الحركة الاهتزازية

(Oscillations)

1

Chapter 15
Oscillations

In this chapter we will cover the following topics:

Displacement, velocity, and acceleration of a simple harmonic oscillator

Energy of a simple harmonic oscillator

Examples of simple harmonic oscillators: spring-mass system, simple pendulum, physical pendulum, torsion pendulum

Damped harmonic oscillator

Forced oscillations/resonance

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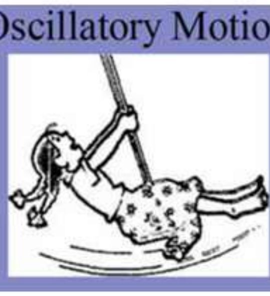
15-2: Simple Harmonic Motion (SHM)

Types of Motion

Translatory Motion	Rotatory Motion	Oscillatory Motion
		

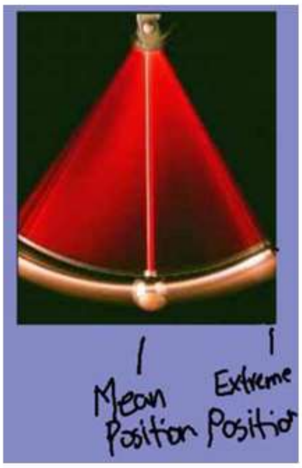
What means Oscillation?
Oscillations could also be called vibrations and cycles.

Oscillation is the *periodic variation, typically in time, of some measure.* as seen, for example, in a swinging pendulum.
Many things oscillate/vibrate: Periodic motion
(a motion that repeats itself over and over)

15-2: Simple Harmonic Motion (SHM)

Oscillatory Motion

- To & fro motion
- Motion along a mean position

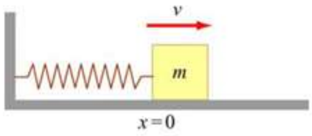


Why does something vibrate/oscillate?
Whenever the system is displaced from equilibrium, a restoring force pulls it back, but it overshoots the equilibrium position.

- Examples of oscillatory motion
  - Simple pendulum
  - Mass attached to a spring
  - Motion of atoms in a solid

نماذج للحركة التوافقية البسيطة :
١/ الحركة الأفقية لكتلة معلقة في خيط (البندول البسيط)
٢/ حركة كتلة معلقة علي نابض .
٣/ حركة قطع الفلين علي سطح الماء .
٤/ حركة الأرجوحة .
٥/ حركة وتر مشدود .
٦/ حركة مسطرة مثبتة من أحد طرفيها .

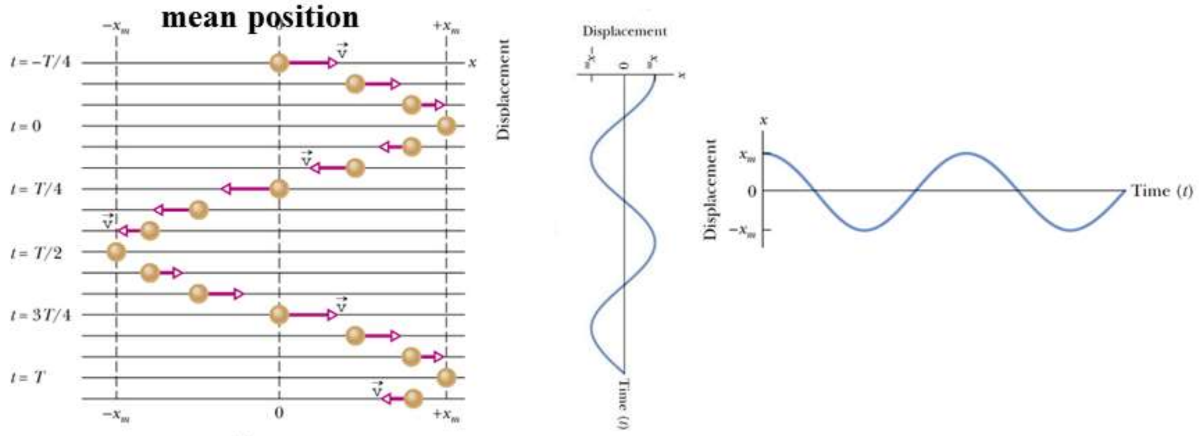
15-2: Simple Harmonic Motion (SHM)



Extreme position

equilibrium position

mean position

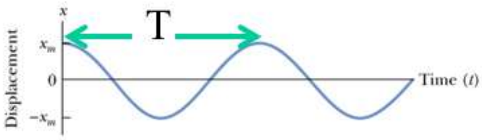


•The motion of a spring mass system is an example of simple harmonic motion

15-2: Simple Harmonic Motion (SHM)

Parameter used to describe vibrations

Important properties of oscillatory motion



Frequency

Number of vibration cycles per second.

$$Frequency = f = \frac{\text{cycles}}{\text{seconds}}$$

1 hertz = 1 Hz = 1 oscillation per second = 1 s^{-1}.

