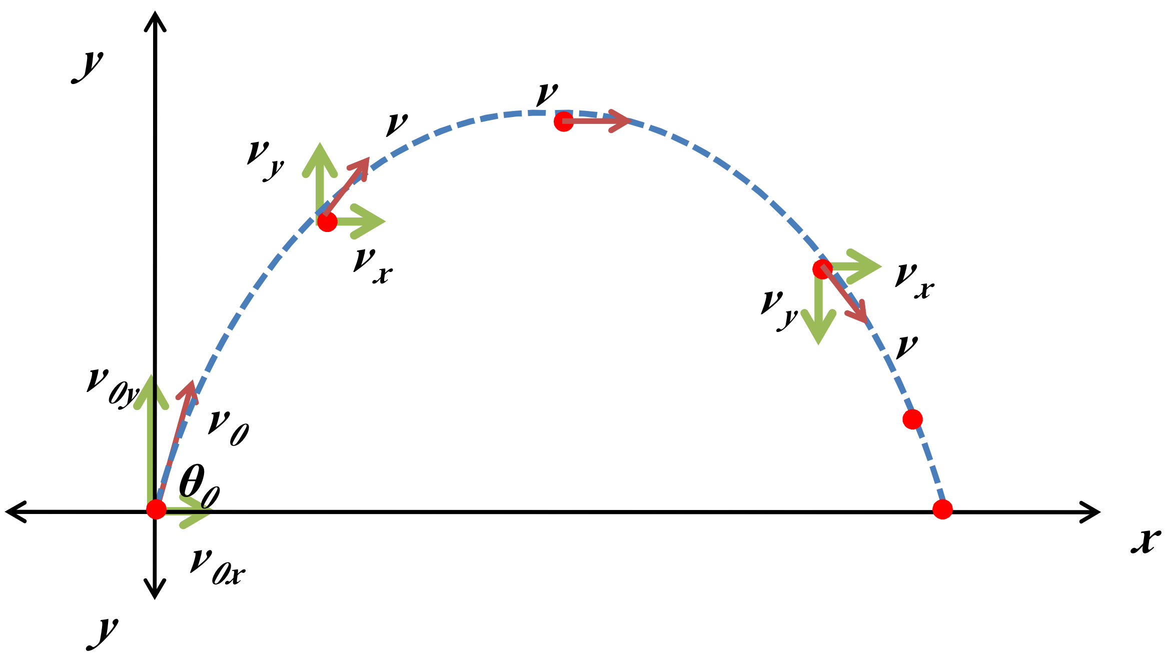
$$v_{0y} = v_0 \sin\theta_0$$

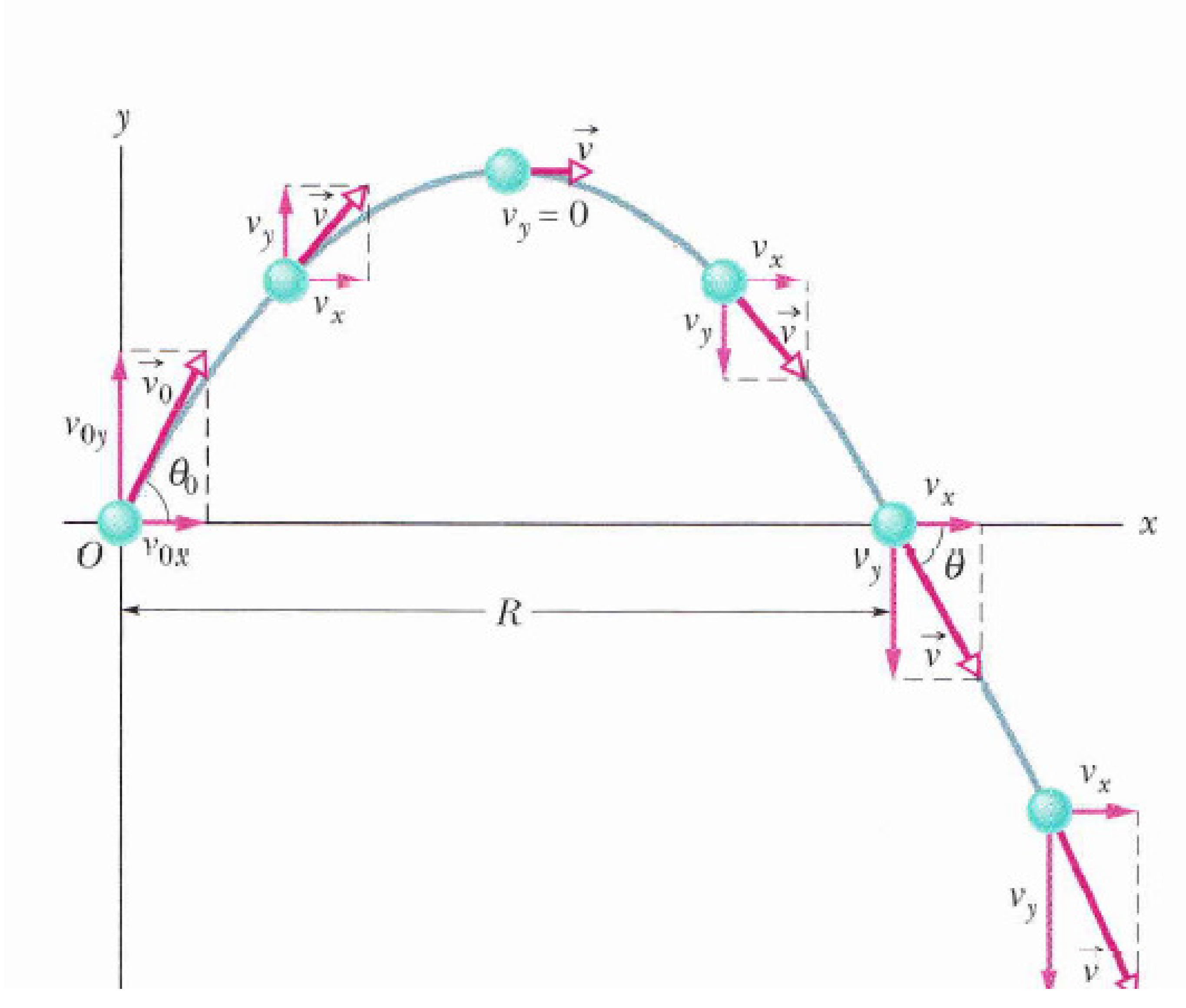


4-6 | Projectile Motion Analyzed



4-6 | Projectile Motion Analyzed





y
x
O
$\vec{v}_0$
v_{0y}
v_{0x}
θ_0
v_y
$\vec{v}$
v_x
$v_y = 0$
θ
R

Projectile motion

Horizontal Motion

Vertical Motion

$$a_x = 0$$

$$v_{0x} = v_0 \cos \theta_0$$

$$v_x = v_{0x}$$

$$x - x_0 = v_{0x} t.$$

$$x - x_0 = (v_0 \cos \theta_0) t.$$

$$v = v_0 + at$$

$$x - x_0 = v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \tfrac{1}{2}(v_0 + v)t$$

