

Workshop Solutions to Sections 3.4 and 3.5

1) $\lim_{x \rightarrow 3^+} \frac{2}{x-3} =$ <u>Solution:</u> If $x \rightarrow 3^+$, then $x > 3 \Rightarrow x - 3 > 0$ $\therefore \lim_{x \rightarrow 3^+} \frac{2}{x-3} = \infty$	2) $\lim_{x \rightarrow 3^-} \frac{2}{x-3} =$ <u>Solution:</u> If $x \rightarrow 3^-$, then $x < 3 \Rightarrow x - 3 < 0$ $\therefore \lim_{x \rightarrow 3^-} \frac{2}{x-3} = -\infty$
3) $\lim_{x \rightarrow 3^+} \frac{-2}{x-3} =$ <u>Solution:</u> If $x \rightarrow 3^+$, then $x > 3 \Rightarrow x - 3 > 0$ $\therefore \lim_{x \rightarrow 3^+} \frac{-2}{x-3} = -\infty$	4) $\lim_{x \rightarrow 3^-} \frac{-2}{x-3} =$ <u>Solution:</u> If $x \rightarrow 3^-$, then $x < 3 \Rightarrow x - 3 < 0$ $\therefore \lim_{x \rightarrow 3^-} \frac{-2}{x-3} = \infty$
5) $\lim_{x \rightarrow -3^+} \frac{2}{x+3} =$ <u>Solution:</u> If $x \rightarrow -3^+$, then $x > -3 \Rightarrow x + 3 > 0$ $\therefore \lim_{x \rightarrow -3^+} \frac{2}{x+3} = \infty$	6) $\lim_{x \rightarrow -3^-} \frac{2}{x+3} =$ <u>Solution:</u> If $x \rightarrow -3^-$, then $x < -3 \Rightarrow x + 3 < 0$ $\therefore \lim_{x \rightarrow -3^-} \frac{2}{x+3} = -\infty$
7) $\lim_{x \rightarrow 2^+} \frac{3x-1}{x-2} =$ <u>Solution:</u> If $x \rightarrow 2^+$, then $x > 2 \Rightarrow x - 2 > 0$ and $3x - 1 > 0$ $\therefore \lim_{x \rightarrow 2^+} \frac{3x-1}{x-2} = \infty$	8) $\lim_{x \rightarrow 2^-} \frac{3x-1}{x-2} =$ <u>Solution:</u> If $x \rightarrow 2^-$, then $x < 2 \Rightarrow x - 2 < 0$ and $3x - 1 > 0$ $\therefore \lim_{x \rightarrow 2^-} \frac{3x-1}{x-2} = -\infty$
9) $\lim_{x \rightarrow -2^+} \frac{1-x}{(x+2)^2} =$ <u>Solution:</u> If $x \rightarrow -2^+$, then $x > -2$ $\Rightarrow 1 - x > 0$ and $(x+2)^2 > 0$ $\therefore \lim_{x \rightarrow -2^+} \frac{1-x}{(x+2)^2} = \infty$	10) $\lim_{x \rightarrow -2^-} \frac{1-x}{(x+2)^2} =$ <u>Solution:</u> If $x \rightarrow -2^-$, then $x < -2$ $\Rightarrow 1 - x > 0$ and $(x+2)^2 > 0$ $\therefore \lim_{x \rightarrow -2^-} \frac{1-x}{(x+2)^2} = \infty$
11) $\lim_{x \rightarrow -2^+} \frac{x-1}{(x+2)^2} =$ <u>Solution:</u> If $x \rightarrow -2^+$, then $x > -2$ $\Rightarrow x - 1 < 0$ and $(x+2)^2 > 0$ $\therefore \lim_{x \rightarrow -2^+} \frac{x-1}{(x+2)^2} = -\infty$	12) $\lim_{x \rightarrow -2^-} \frac{x-1}{(x+2)^2} =$ <u>Solution:</u> If $x \rightarrow -2^-$, then $x < -2$ $\Rightarrow x - 1 < 0$ and $(x+2)^2 > 0$ $\therefore \lim_{x \rightarrow -2^-} \frac{x-1}{(x+2)^2} = -\infty$
13) $\lim_{x \rightarrow 2^+} \frac{6x-1}{x^2-4} =$ <u>Solution:</u> If $x \rightarrow 2^+$, then $x^2 > 4$ $\Rightarrow x^2 - 4 > 0$ and $6x - 1 > 0$ $\therefore \lim_{x \rightarrow 2^+} \frac{6x-1}{x^2-4} = \infty$	14) $\lim_{x \rightarrow 2^-} \frac{6x-1}{x^2-4} =$ <u>Solution:</u> If $x \rightarrow 2^-$, then $x^2 < 4$ $\Rightarrow x^2 - 4 < 0$ and $6x - 1 > 0$ $\therefore \lim_{x \rightarrow 2^-} \frac{6x-1}{x^2-4} = -\infty$