SI Units: 1/s **(Hz, Hertz)**

Period T : Time taken to complete one cycle of the vibration.

$$Period = T = \frac{\text{seconds}}{\text{cycles}} \qquad T = \frac{1}{f} \quad f = \frac{1}{T}$$

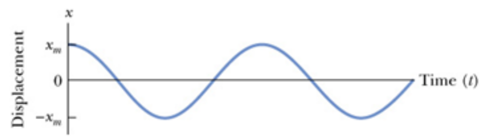
Units: s

15-2: Simple Harmonic Motion (SHM)

Parameter used to describe vibrations
Important properties of oscillatory motion

Any motion that repeats at regular intervals is called **periodic motion** or **harmonic motion**. However, here we are interested in a particular type of periodic motion called **simple harmonic motion** (SHM).

Any oscillation that can be expressed with a sinusoidal function is a harmonic oscillation. When its amplitude is constant, it is a simple harmonic oscillation.

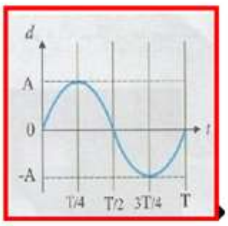


15-2: Simple Harmonic Motion (SHM)

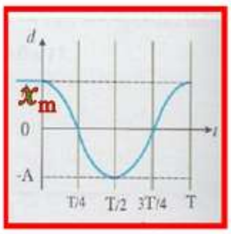
Parameter used to describe vibrations
Important properties of oscillatory motion

Displacement Curve

Starting from mean position — **Starting from Extreme position**



$$x(t) = x_m \sin(\omega t + \phi)$$



$$x(t) = x_m \cos(\omega t + \phi)$$

The displacement from equilibrium is given as function of time by

$$x(t) = x_m \cos(\omega t + \emptyset) \qquad \text{(displacement)}$$

In which x_m, ω, and $\emptyset$ are quantities

15-2: Simple Harmonic Motion (SHM)

Important properties of oscillatory motion

Phase argument

$$x(t) = x_m \cos(\omega t + \emptyset)$$

Displacement at time

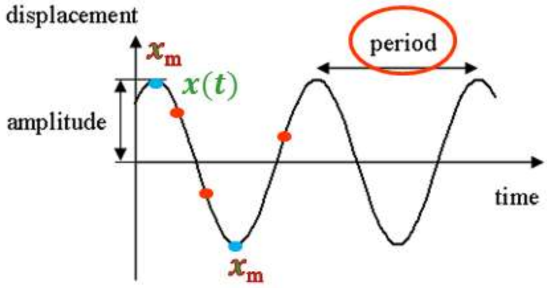
Amplitude

time

Phase constant
Or phase angle

Angular frequency

$$\omega = \frac{2\pi}{T} = 2\pi f$$



kinds of frequency

f
Frequency is measured in **hertz**

ω
angular frequency is measured in **radians per second**.

15-2: Simple Harmonic Motion (SHM)

Important properties of oscillatory motion

CHECKPOINT 1 A particle undergoing simple harmonic oscillation of period T (like that in Fig. 15-1) is at $-x_m$ at time $t = 0$. Is it at $-x_m$, at $+x_m$, at 0, between $-x_m$ and 0, or between 0 and $+x_m$ when (a) $t = 2.00T$, (b) $t = 3.50T$, and (c) $t = 5.25T$?

15-2: Simple Harmonic Motion (SHM)

Important properties of oscillatory motion

The velocity of SHM

$$x(t) = x_m \cos(\omega t + \phi)$$

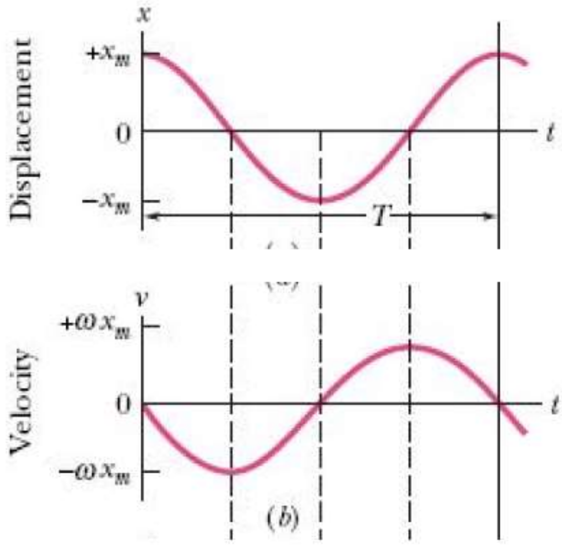
$$v(t) = \frac{dx(t)}{dt} = \frac{d}{dt}[x_m \cos(\omega t + \phi)]$$

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

$$v_m = \omega x_m$$

Velocity amplitude

$$v(t) = -v_m \sin(\omega t + \emptyset)$$



- The speed is zero at extreme points.
- The speed is greatest at $x = 0$.