$$x - x_0 = vt - \tfrac{1}{2} a t^2$$

$$a_y = -g$$

$$v_{0y} = v_0 \sin \theta_0$$

$$v_y = v_{0y} - g\,t$$

$$v_y = v_0 \sin \theta_0 - g\,t$$

$$y - y_0 = v_{0y} t - \tfrac{1}{2} g t^2$$

$$= (v_0 \sin \theta_0) t - \tfrac{1}{2} g t^2,$$

Projectile motion

Horizontal Motion

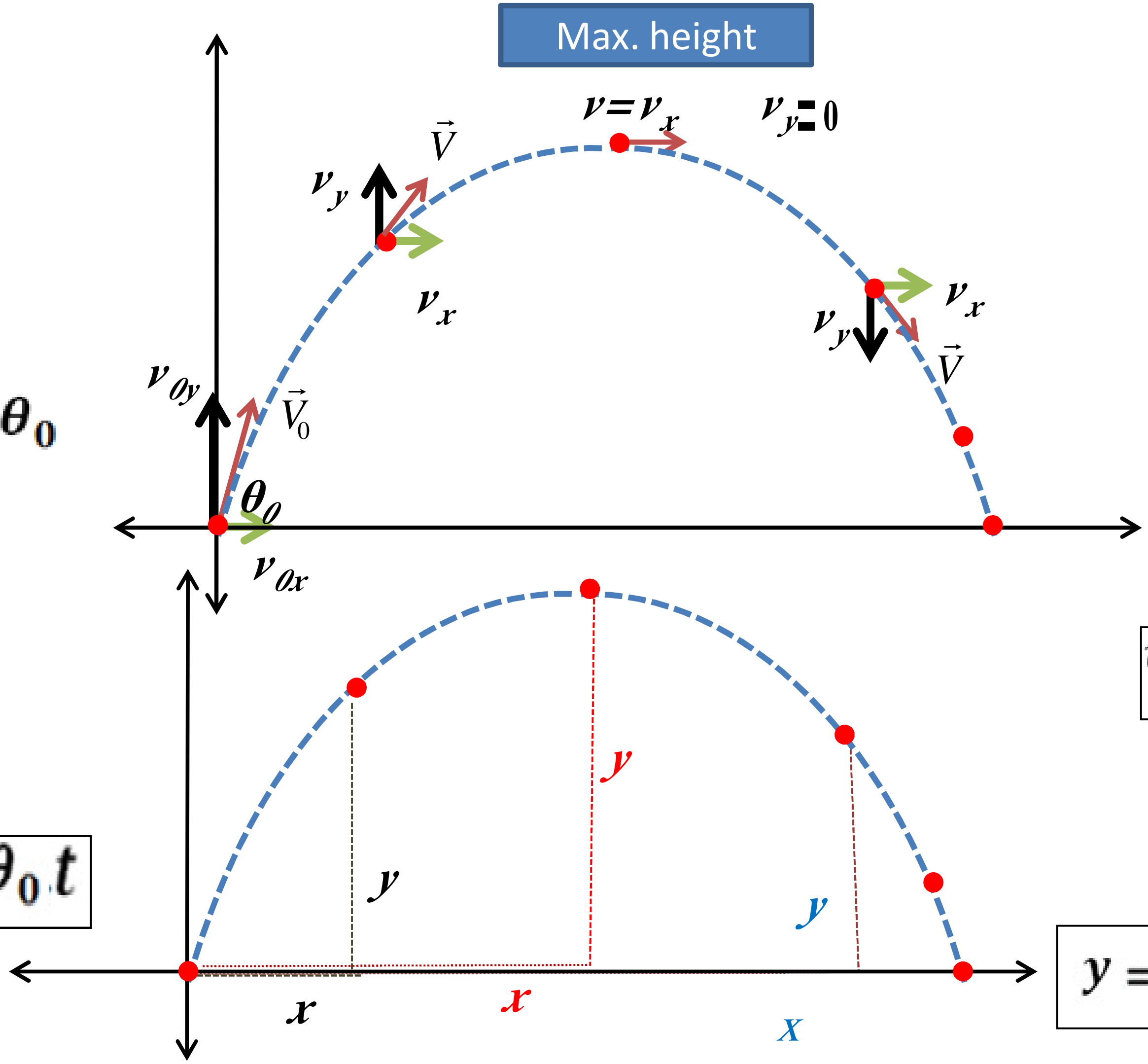
$$a_x = 0$$

$$v_{0x} = v_0 \cos\theta_0$$

$$v_x = v_{0x}$$

$$x = v_{0x} t$$

$$x = v_0 \cos\theta_0 \, t$$



Vertical Motion

$$a_y = -g$$

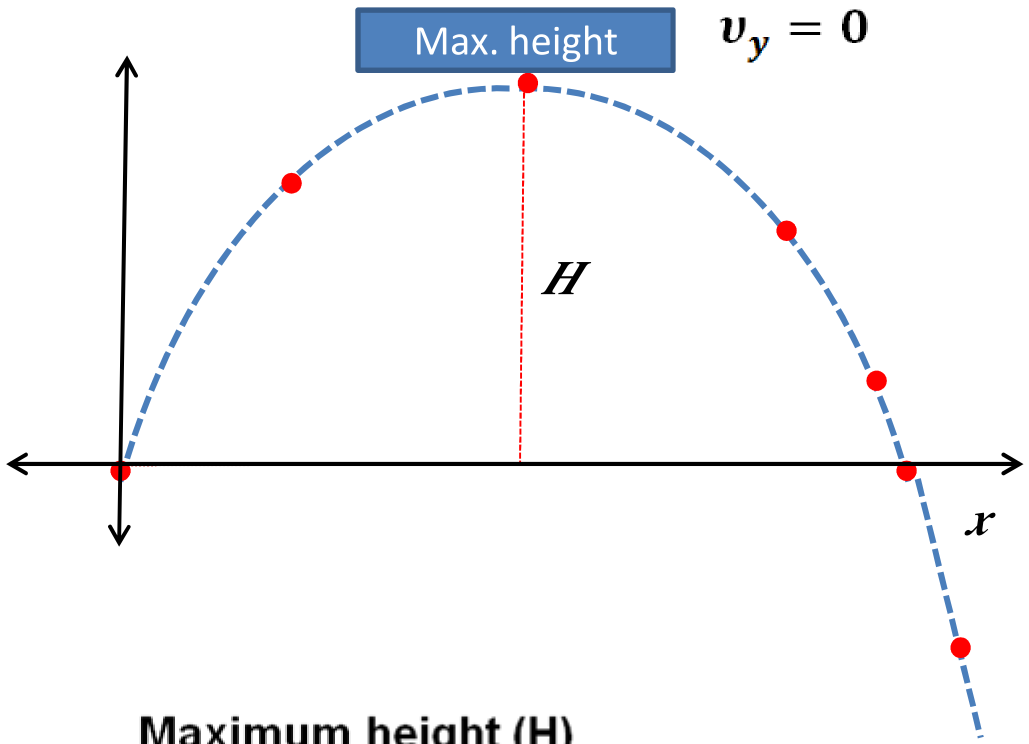
$$v_{0y} = v_0 \sin\theta_0$$

$$v_y = v_{0y} - g\,t$$

$$v_y = v_0 \sin\theta_0 - g\,t$$

$$y = v_{0y} t - \frac{1}{2} g t^2$$

$$y = v_0 \sin\theta_0 \, t - \frac{1}{2} g t^2$$



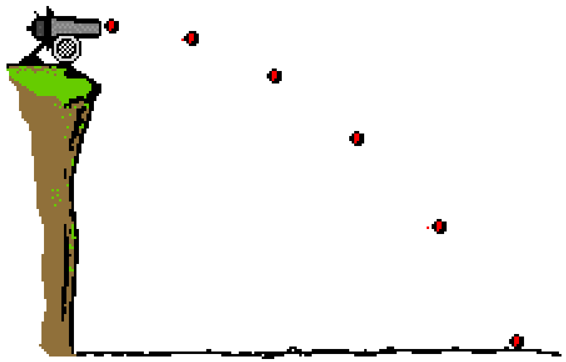
Maximum height (H)

$$H = \frac{(v_0 \sin\theta_0)^2}{2g}$$

PROBLEMS

•21 A projectile is fired horizontally from a gun that is 45.0 m above flat ground, emerging from the gun with a speed of 250 m/s. (a) How long does the projectile remain in the air?

(b) At what horizontal distance from the firing point does it strike the ground?



(c) What is the magnitude of the vertical component of its velocity as it strikes the ground?

••38 You throw a ball toward a wall at speed 25.0 m/s and at angle $\theta_0 = 40.0°$ above the horizontal (Fig. 4-38). The wall is distance $d = 22.0$ m from the release point of the ball. (a) How far above the release point does the ball hit the wall?

