

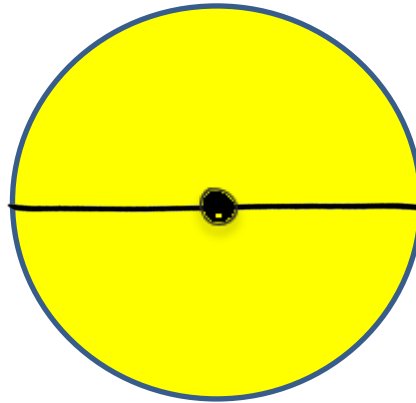


CALCULUS IIO

APPANDIX – D Trigonometry

Dr. Rola Asaad Hijazi

What is π ?



The number π is a mathematical constant. Originally defined as the ratio of a circle's circumference to its diameter.

(π) هو ثابت الدائرة أو ثابت رياضي يستخدم في علوم الرياضيات والفيزياء.

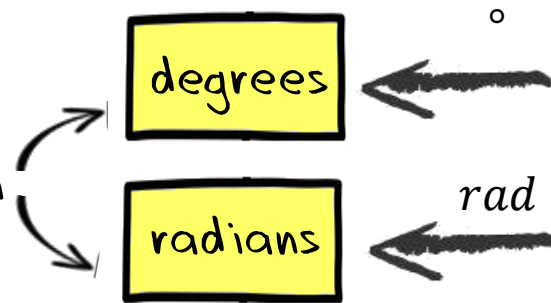
هو عدد حقيقي غير كسري
أي لا يمكن كتابته على
شكل $\frac{a}{b}$ حيث a, b عددان
صحيحان.

Irrational
number

لا يمكن تمثيله بكسر
عشري منته ولكن يساوي
تقريباً $3.14 \approx \frac{22}{7}$

Angles:

Angles can be measured in



The angle given by a complete revolution contains

$$360^\circ = 2\pi \text{ rad.}$$

① $\pi \text{ rad} = 180^\circ$

② $1 \text{ rad} = \frac{180^\circ}{\pi} \approx 57.3^\circ$

③ $1^\circ = \frac{\pi}{180} \text{ rad} = 0.017 \text{ rad}$

what is radian?

هي وحدة قياس للزوايا المستوية وهي الوحدة الرسمية المعتمدة ضمن النظام الدولي للوحدات المستخدمة في الرياضيات والفيزياء.

Examples:

1. Convert from degrees to radian.

① 60°

② $-315^\circ =$

2. Convert from radian to degrees.

① $\frac{5\pi}{4} \text{ rad}$

② $\frac{-7\pi}{2} \text{ rad}$

③ $\frac{-3}{8} \pi \text{ rad}$

**In Calculus we use radians to measure angles
except when otherwise indicated.**

Degrees	0°	30	45°	60°	90	120	135	150	180	270	360
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π

The Trigonometric Functions

For an acute angle θ the six trigonometric Functions are defined as ratios of lengths of sides of a right triangle as follows:

$$1. \sin \theta = \frac{opp}{hyp}$$

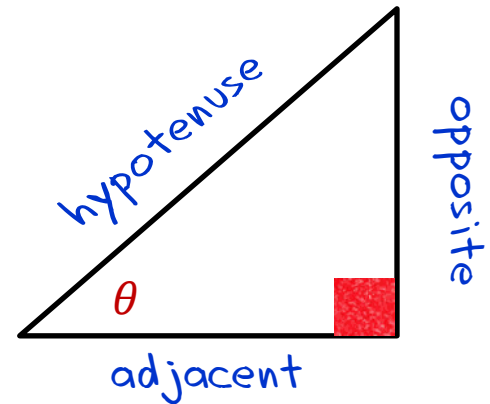
$$2. \cos \theta = \frac{adj}{hyp}$$

$$3. \tan \theta = \frac{opp}{adj}$$

$$4. \csc \theta = \frac{hyp}{opp}$$

$$5. \sec \theta = \frac{hyp}{adj}$$

$$6. \cot \theta = \frac{adj}{opp}$$

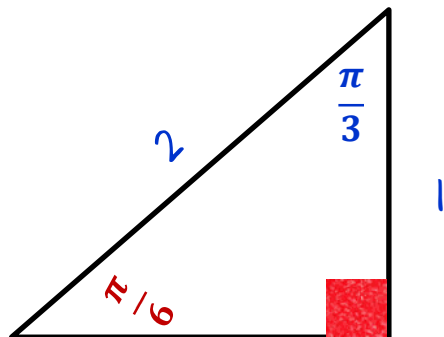


The Trigonometric Functions

$$1. \sin \frac{\pi}{6} = \frac{1}{2}$$

$$2. \cos \frac{\pi}{6} =$$

$$3. \tan \frac{\pi}{6} =$$



$$1. \sin \frac{\pi}{3} =$$

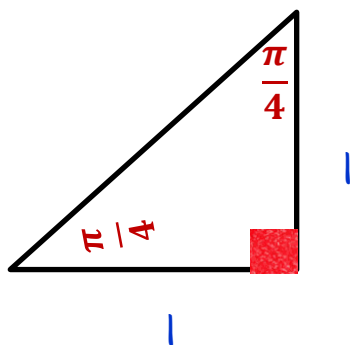
$$2. \cos \frac{\pi}{3} =$$

$$3. \tan \frac{\pi}{3} =$$

$$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\cos \frac{\pi}{4} =$$

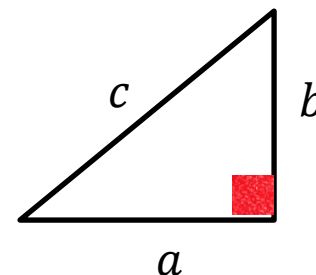
$$\tan \frac{\pi}{4} =$$



Recall that:

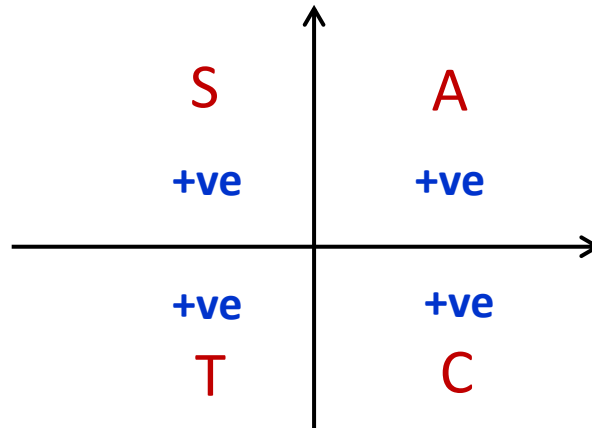
Pythagoras' Theorem

$$a^2 + b^2 = c^2$$



Remark:

The sign of the trigonometric functions for angles in each of the four quadrants can be remembered by the quote :



“All Students **T**ake **C**alculus”.

The following table gives some value of
 $\sin \theta$ and $\cos \theta$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{3\pi}{2}$	2π
$\sin$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\cos$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{1}{2}$	-1	0	1

Examples:

① If $\cos \theta = \frac{2}{5}$, $0 < \theta < \frac{\pi}{2}$, find the other five trigonometric functions of θ .

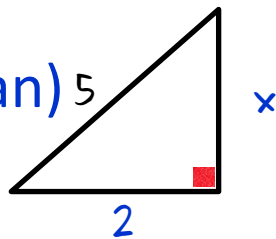
Solution

θ is in the first quadrant

$$x^2 + 4 = 25 \quad (\text{Pythagorean})$$

$$x^2 = 25 - 4 = 21$$

$$\Rightarrow x = \pm\sqrt{21}$$



θ in the 1st quadrant

$\Rightarrow$ all trigonometry functions are positive:

$$\sin \theta = \frac{\sqrt{21}}{5} \quad \cos \theta = \frac{2}{5} \quad \sec \theta = \frac{5}{2}$$

$$\tan \theta = \frac{\sqrt{21}}{2} \quad \csc \theta = \frac{5}{\sqrt{21}} \quad \cot \theta = \frac{2}{\sqrt{21}}$$

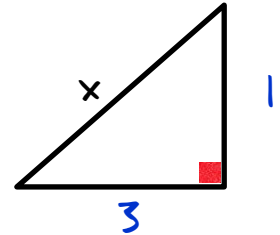
②

If $\cot \beta = 3$, $\pi < \beta < 2\pi$, find the other five trigonometric functions of θ .