15-2: Simple Harmonic Motion (SHM)

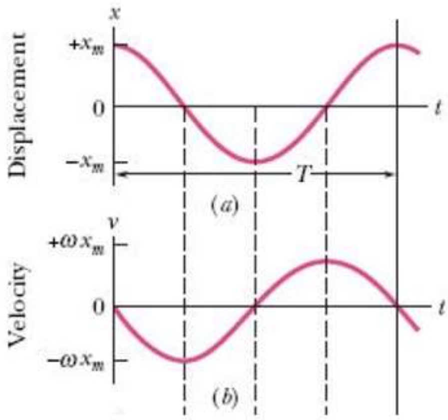
Important properties of oscillatory motion

The velocity of SHM

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

From figure:

- The velocity oscillating particle varies between the limits $\pm v_m = \pm \omega x_m$.
- The curve of $v(t)$ is shifted (to the left) from the curve of $x(t)$ by one-quarter period.
- When the magnitude of the displacment is greatest ($x(t) = x_m$), the magnitude of the velocity is least ($v(t) = 0$)
- When the magnitude of the displacment is least ($x(t) = 0$), the magnitude of the velocity is greatest ($v(t) = \omega x_m 0$)



15-2: Simple Harmonic Motion (SHM)
Important properties of oscillatory motion

The acceleration of SHM

$$v(t) = -\omega\, x_m \sin(\omega t + \phi)$$

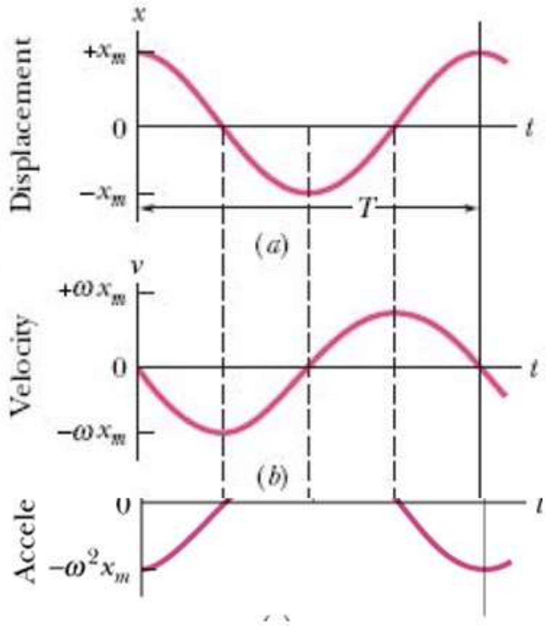
$$a(t) = \frac{dv(t)}{dt} = \frac{d}{dt}[-\omega x_m \sin(\omega t + \phi)]$$

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

$$a_m = \omega^2 x_m$$

acceleration amplitude

$$a(t) = -\omega^2 x(t)$$



In SHM, the acceleration a is proportional to the displacement x but opposite in sign, and the two quantities are related by the square of the angular frequency ω.

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15-2: Simple Harmonic Motion (SHM)
Important properties of oscillatory motion

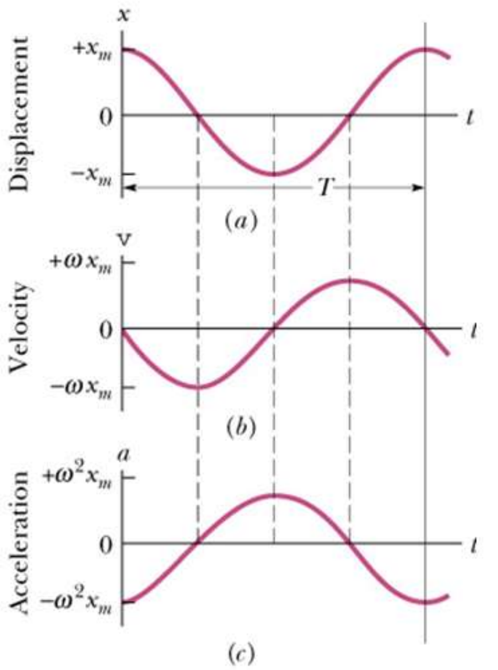
The acceleration of SHM

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$

From figure:

- **The acceleration of particle varies between the limits $\pm a_m = \pm \omega^2 x_m$.**
- **The curve of $v(t)$ is shifted (to the left) by one-quarter period (¼ T) relative the velocity curve $v(t)$.**
- **When the displacment has its greatest positive value, the acceleration has its greatest negative value.**

•When the displacement is zero, the acceleration is also zero.



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15-2: Simple Harmonic Motion (SHM)

A particle oscillates with simple harmonic motion, so that its displacement varies according to the expression x = (5 cm)cos(2t + π/6) where x is in centimeters and t is in seconds. At t = 0 find

(a) the displacement of the particle, (b) its velocity, and

(c) its acceleration. (d) Find the period and amplitude of the motion.

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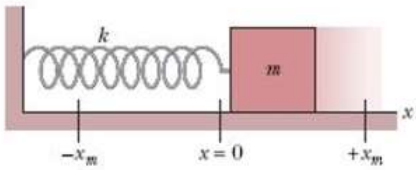
15-3: The Force Law for Simple Harmonic Motion (SHM)

The block-spring system Linear Simple Harmonic oscillator

For simple harmonic motion, the force is proportional to the displacement

- **Hooke's law:**

$$F = -kx$$

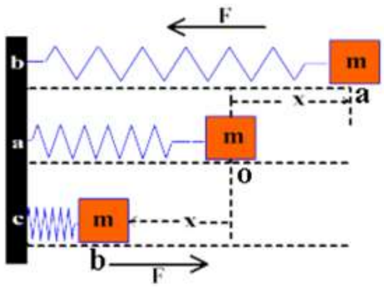


- **From the Newton's Second Law:**

$$F = ma$$

$$F = -m\,\omega^2 x$$

$$k = m\omega^2$$



Angular frequency

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega = \frac{2\pi}{T} \qquad T = \frac{2\pi}{\omega} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Simple harmonic motion is the motion of a particle when the force acting on it is proportional to the particle's displacement but in the opposite direction.