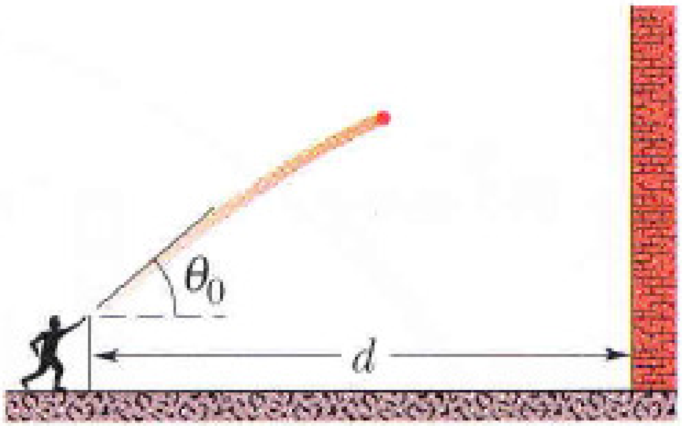


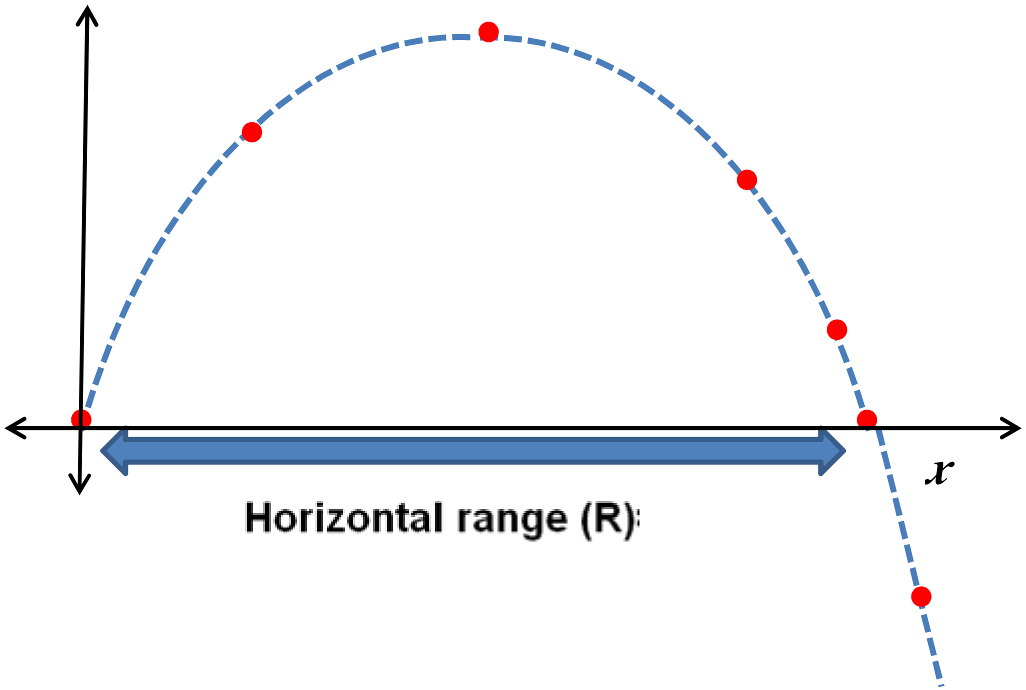
FIG. 4-38 Problem 38.

What are the (b) horizontal and (c) vertical components of its velocity as it hits the wall?

(d) When it hits, has it passed the highest point on its trajectory?

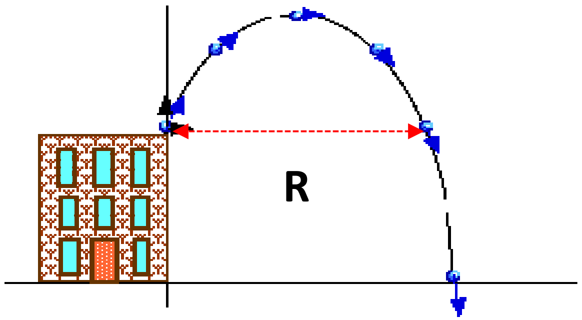
horizontal range R, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