15) $\lim_{x \rightarrow -2^+} \frac{6x-1}{x^2-4} =$ <u>Solution:</u> If $x \rightarrow -2^+$, then $x^2 < 4$ $\Rightarrow x^2 - 4 < 0$ and $6x - 1 < 0$ $\therefore \lim_{x \rightarrow -2^+} \frac{6x-1}{x^2-4} = \infty$	16) $\lim_{x \rightarrow -2^-} \frac{6x-1}{x^2-4} =$ <u>Solution:</u> If $x \rightarrow -2^-$, then $x^2 > 4$ $\Rightarrow x^2 - 4 > 0$ and $6x - 1 < 0$ $\therefore \lim_{x \rightarrow -2^-} \frac{6x-1}{x^2-4} = -\infty$
17) $\lim_{x \rightarrow -2^-} \frac{6x-1}{x^2-x-6} =$ <u>Solution:</u> $f(x) = \frac{6x-1}{x^2-x-6} = \frac{6x-1}{(x-3)(x+2)}$ If $x \rightarrow -2^-$, then $x < -2$ $\Rightarrow x-3 < 0$, $x+2 < 0$ and $6x-1 < 0$ $\therefore \lim_{x \rightarrow -2^-} \frac{6x-1}{x^2-x-6} = -\infty$	18) $\lim_{x \rightarrow -2^+} \frac{6x-1}{x^2-x-6} =$ <u>Solution:</u> $f(x) = \frac{6x-1}{x^2-x-6} = \frac{6x-1}{(x-3)(x+2)}$ If $x \rightarrow -2^+$, then $x > -2$ $\Rightarrow x-3 < 0$, $x+2 > 0$ and $6x-1 < 0$ $\therefore \lim_{x \rightarrow -2^+} \frac{6x-1}{x^2-x-6} = \infty$
19) $\lim_{x \rightarrow 3^+} \frac{-1}{x^2-x-6} =$ <u>Solution:</u> $f(x) = \frac{-1}{x^2-x-6} = \frac{-1}{(x-3)(x+2)}$ If $x \rightarrow 3^+$, then $x > 3$ $\Rightarrow x-3 > 0$, $x+2 > 0$ and $-1 < 0$ $\therefore \lim_{x \rightarrow 3^+} \frac{-1}{x^2-x-6} = -\infty$	20) $\lim_{x \rightarrow 3^-} \frac{-1}{x^2-x-6} =$ <u>Solution:</u> $f(x) = \frac{-1}{x^2-x-6} = \frac{-1}{(x-3)(x+2)}$ If $x \rightarrow 3^-$, then $x < 3$ $\Rightarrow x-3 < 0$, $x+2 > 0$ and $-1 < 0$ $\therefore \lim_{x \rightarrow 3^-} \frac{-1}{x^2-x-6} = \infty$
21) $\lim_{x \rightarrow (\pi/2)^+} \tan x =$ <u>Solution:</u> $\lim_{x \rightarrow (\pi/2)^+} \tan x = -\infty$	22) $\lim_{x \rightarrow (\pi/2)^-} \tan x =$ <u>Solution:</u> $\lim_{x \rightarrow (\pi/2)^-} \tan x = \infty$
23) The vertical asymptote of $f(x) = \frac{1-x}{2x+1}$ is <u>Solution:</u> We see that the function $f(x)$ is not defined when $2x+1=0 \Rightarrow x = -\frac{1}{2}$. Since $\lim_{x \rightarrow (-\frac{1}{2})^+} \frac{1-x}{2x+1} = \infty$ and $\lim_{x \rightarrow (-\frac{1}{2})^-} \frac{1-x}{2x+1} = -\infty$ then, $x = -\frac{1}{2}$ is a vertical asymptote.	24) The vertical asymptote of $f(x) = \frac{3-x}{x^2-4}$ is <u>Solution:</u> We see that the function $f(x)$ is not defined when $x^2-4=0 \Rightarrow x = \pm 2$. Since $\lim_{x \rightarrow 2^+} \frac{3-x}{x^2-4} = \infty, \quad \lim_{x \rightarrow 2^-} \frac{3-x}{x^2-4} = -\infty$ and $\lim_{x \rightarrow -2^+} \frac{3-x}{x^2-4} = -\infty, \quad \lim_{x \rightarrow -2^-} \frac{3-x}{x^2-4} = \infty$ then, $x = \pm 2$ are vertical asymptotes.