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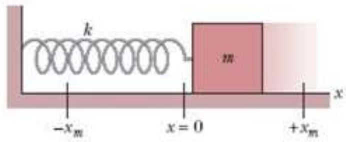
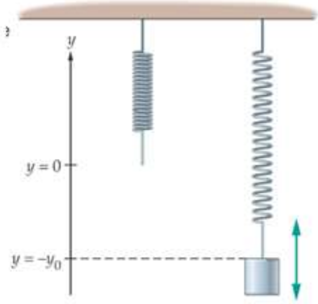
15-3: The Force Law for Simple Harmonic Motion (SHM)

The block-spring system in vertical setup



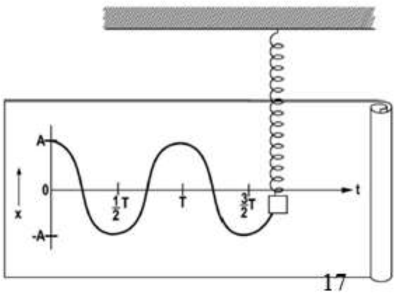
When a mass-spring system is oriented vertically, it will exhibit SHM with the same period and frequency as a horizontally placed system.



Same formulae as for the horizontal setup but the system oscillates around a new equilibrium position y_0.

Checkpoint 3

Which of the following relationships between the force F on a particle and the particle's position x gives SHM:



(a) $F = -5x$, **(b)** $F = -400x^2$,

(c) $F \;= 10x$, **(d)** $F \;\; 3x^2$?

15-3: The Force Law for Simple Harmonic Motion (SHM)

Sample Problem 15-1:

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(a) What are the angular frequency, the frequency, and the period of the resulting motion?

15-3: The Force Law for Simple Harmonic Motion (SHM)

Sample Problem 15-1:

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(b) What is the amplitude of the oscillation?

(c) What is the maximum speed v_m of the oscillating block, and where is the block when it has this speed?

15-3: The Force Law for Simple Harmonic Motion (SHM)

Sample Problem 15-1:

A block whose mass m is 680 g is fastened to a spring whose spring constant k is 65 N/m. The block is pulled a distance $x = 11$ cm from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$.

(d) What is the amplitude of the oscillation?

(e) What is the amplitude of the oscillation?

15-3: The Force Law for Simple Harmonic Motion (SHM)

Sample Problem 15-2:

At $t = 0$, the displacement $x(0)$ of the block in a linear oscillator is -8.50 cm. (Read $x(0)$ as "x at time zero.") The block's velocity $v(0)$ then is 0.920 m/s, and its acceleration $a(0)$ is 47.0 m/s^2.

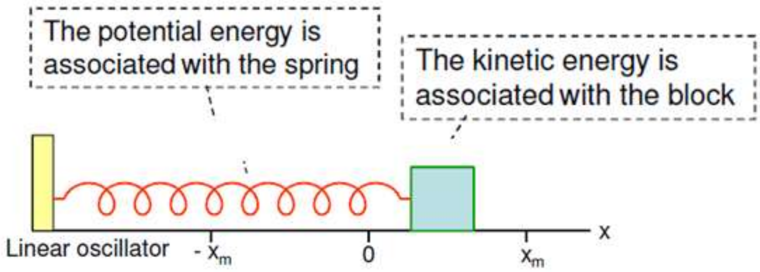
(a) What is the angular frequency ω of this system?

(b) What are the phase constant f and amplitude x_m?

15-4: Energy in Simple Harmonic Motion (SHM)

E= P.E + K.E

$$E = U + K$$



U: Potential Energy

$$U(t) = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kx_m^2 \cos^2(\omega t + \phi)$$