Solution

$$1^2 + 3^2 = x^2$$

$$\Rightarrow x = \pm\sqrt{10}$$



β is in the 3rd or 4th quadrant.

$$\tan \beta = \frac{1}{3}, \text{ as } \tan \beta \text{ is positive.}$$

$\Rightarrow \beta$ is in the 3rd quadrant.

$$\sin \beta = -\frac{1}{\sqrt{10}} \quad \cos \beta = \frac{-3}{\sqrt{10}}$$

$$\tan \beta = \frac{1}{3} \quad \csc \beta = -\sqrt{10}$$

$$\sec \beta = -\frac{\sqrt{10}}{3} \quad \cot \beta = 3$$

In Calculus we use radians to measure angles except when otherwise indicated.

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Trigonometric Identities

$$\textcircled{1} \quad \csc \theta = \frac{1}{\sin \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\textcircled{2} \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\textcircled{3} \quad \sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos(\theta)$$

$$\textcircled{4} \quad \sin(2\pi + \theta) = \sin \theta$$

$$\cos(2\pi + \theta) = \cos \theta$$

$$\textcircled{5} \quad \sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

All Trigonometric Functions are odd functions except $\cos x$ and $\sec x$ are even functions.

Extra Examples:

① Evaluate $\sin\left(-\frac{3\pi}{2}\right)$

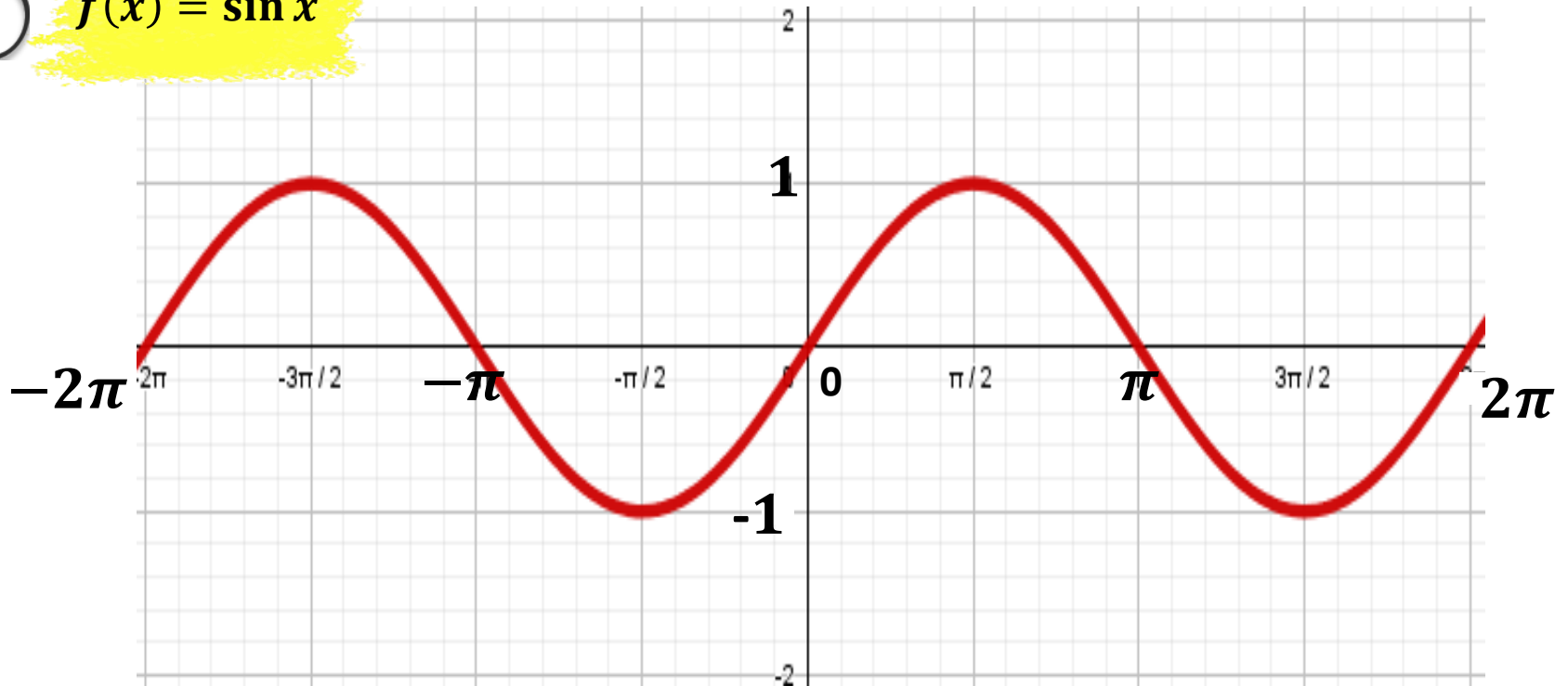
② Evaluate $\sec\left(-\frac{\pi}{3}\right)$