25) The vertical asymptote of $f(x) = \frac{3-x}{x^2-x-6}$ is

Solution:

$$f(x) = \frac{3-x}{x^2-x-6} = \frac{3-x}{(x-3)(x+2)} = \frac{-(x-3)}{(x-3)(x+2)} = -\frac{1}{x+2}$$

We see that the function $f(x)$ is not defined when

$$x^2 - x - 6 = 0 \Rightarrow (x-3)(x+2) = 0$$

$$\Rightarrow x = 3 \text{ or } x = -2. \text{ Since}$$

$$\lim_{x \rightarrow 3} \frac{3-x}{x^2-x-6} = \lim_{x \rightarrow 3} \frac{3-x}{(x-3)(x+2)} = \lim_{x \rightarrow 3} \frac{-(x-3)}{(x-3)(x+2)} = \lim_{x \rightarrow 3} \frac{-1}{x+2} = -\frac{1}{5}$$

then, $x = 3$ is a removable discontinuity.

$$\lim_{x \rightarrow -2^+} \frac{3-x}{x^2-x-6} = \lim_{x \rightarrow -2^+} \frac{3-x}{(x-3)(x+2)} = \infty$$

and

$$\lim_{x \rightarrow -2^-} \frac{3-x}{x^2-x-6} = \lim_{x \rightarrow -2^-} \frac{3-x}{(x-3)(x+2)} = -\infty$$

then, $x = -2$ is a vertical asymptote only.

26) The vertical asymptote of $f(x) = \frac{7-x}{x^2-5x+6}$ is

Solution:

$$f(x) = \frac{7-x}{x^2-5x+6} = \frac{7-x}{(x-3)(x-2)}$$

We see that the function $f(x)$ is not defined when $x-3=0$ or $x-2=0 \Rightarrow x=3$ or $x=2$.

Since

$$\lim_{x \rightarrow 3^+} \frac{7-x}{x^2-5x+6} = \lim_{x \rightarrow 3^+} \frac{7-x}{(x-3)(x-2)} = \infty$$

$$\lim_{x \rightarrow 3^-} \frac{7-x}{x^2-5x+6} = \lim_{x \rightarrow 3^-} \frac{7-x}{(x-3)(x-2)} = -\infty$$

and

$$\lim_{x \rightarrow 2^+} \frac{7-x}{x^2-5x+6} = \lim_{x \rightarrow 2^+} \frac{7-x}{(x-3)(x-2)} = -\infty$$

$$\lim_{x \rightarrow 2^-} \frac{7-x}{x^2-5x+6} = \lim_{x \rightarrow 2^-} \frac{7-x}{(x-3)(x-2)} = \infty$$

then, $x = 3$ and $x = 2$ are vertical asymptotes.

27) The vertical asymptote of $f(x) = \frac{x-7}{x^2+5x+6}$ is

Solution:

$$f(x) = \frac{x-7}{x^2+5x+6} = \frac{x-7}{(x+3)(x+2)}$$

We see that the function $f(x)$ is not defined when

$$x+3=0 \text{ or } x+2=0 \Rightarrow x=-3 \text{ or } x=-2.$$

Since

$$\lim_{x \rightarrow -3^+} \frac{x-7}{x^2+5x+6} = \lim_{x \rightarrow -3^+} \frac{x-7}{(x+3)(x+2)} = \infty$$

$$\lim_{x \rightarrow -3^-} \frac{x-7}{x^2+5x+6} = \lim_{x \rightarrow -3^-} \frac{x-7}{(x+3)(x+2)} = -\infty$$

and

$$\lim_{x \rightarrow -2^+} \frac{x-7}{x^2+5x+6} = \lim_{x \rightarrow -2^+} \frac{x-7}{(x+3)(x+2)} = -\infty$$

$$\lim_{x \rightarrow -2^-} \frac{x-7}{x^2+5x+6} = \lim_{x \rightarrow -2^-} \frac{x-7}{(x+3)(x+2)} = \infty$$

then, $x = -3$ and $x = -2$ are vertical asymptotes.