K: Kinetic Energy

$$K(t) = \tfrac{1}{2}mv^2 = \tfrac{1}{2}kx_m^2 \sin^2(\omega t + \phi)$$

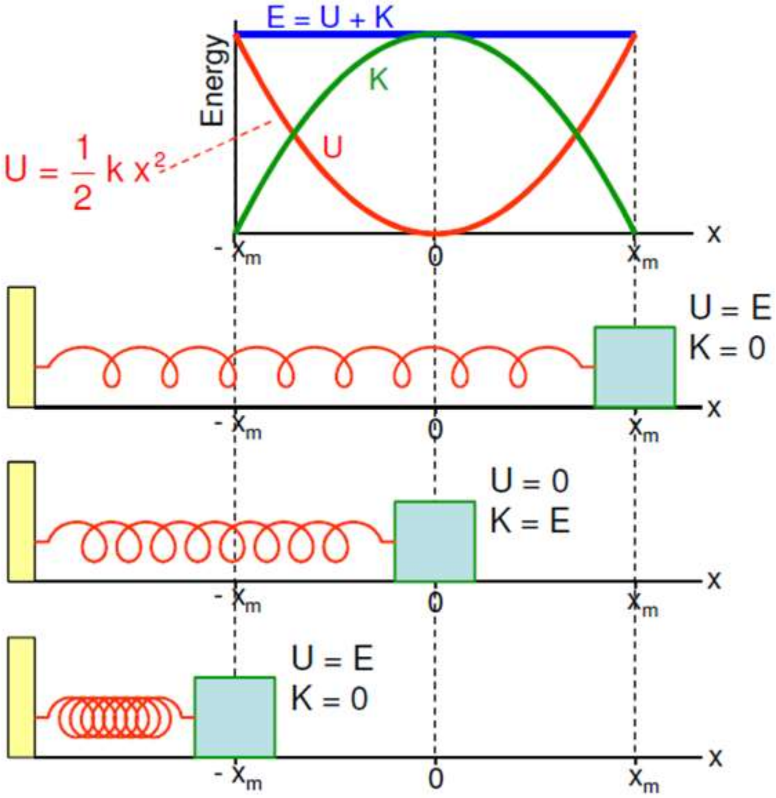
The mechanical energy

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2,$$

$$E = \frac{1}{2} k x_m^2 = \text{constant}$$

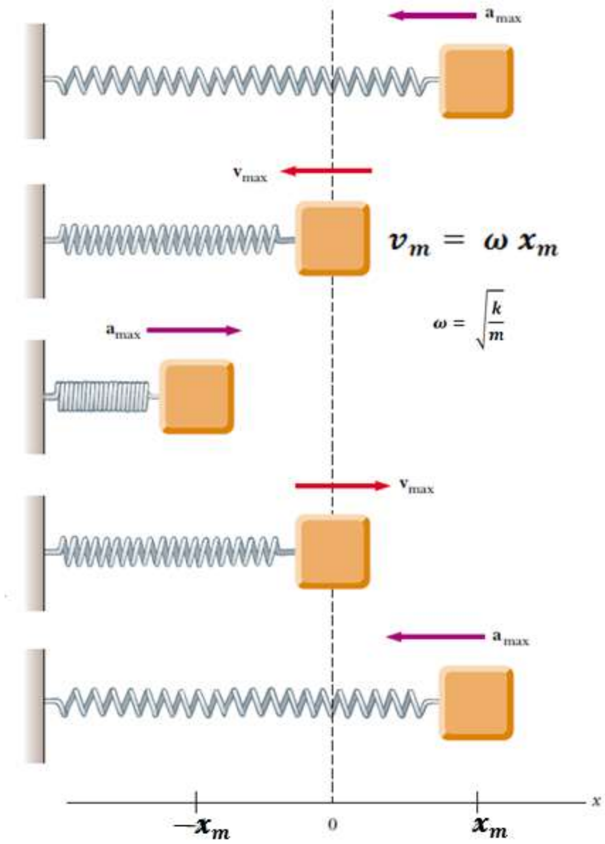
The total mechanical energy of a simple harmonic oscillator is a constant of the motion and is proportional to the square of the amplitude.

15-4: Energy in Simple Harmonic Motion (SHM)



The mechanical energy of a linear oscillator is indeed constant and independent of time.

15-4: Energy in Simple Harmonic Motion (SHM)

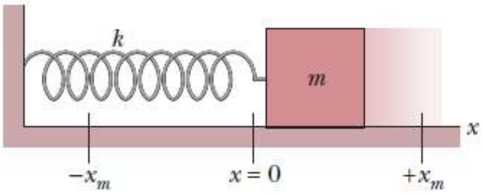


t	x	v	a	K	U
0	x_m	0	$-\omega^2 x_m$	0	$\frac{1}{2}kx_m^2$
$T/4$	0	$-\omega x_m$	0	$\frac{1}{2}kx_m^2$	0
$T/2$	$-xm$	0	$\omega^2 x_m$	0	$\frac{1}{2}kx_m^2$
$3T/4$	0	ωx_m	0	$\frac{1}{2}kx_m^2$	0
T	x_m	0	$-\omega^2 x_m$	0	$\frac{1}{2}kx_m^2$

15-4: Energy in Simple Harmonic Motion (SHM)

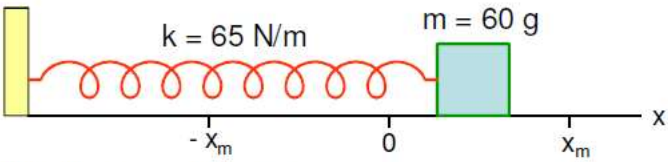
Checkpoint 4

In Fig. 15-7, the block has a kinetic energy of 3 J and the spring has an elastic potential energy of 2 J when the block is at $x = +2.0$ cm. (a) What is the kinetic energy when the block is at $x = 0$? What is the elastic potential energy when the block is at (b) $x = -2.0$ cm and (c) $x = -x_m$?



Example

Mass of the block m = 60 g.
Spring constant k = 65 N/m.
Initially, the block position is at x = 11 cm and its speed is v = 0.
What is the mechanical energy of this linear oscillator?



What is the potential energy U and the kinetic energy K of the oscillator when the block is at $x = \frac{1}{2} x_m$?