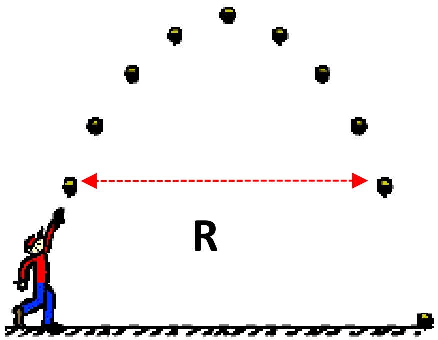
$$R = \frac{v_0^2}{g}\sin 2\theta_0$$



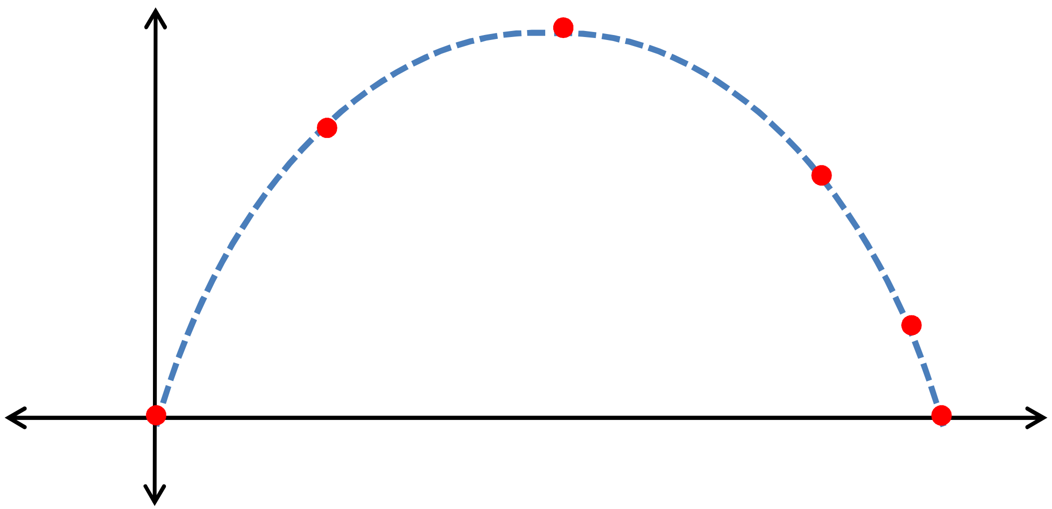
Maximum range

$$\theta_0 = 45^0 \rightarrow R_{max} = \frac{v_0^2}{g}$$





The equation of the projectile path (TRAJECTORY)

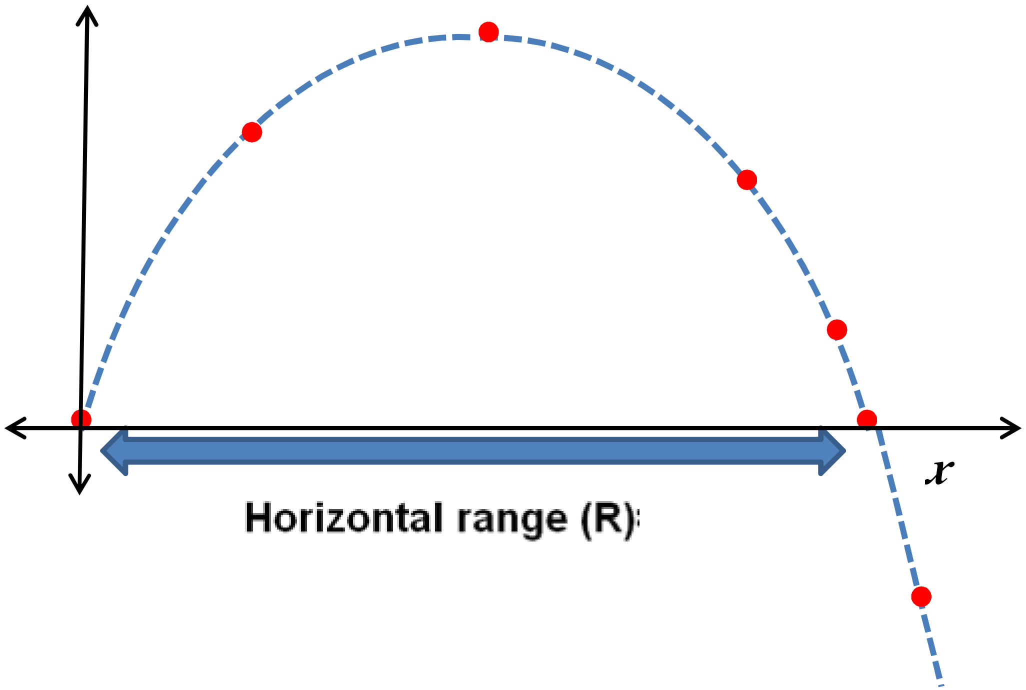


$$y = (\tan\theta_0)x - \frac{gx^2}{2(v_0\cos\theta_0)^2}$$

This is the equation of a parabola, so the projectile path is parabolic

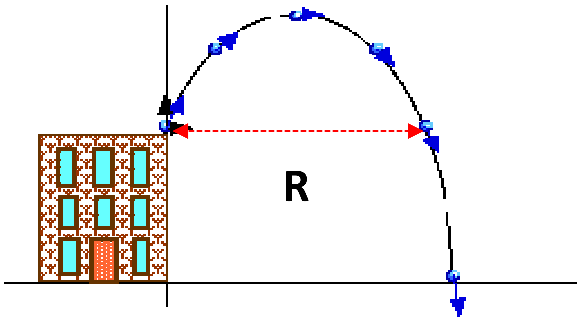
horizontal range R, which is the horizontal distance from the launch point to the point at which the particle returns to the launch height, is