28) The vertical asymptote of $f(x) = \frac{x-7}{x^2+3x}$ is

Solution:

$$f(x) = \frac{x-7}{x^2+3x} = \frac{x-7}{x(x+3)}$$

We see that the function $f(x)$ is not defined when

$$x=0 \text{ or } x+3=0 \Rightarrow x=0 \text{ or } x=-3. \text{ Since}$$

$$\lim_{x \rightarrow -3^+} \frac{x-7}{x^2+3x} = \lim_{x \rightarrow -3^+} \frac{x-7}{x(x+3)} = \infty$$

$$\lim_{x \rightarrow -3^-} \frac{x-7}{x^2+3x} = \lim_{x \rightarrow -3^-} \frac{x-7}{x(x+3)} = -\infty$$

and

$$\lim_{x \rightarrow 0^+} \frac{x-7}{x^2+3x} = \lim_{x \rightarrow 0^+} \frac{x-7}{x(x+3)} = -\infty$$

$$\lim_{x \rightarrow 0^-} \frac{x-7}{x^2+3x} = \lim_{x \rightarrow 0^-} \frac{x-7}{x(x+3)} = \infty$$

then, $x = -3$ and $x = 0$ are vertical asymptotes.

29) The vertical asymptote of $f(x) = \frac{x-7}{x^2-3x}$ is

Solution:

$$f(x) = \frac{x-7}{x^2-3x} = \frac{x-7}{x(x-3)}$$

We see that the function $f(x)$ is not defined when

$$x=0 \text{ or } x-3=0 \Rightarrow x=0 \text{ or } x=3. \text{ Since}$$

$$\lim_{x \rightarrow 3^+} \frac{x-7}{x^2-3x} = \lim_{x \rightarrow 3^+} \frac{x-7}{x(x-3)} = -\infty$$

$$\lim_{x \rightarrow 3^-} \frac{x-7}{x^2-3x} = \lim_{x \rightarrow 3^-} \frac{x-7}{x(x-3)} = \infty$$

and

$$\lim_{x \rightarrow 0^+} \frac{x-7}{x^2-3x} = \lim_{x \rightarrow 0^+} \frac{x-7}{x(x-3)} = \infty$$

$$\lim_{x \rightarrow 0^-} \frac{x-7}{x^2-3x} = \lim_{x \rightarrow 0^-} \frac{x-7}{x(x-3)} = -\infty$$

then, $x = 3$ and $x = 0$ are vertical asymptotes.

30) The vertical asymptotes of $f(x) = \frac{2x^2+1}{x^2-9}$ are

Solution:

$$f(x) = \frac{2x^2+1}{x^2-9} = \frac{2x^2+1}{(x+3)(x-3)}$$

We see that the function $f(x)$ is not defined when

$$x^2-9=0 \Rightarrow x=\pm 3. \text{ Since}$$

$$\lim_{x \rightarrow 3^+} \frac{2x^2+1}{x^2-9} = \lim_{x \rightarrow 3^+} \frac{2x^2+1}{(x+3)(x-3)} = \infty$$

$$\lim_{x \rightarrow 3^-} \frac{2x^2+1}{x^2-9} = \lim_{x \rightarrow 3^-} \frac{2x^2+1}{(x+3)(x-3)} = -\infty$$

and

$$\lim_{x \rightarrow -3^+} \frac{2x^2+1}{x^2-9} = \lim_{x \rightarrow -3^+} \frac{2x^2+1}{(x+3)(x-3)} = -\infty$$

$$\lim_{x \rightarrow -3^-} \frac{2x^2+1}{x^2-9} = \lim_{x \rightarrow -3^-} \frac{2x^2+1}{(x+3)(x-3)} = \infty$$

then, $x = \pm 3$ are vertical asymptotes.