15-4: Energy in Simple Harmonic Motion (SHM)

Sample Problem 15-3:

Suppose the block has mass m =2.72x10^{5} kg and is designed to oscillate at frequency f=10.0 Hz and with amplitude x_m=20 cm.

(a) What is the total mechanical energy E of the spring block system?

(b) What is the block's speed as it passes through the equilibrium point?

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15-4: Energy in Simple Harmonic Motion (SHM)

Example:

A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless air track.

(A) Calculate the total energy of the system and the maximum speed of the cart if the amplitude of the motion is 3.00 cm.

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15-4: Energy in Simple Harmonic Motion (SHM)

Example:

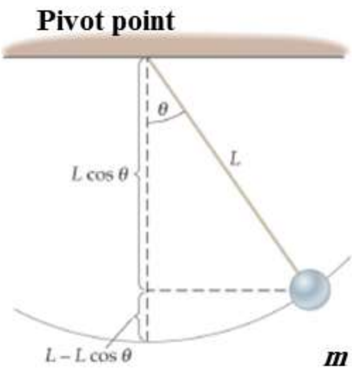
A 0.500-kg cart connected to a light spring for which the force constant is 20.0 N/m oscillates on a horizontal, frictionless air track.

(b) What is the velocity of the cart when the position is 2.00 cm? (the amplitude of the motion is 3.00 cm.)

15-6: Pendulums

The simple pendulum

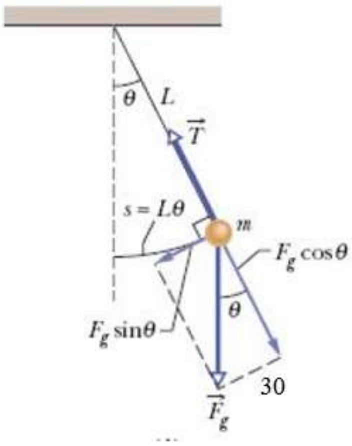
A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.(the angle is small, sin $\theta \approx \theta$.



$$T = 2\pi\sqrt{\frac{L}{g}}$$

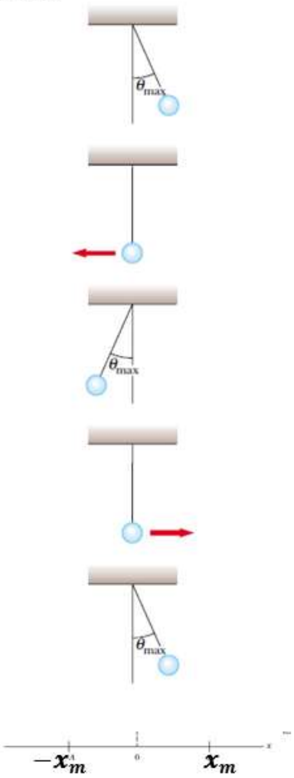
(simple pendulum, small amplitude)

[θ small]



15-6: Pendulums

Energy in Pendulu



t	x	v	a	K	U
0	x_m	0	$-\omega^2 x_m$	0	$\frac{1}{2}kx_m^2$
$T/4$	0	$-\omega x_m$	0	$\frac{1}{2}kx_m^2$	0
$T/2$	$-xm$	0	$\omega^2 x_m$	0	$\frac{1}{2}kx_m^2$
$3T/4$	0	ωx_m	0	$\frac{1}{2}kx_m^2$	0
T	x_m	0	$-\omega^2 x_m$	0	$\frac{1}{2}kx_m^2$

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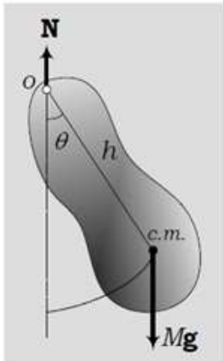
15-6: Pendulums

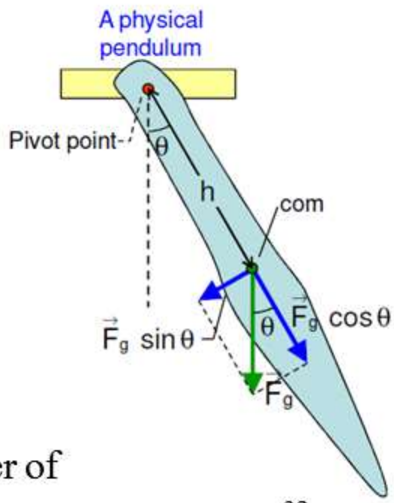


The physical pendulum

A real pendulum, usually called a **physical pendulum,** can have a complicated distribution of mass.

The period of a physical pendulum swinging through small angles is

$$T = 2\pi\sqrt{\frac{I}{m\,g\,h}}$$



I : the pendulum the rotational inertia about the pivot point.
h : a distance from the pivot point *O* to the center of mass *COM.*

Note for the physical pendulum the rotational inertia I depends on the shape of the pendulum and it is not mh^2.

A physical pendulum will not swing if it pivoted at its center of mass. When $h \rightarrow 0$, $T \rightarrow \infty$.