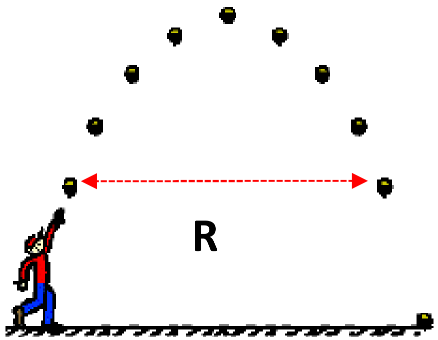
$$R = \frac{v_0^2}{g}\sin 2\theta_0$$



Maximum range

$$\theta_0 = 45^0 \rightarrow R_{max} = \frac{v_0^2}{g}$$

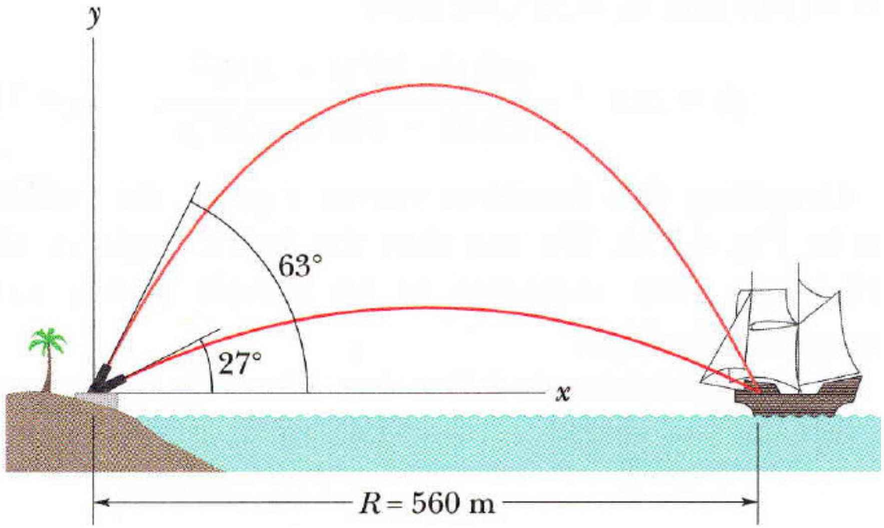




Sample Problem 4-7

Figure 4-16 shows a pirate ship 560 m from a fort defending a harbor entrance. A defense cannon, located at sea level, fires balls at initial speed $v_0 = 82$ m/s.

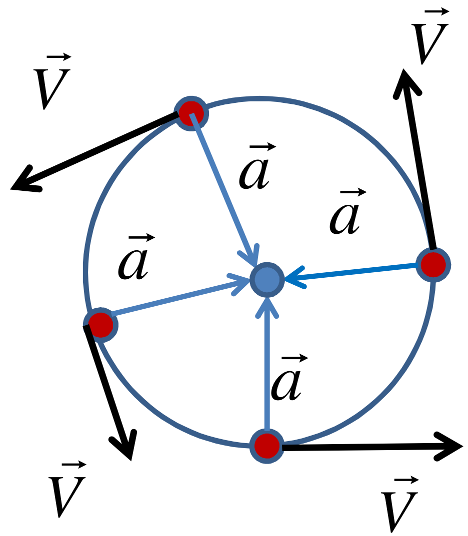
(a) At what angle θ_0 from the horizontal must a ball be fired to hit the ship?



(b) What is the maximum range of the cannonballs?

4-7 | Uniform Circular Motion

A particle is in uniform circular motion if it travels around a circle or circular arc at **constant speed.**



1-Velocity :

-magnitude constant v.

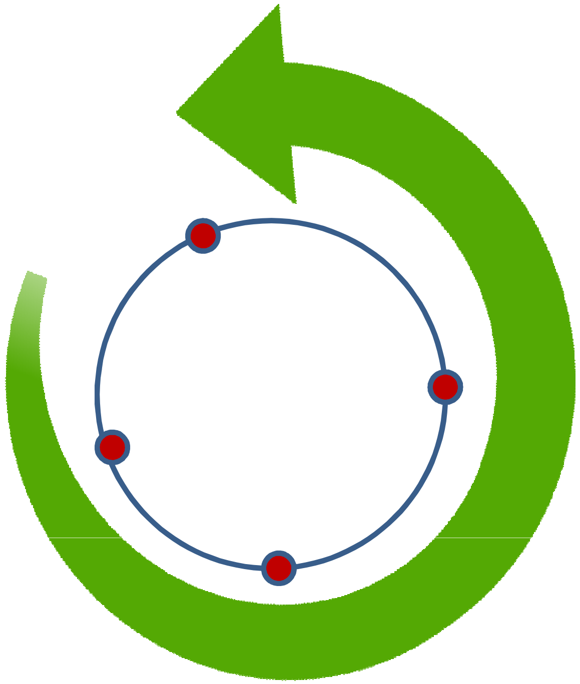
-direction :tangent to the circle in the direction of motion.