31) The function $f(x) = \frac{x+1}{x^2-9}$ is continuous at $a = 2$ because 1- $f(2) = \frac{(2)+1}{(2)^2-9} = \frac{3}{-5} = -\frac{3}{5}$ 2- $\lim_{x \rightarrow 2} \frac{x+1}{x^2-9} = \lim_{x \rightarrow 2} \frac{(2)+1}{(2)^2-9} = \frac{3}{-5} = -\frac{3}{5}$ 3- $\lim_{x \rightarrow 2} \frac{x+1}{x^2-9} = f(2)$ OR We know that $D_f = \mathbb{R} \setminus \{\pm 3\}$, so $\{2\} \in D_f$. Note: Any function is continuous on its domain.	32) The function $f(x) = \frac{x+1}{x^2-9}$ is discontinuous at $a = \pm 3$ because we know that $D_f = \mathbb{R} \setminus \{\pm 3\}$, so $\{\pm 3\} \notin D_f$. 33) The function $f(x) = \frac{x+1}{x^2-9}$ is discontinuous at ± 3 because $\{\pm 3\} \notin D_f$.
34) The function $f(x) = \frac{x+1}{x^2-9}$ is continuous on its domain which is $D_f = \mathbb{R} \setminus \{\pm 3\}$.	35) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ 3, & x = 0 \end{cases}$ is continuous at $a = 0$ because 1- $f(0) = 3$ 2- $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3- $\lim_{x \rightarrow 0} f(x) = f(0)$
36) The function $f(x) = \begin{cases} \frac{\sin 3x}{x}, & x \neq 0 \\ 5, & x = 0 \end{cases}$ is discontinuous at $a = 0$ because 1- $f(0) = 5$ 2- $\lim_{x \rightarrow 0} \frac{\sin 3x}{x} = 3 \lim_{x \rightarrow 0} \frac{\sin 3x}{3x} = 3(1) = 3$ 3- $\lim_{x \rightarrow 0} f(x) \neq f(0)$	37) The function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}, & x \neq 1 \\ 7, & x = 1 \end{cases}$ is discontinuous at $a = 1$ because 1- $f(1) = 7$ 2- $\lim_{x \rightarrow 1} \frac{2x^2-3x+1}{x-1} = \lim_{x \rightarrow 1} \frac{(2x-1)(x-1)}{x-1} = \lim_{x \rightarrow 1} (2x-1) = 1$ 3- $\lim_{x \rightarrow 1} f(x) \neq f(1)$
38) The function $f(x) = \begin{cases} \frac{2x^2-3x+1}{x-1}, & x \neq 1 \\ 1, & x = 1 \end{cases}$ is continuous at $a = 1$ because 1- $f(1) = 1$ 2- $\lim_{x \rightarrow 1} \frac{2x^2-3x+1}{x-1} = \lim_{x \rightarrow 1} \frac{(2x-1)(x-1)}{x-1} = \lim_{x \rightarrow 1} (2x-1) = 1$ 3- $\lim_{x \rightarrow 1} f(x) = f(1)$	39) The function $f(x) = \frac{x^2-x-2}{x-2}$ is discontinuous at $a = 2$ because $\{2\} \notin D_f$.
40) The function $f(x) = \begin{cases} 2x+3, & x > 2 \\ 3x+1, & x \leq 2 \end{cases}$ is continuous at $a = 2$ because 1- $f(2) = 3(2)+1 = 7$ 2- $\lim_{x \rightarrow 2^+} (2x+3) = 2(2)+3 = 7$ $\lim_{x \rightarrow 2^-} (3x+1) = 3(2)+1 = 7$ $\therefore \lim_{x \rightarrow 2} f(x) = 7$ 3- $\lim_{x \rightarrow 2} f(x) = f(2)$	41) The function $f(x) = \frac{x+3}{\sqrt{x^2-4}}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 > 0 \Rightarrow x^2 > 4 \Rightarrow \sqrt{x^2} > \sqrt{4}$ $\Rightarrow x > 2 \Leftrightarrow x > 2 \text{ or } x < -2$ Hence, $D_f = (-\infty, -2) \cup (2, \infty).$
42) The function $f(x) = \sqrt{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \geq 0 \Rightarrow x^2 \geq 4 \Rightarrow \sqrt{x^2} \geq \sqrt{4}$ $\Rightarrow x \geq 2 \Leftrightarrow x \geq 2 \text{ or } x \leq -2$ Hence, $D_f = (-\infty, -2] \cup [2, \infty).$	43) The function $f(x) = \sqrt{4-x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that $4 - x^2 \geq 0 \Rightarrow -x^2 \geq -4 \Rightarrow x^2 \leq 4$ $\Rightarrow \sqrt{x^2} \leq \sqrt{4} \Rightarrow x \leq 2 \Leftrightarrow -2 \leq x \leq 2$ Hence, $D_f = [-2, 2].$
44) The function $f(x) = \frac{x+3}{\sqrt{4-x^2}}$ is continuous on its domain where $f(x)$ is defined, we mean that $4 - x^2 > 0 \Rightarrow -x^2 > -4 \Rightarrow x^2 < 4$ $\Rightarrow \sqrt{x^2} < \sqrt{4} \Rightarrow x < 2 \Leftrightarrow -2 < x < 2$ Hence, $D_f = (-2, 2).$	45) The function $f(x) = \frac{x+1}{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x^2 - 4 \neq 0 \Rightarrow x^2 \neq 4 \Rightarrow x \neq \pm 2$ Hence, $D_f = \mathbb{R} \setminus \{\pm 2\}$ $= (-\infty, -2) \cup (-2, 2) \cup (2, \infty) = \{x \in \mathbb{R} : x \neq \pm 2\}.$