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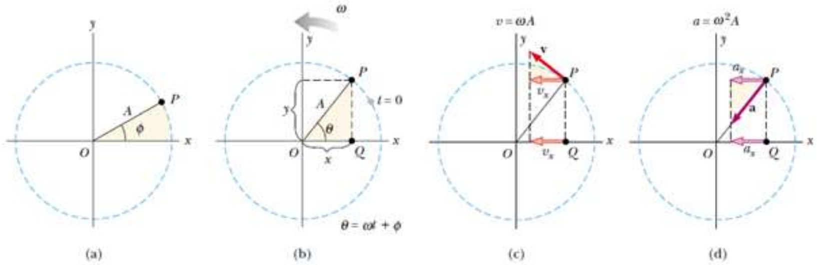
15-6: Pendulums

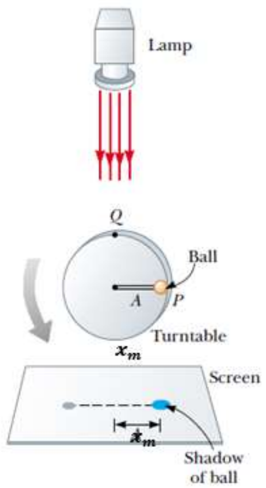
The physical pendulum

Example

A meter stick swings about a point at one end at a distance h from the stick's COM. What is the period of oscillation? ? (Hint I = $\frac{1}{3}mL^2$)

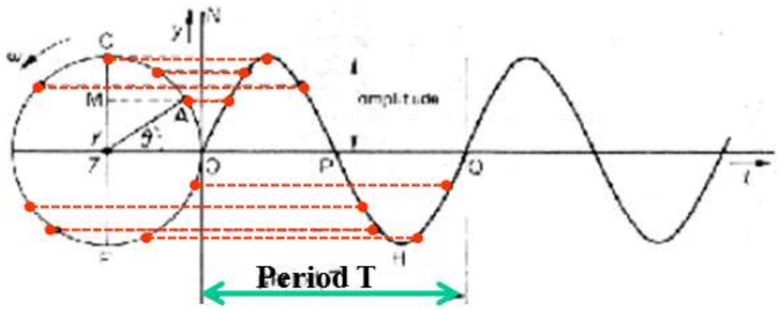
15-7: Simple Harmonic Motion and Uniform Circular Motion

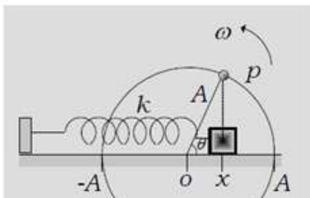




$x(t) = x_m \cos(\omega t + \phi)$ $y(t) = y_m \cos(\omega t + \phi)$

Simple harmonic motion is the projection of uniform circular motion on the diameter of the circle in which the circular motion occurs

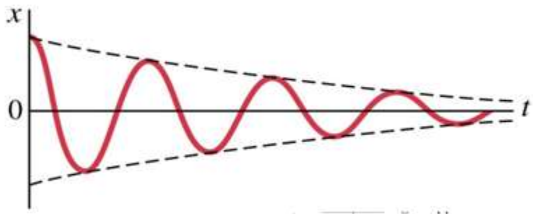




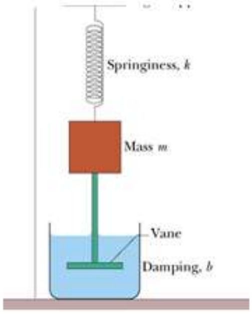
15-8: Damped Simple Harmonic Motion

When the motion of an oscillator is reduced by an external force, the oscillator and its motion are said to be damped.

Damped harmonic motion is harmonic motion with a frictional or drag force. If the damping is small, we can treat it as an "envelope" that modifies the undamped oscillation.



An example is given in the figure. A mass m attached to a spring of spring constant k oscillates vertically. The oscillating mass is attached to a vane submerged in a liquid. The liquid exerts a damping force $\vec{F}_d$ whose magnitude is given by the equation $F_d = -bv$.



The negative sign indicates that $\vec{F}_d$ opposes the motion of the oscillating mass.

The parameter b is called the damping constant.

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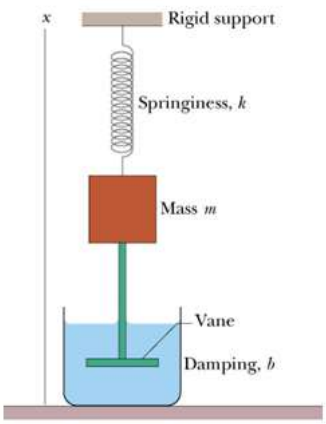
15-8: Damped Simple Harmonic Motion

The net force on m is $F_{net} = -kx - bv.$

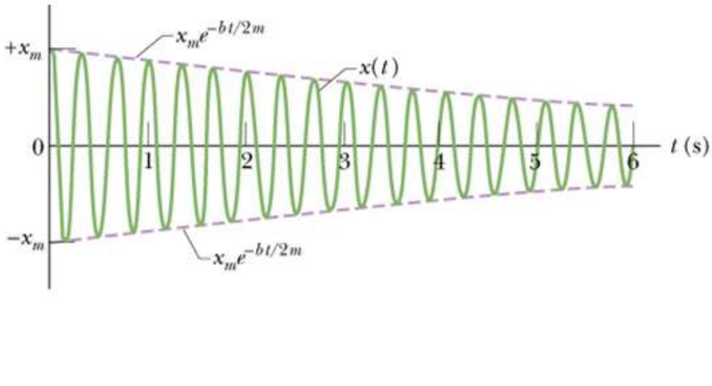


The solution of this equation is

$$x(t) = \underbrace{x_m e^{-bt/2m}}_{\text{time-dependent amplitude}} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

where x_m is the amplitude and ω' is the angular frequency of the damped oscillator.