③ If $\sin x = \frac{1}{\sqrt{5}}$, and $\cos x = \frac{2}{\sqrt{5}}$

Find $\sin 2x$ and $\cos 2x$

Graphs of the Trigonometric Functions

① $f(x) = \sin x$



Period of $\sin \theta =$

Domain =

Range =

odd or even function

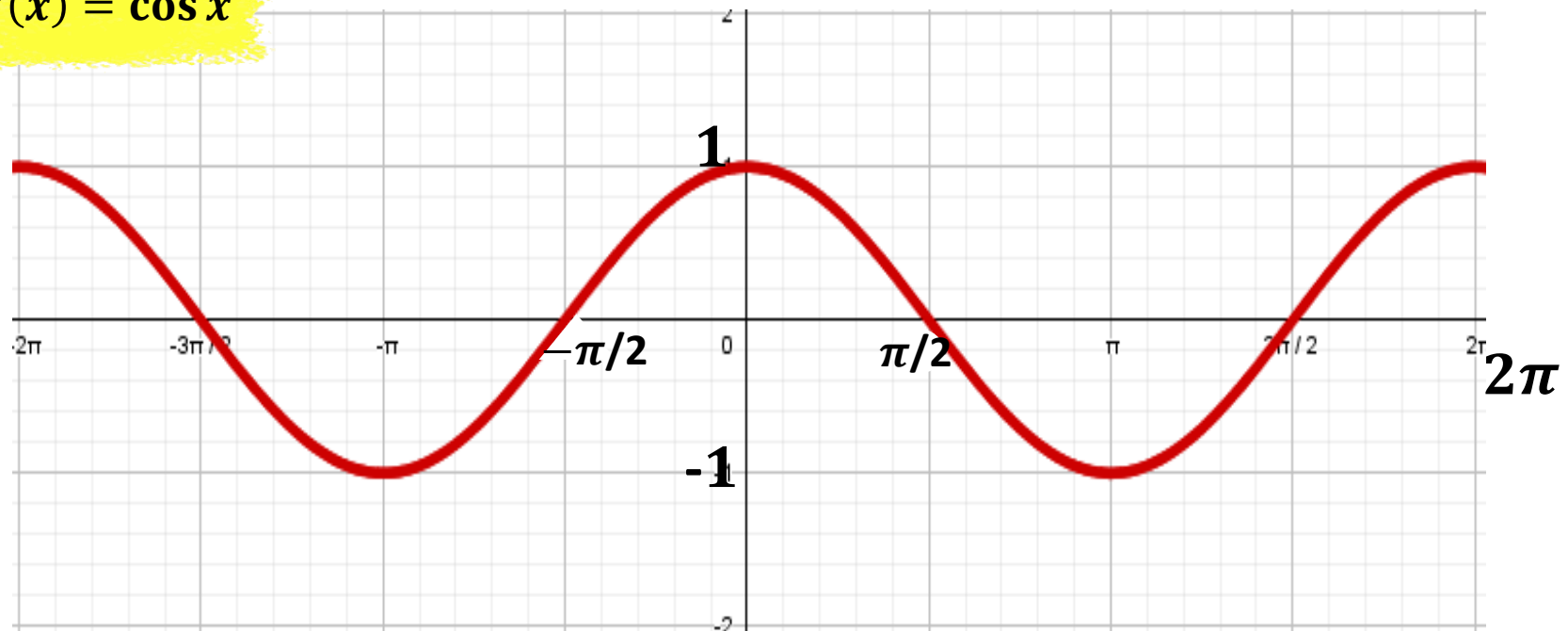
Symmetric about _ _ _

Is it 1-1 ?