2- Acceleration:

Why is the particle accelerating even though the speed does not vary?

- magnitude $a = \frac{v^2}{r}$
- direction: toward the center.

- It is called **Centripetal acceleration(meaning seeking center)**



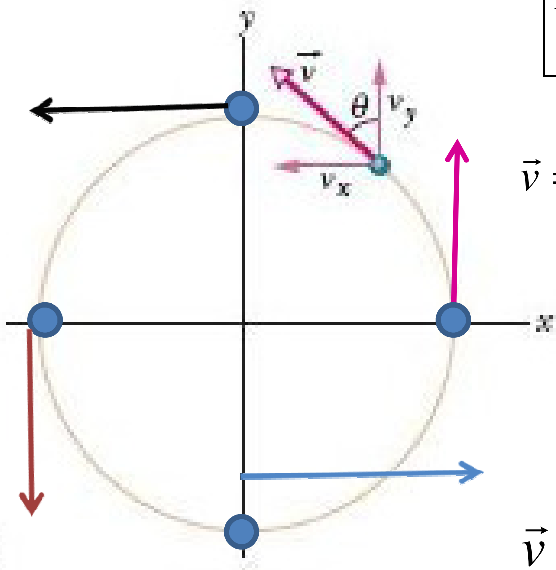
3- <u>Period:</u> is the time for a particle go around the circle once.

$$\text{Time} = \frac{\text{distance}}{\text{velocity}}$$

For one round $\Rightarrow$ distance = circumference of the circle

$$\boxed{T = \frac{2\pi r}{v}}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$



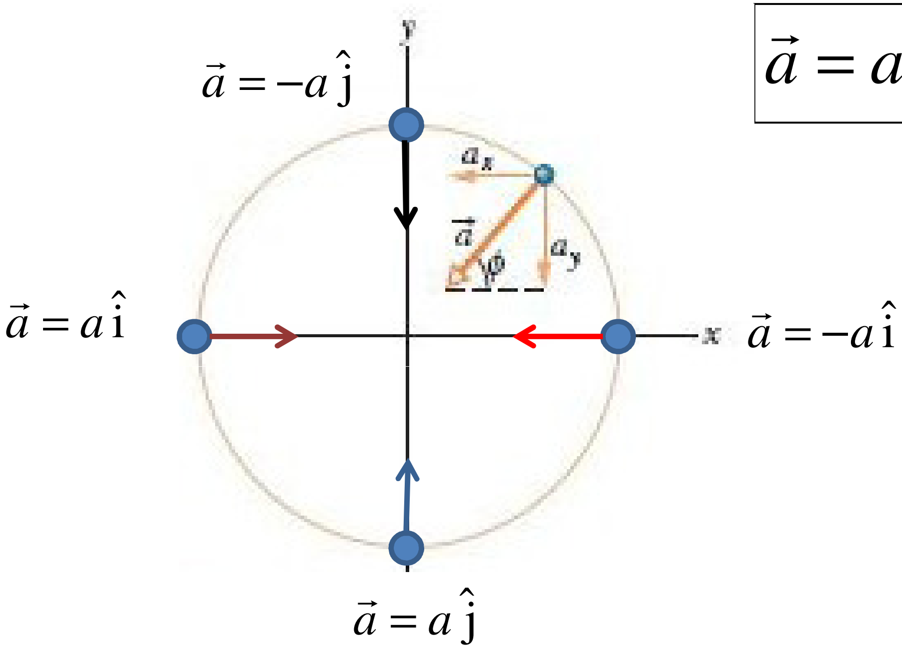
$\vec{v} = -v\,\hat{\mathrm{i}}$

$\vec{v} = v\,\hat{\mathrm{j}}$

$\vec{v} = -v\,\hat{\mathrm{j}}$

$\vec{v} = v\,\hat{\mathrm{i}}$

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$



Sample Problem 4-10

What is the magnitude of the acceleration, in g units, of a pilot whose aircraft enters a horizontal circular turn with a velocity of $\vec{v}_i = (400\hat{i} + 500\hat{j})$ m/s and 24.0 s later leaves the turn with a velocity of $\vec{v}_f = (-400\hat{i} - 500\hat{j})$ m/s?