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15-8: Damped Simple Harmonic Motion

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}.$$

If $b = 0$ (there is no damping),

$$\omega' = \sqrt{\frac{k}{m}} \qquad \omega' \approx \omega$$

$$x(t) = x_m \cos(\omega t + \phi)$$

the displacement of an undamped oscillator

15-8: Damped Simple Harmonic Motion

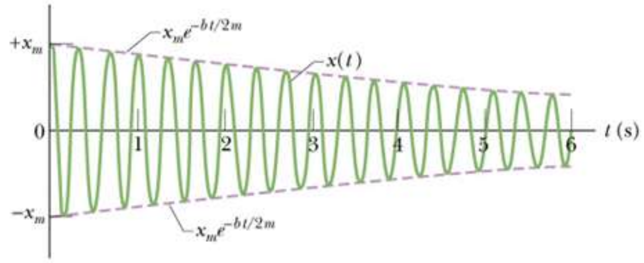
Damped Energy

If the oscillator is damped, amplitude gradually decreases with time. Then, the mechanical energy is not constant but decreases with time.

$$x(t) = \boxed{x_m e^{-bt/2m}} \cos(\omega' t + \phi)$$

If the damping is small

$$E = \frac{1}{2} k x_m^2 \text{ = constant} \qquad x_m \rightarrow x_m e^{-bt/2m}$$



$$E(t) \approx \frac{1}{2} k x_m^2 e^{-bt/m}.$$

The mechanical energy decreases exponentially with time.

15-9: Forced Oscillations and Resonance

We have seen that the mechanical energy of a damped oscillator decreases in time as a result of the resistive force. It is possible to compensate for this energy decrease by applying an external force that does positive work on the system. At any instant, energy can be transferred into the system by an applied force that acts in the direction of motion of the oscillator.

For example, a child on a swing can be kept in motion by appropriately timed "pushes." The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the decrease in mechanical energy in each cycle that results from resistive forces.

15-9: Forced Oscillations and Resonance

A person swinging in a swing without anyone pushing it is an example of *free oscillation*. However, if someone pushes the swing periodically, the swing has *forced*, or *driven, oscillations*.

Two angular frequencies are associated with a system undergoing driven oscillations:

(1) the *natural* angular frequency ω of the system, which is the angular frequency at which it would oscillate if it were suddenly disturbed and then left to oscillate freely.

(2) the angular frequency ω_d of the external driving force causing the driven oscillations.

15-9: Forced Oscillations and Resonance

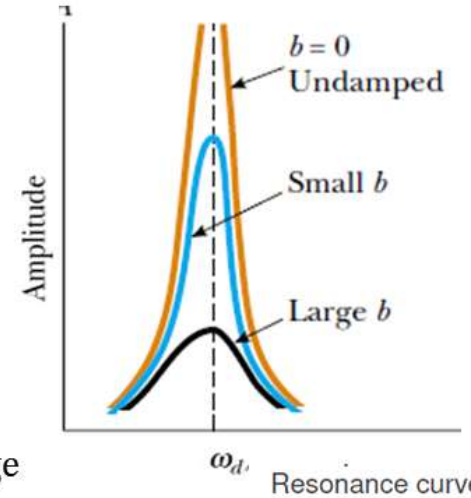
its displacement $x(t)$ is given by $\boldsymbol{x(t) = x_m \cos(\omega_d t + \varphi)}$

Where x_m (Amplitude of a driven oscillator) is

$$x_m = \frac{F_0/m}{\sqrt{(\omega_d^2 - \omega^2)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

and where $\omega = \sqrt{k/m}$ is the natural frequency of the undamped oscillator ($b = 0$).

When the driven frequency is close to natural frequency, its amplitude of motion can be quite large



Resonance curve

x_m is approximately greatest when $\omega_d = \omega$

$$\omega_d = \omega \quad \text{(resonance)}.$$

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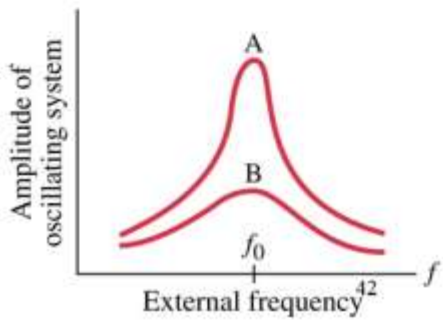
15-9: Forced Oscillations and Resonance

Forced vibrations occur when there is a periodic driving force. This force may or may not have the same period as the natural frequency of the system.

If the frequency is the same as the natural frequency, the amplitude becomes quite large. This is called resonance.

The sharpness of the resonant peak depends on the damping. If the damping is can be quite sharp; if the damping is larger (B), it is less sharp.

Like damping, resonance can be wanted or unwanted. Musical instruments and TV/radio receivers depend on it.



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