Graphs of the Trigonometric Functions

2

$$f(x) = \cos x$$



Period of $\cos x$

Domain =

Range =



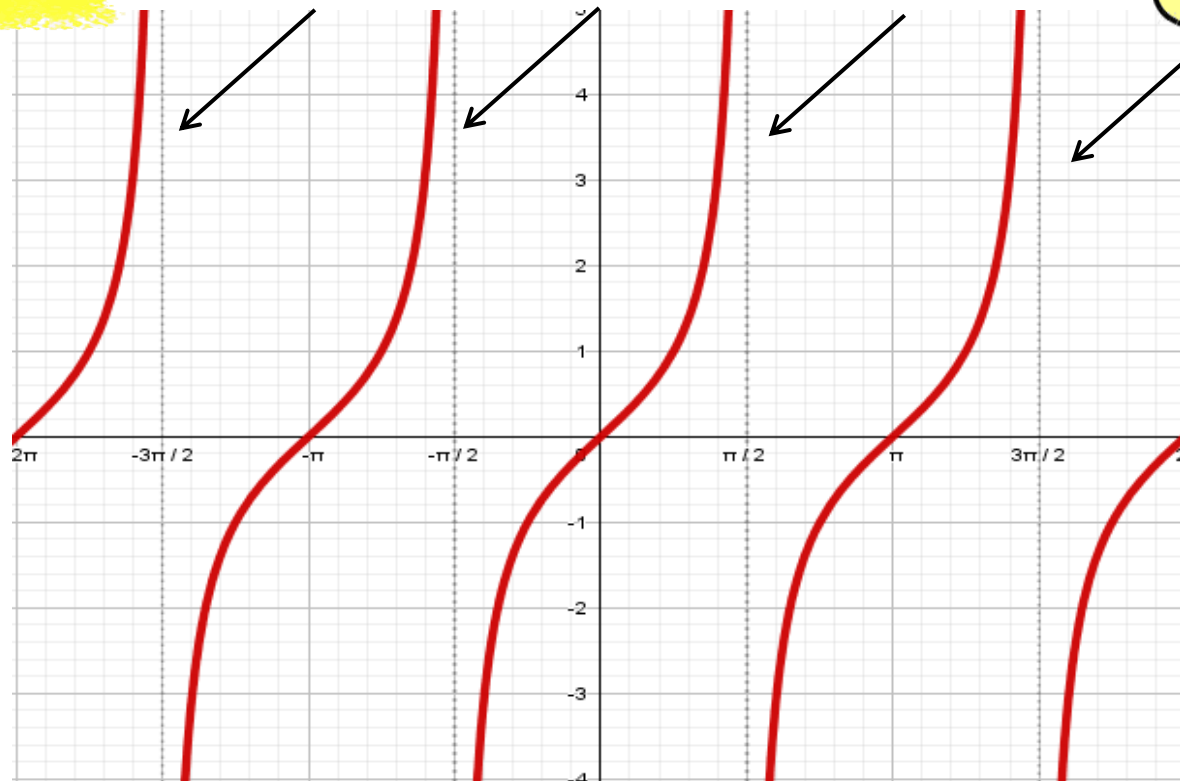
odd or even function

Symmetric about ____

Is it 1-1 ?

Graphs of the Trigonometric Functions

3 $f(x) = \tan x$



Vertical asymptotes

Period of $\tan x$

Domain =

Range =

odd or even function

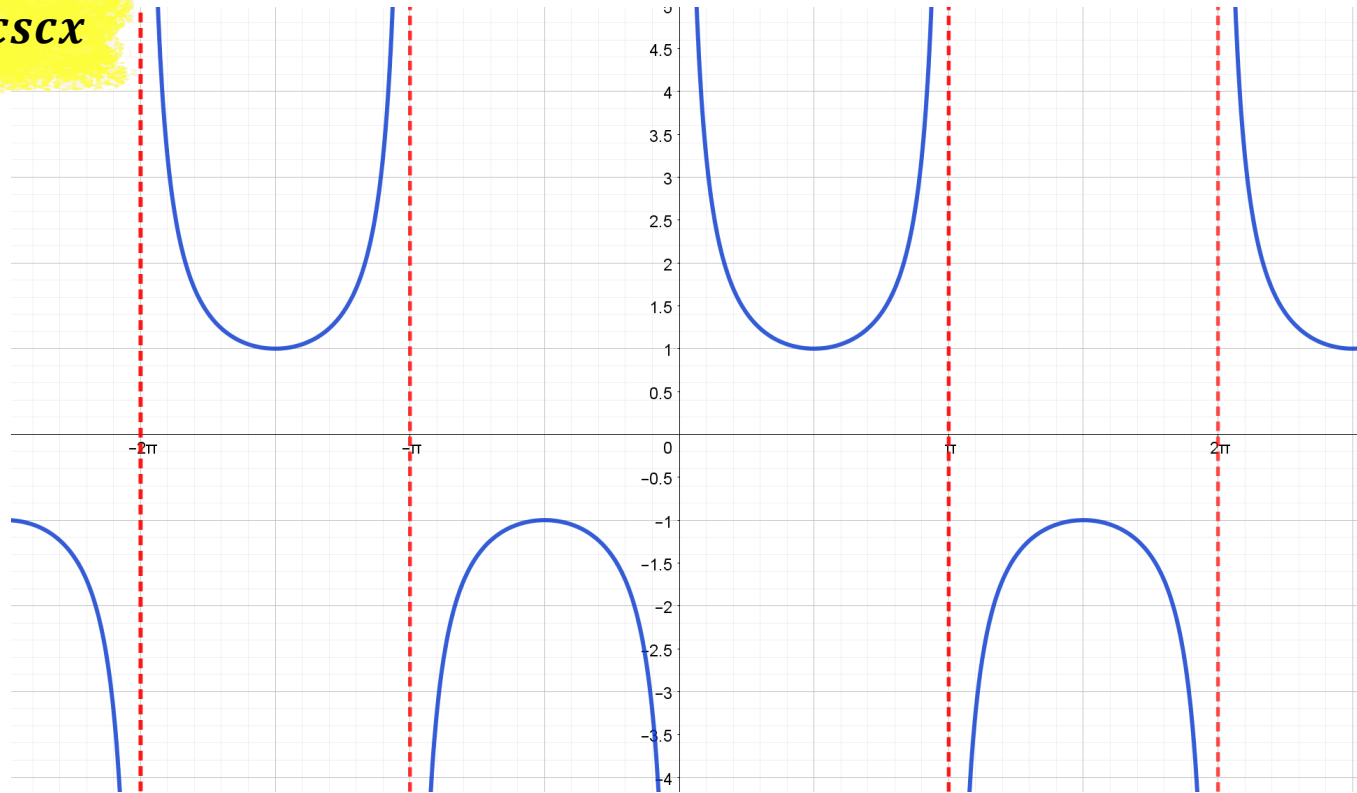
Symmetric about ____

Is it 1-1 ?

Graphs of the Trigonometric Functions

4

$$f(x) = \csc x$$



Period of $\csc x$

Domain =

Range =

Vertical asymptotes:



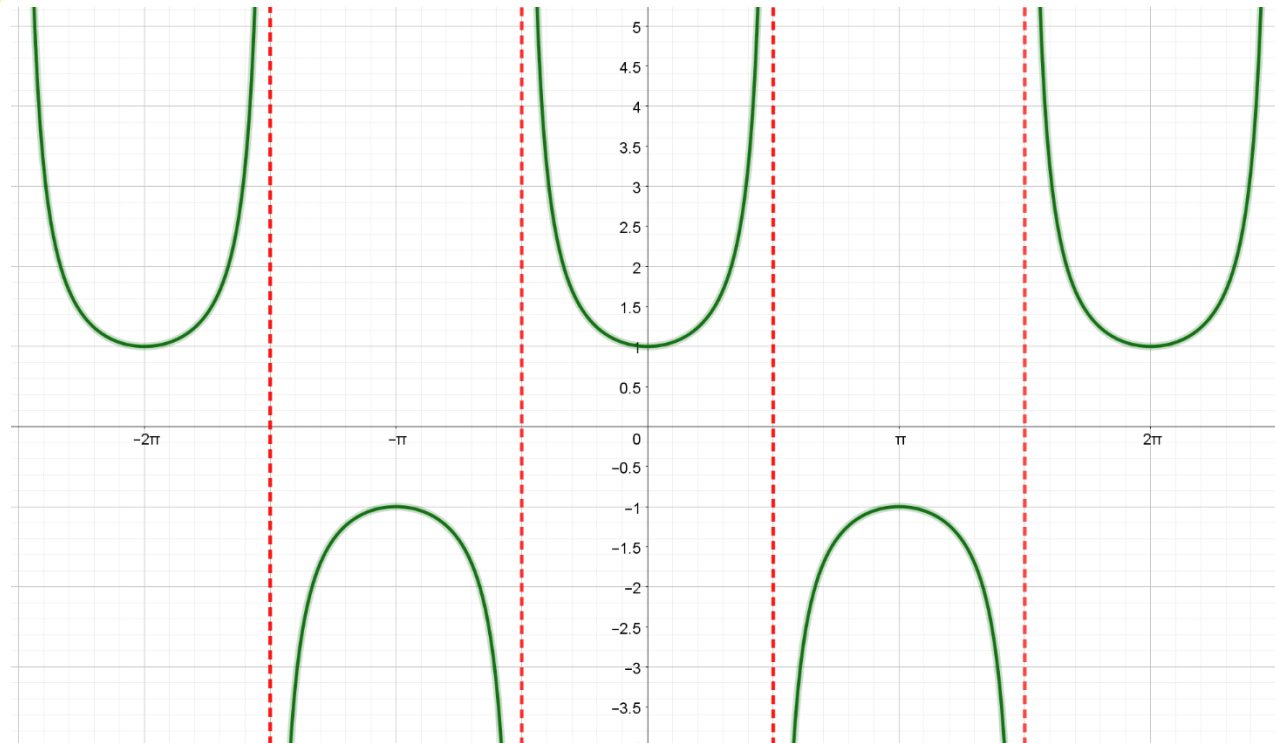
odd or even function

Symmetric about ____

Is it 1-1 ?

Graphs of the Trigonometric Functions

5 $f(x) = \sec x$



Period of $\sec x$

Vertical asymptotes:

Domain =

Range =



odd or even *function*

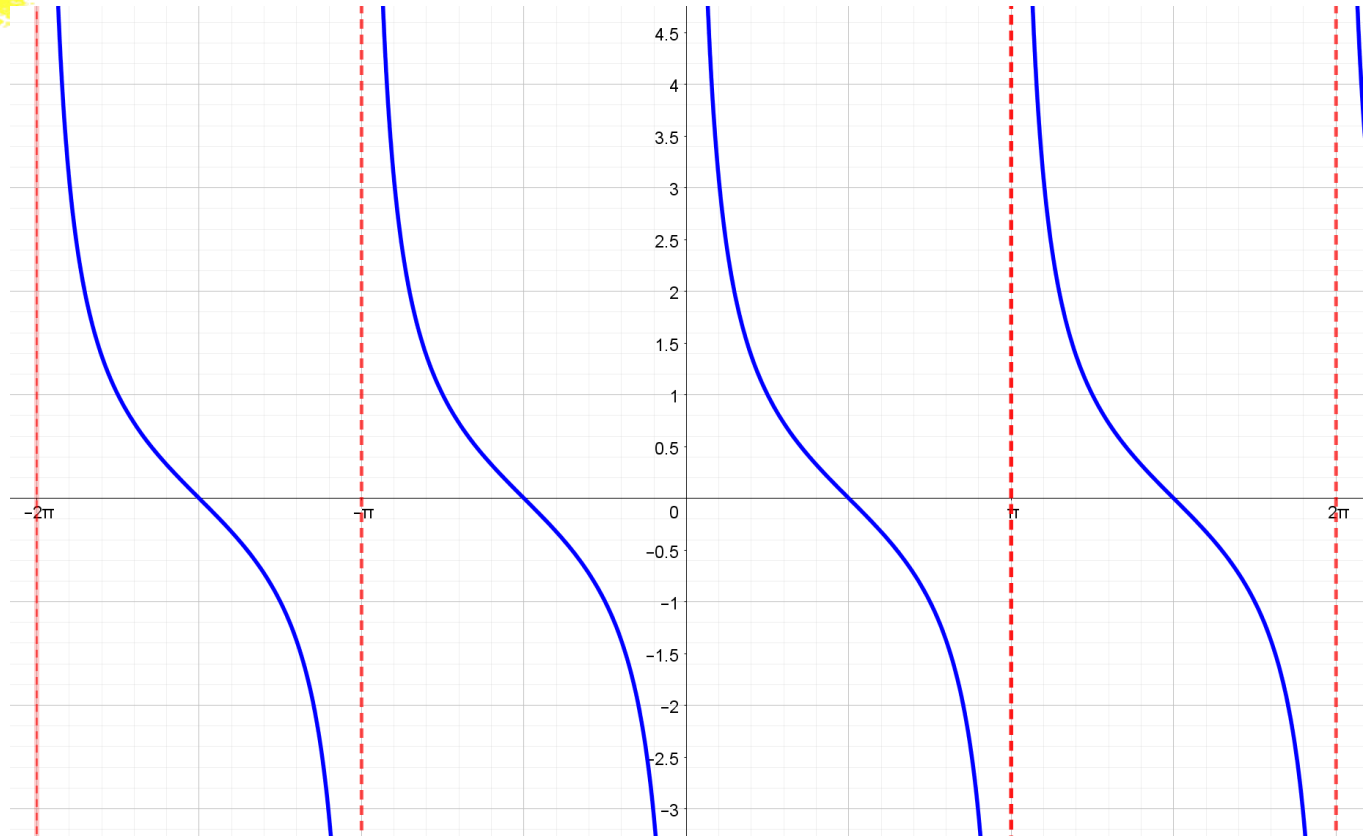
Symmetric with respect to _____

Is it 1-1

Graphs of the Trigonometric Functions

6

$$f(x) = \cot x$$



Period =

Domain =

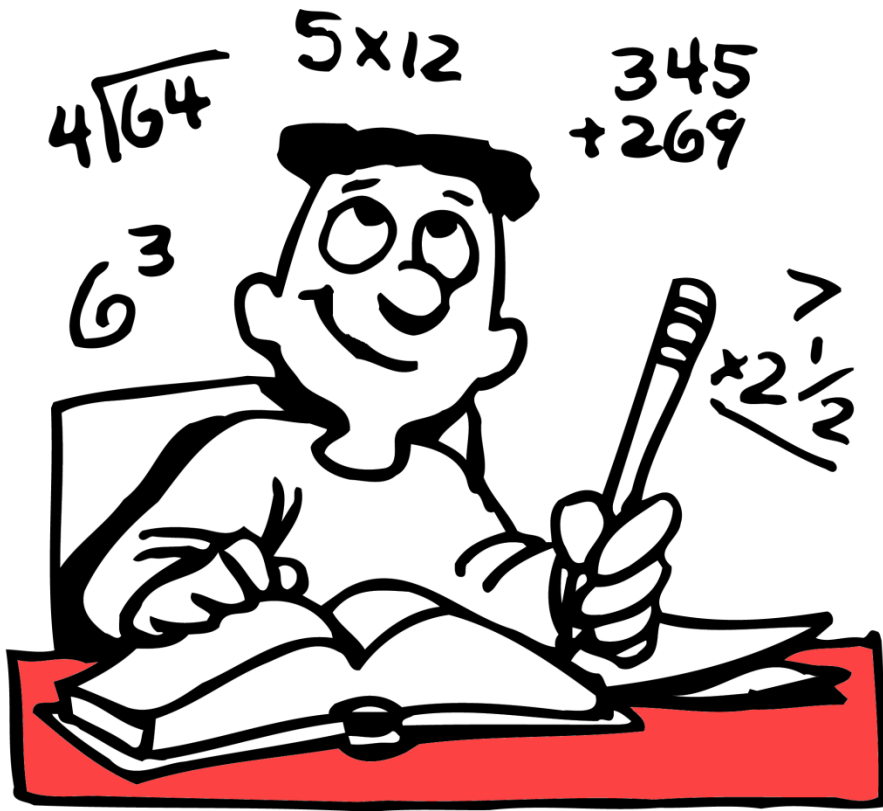
Range =

Vertical asymptotes

Odd or even function

Symmetric about _ _ _ _

Is it 1-1



homework

1 - 11 (odd), 29 - 33 (odd)