46) The function $f(x) = \log_2(x + 2)$ is continuous on its domain where $f(x)$ is defined, we mean that $x + 2 > 0 \Rightarrow x > -2$ Hence, $D_f = (-2, \infty)$.	47) The function $f(x) = \sqrt{x - 1} + \sqrt{x + 4}$ is continuous on its domain where $f(x)$ is defined, we mean that $x - 1 \geq 0$ and $x + 4 \geq 0 \Rightarrow x \geq 1 \cap x \geq -4$ Hence, $D_f = [1, \infty)$.
48) The function $f(x) = 5^x$ is continuous on its domain. Hence, $D_f = \mathbb{R} = (-\infty, \infty)$.	49) The function $f(x) = e^x$ is continuous on its domain. Hence, $D_f = \mathbb{R} = (-\infty, \infty)$.
50) The function $f(x) = \sin^{-1}(3x - 5)$ is continuous on its domain where $f(x)$ is defined, we mean that $-1 \leq 3x - 5 \leq 1 \Leftrightarrow 4 \leq 3x \leq 6 \Leftrightarrow \frac{4}{3} \leq x \leq 2$. Hence, $D_f = \left[\frac{4}{3}, 2\right]$.	51) The function $f(x) = \cos^{-1}(3x + 5)$ is continuous on its domain where $f(x)$ is defined, we mean that $-1 \leq 3x + 5 \leq 1 \Leftrightarrow -6 \leq 3x \leq -4 \Leftrightarrow -2 \leq x \leq -\frac{4}{3}$. Hence, $D_f = \left[-2, -\frac{4}{3}\right]$.
52) The number c that makes $f(x) = \begin{cases} c + x, & x > 2 \\ 2x - c, & x \leq 2 \end{cases}$ is continuous at $x = 2$ is <u>Solution:</u> $\lim_{x \rightarrow 2} f(x)$ exists if $\begin{aligned} \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} (c + x) \\ \lim_{x \rightarrow 2^+} (c + x) &= \lim_{x \rightarrow 2^+} (2x - c) \\ c + 2 &= 4 - c \\ c + c &= 4 - 2 \\ 2c &= 2 \\ c &= 1 \end{aligned}$	53) The number c that makes $f(x) = \begin{cases} cx^2 - 2x + 1, & x \leq -1 \\ 3x + 2, & x > -1 \end{cases}$ is continuous at -1 is <u>Solution:</u> $\lim_{x \rightarrow -1} f(x)$ exists if $\begin{aligned} \lim_{x \rightarrow -1^+} f(x) &= \lim_{x \rightarrow -1^+} (3x + 2) \\ \lim_{x \rightarrow -1^+} (3x + 2) &= \lim_{x \rightarrow -1^-} (cx^2 - 2x + 1) \\ 3(-1) + 2 &= c(-1)^2 - 2(-1) + 1 \\ -1 &= c + 3 \\ c &= -1 - 3 \\ c &= -4 \end{aligned}$
54) The number c that makes $f(x) = \begin{cases} \frac{\sin cx}{x} + 2x - 1, & x < 0 \\ 3x + 4, & x \geq 0 \end{cases}$ is continuous at 0 is <u>Solution:</u> $\lim_{x \rightarrow 0} f(x)$ exists if $\begin{aligned} \lim_{x \rightarrow 0^+} f(x) &= \lim_{x \rightarrow 0^+} (3x + 4) \\ \lim_{x \rightarrow 0^+} (3x + 4) &= \lim_{x \rightarrow 0^-} \left(\frac{\sin cx}{x} + 2x - 1 \right) \\ 3(0) + 4 &= c(1) + 2(0) - 1 \\ 4 &= c - 1 \\ c &= 4 + 1 \\ c &= 5 \end{aligned}$	55) The value c that makes $f(x) = \begin{cases} cx^2 + 2x, & x \leq 2 \\ x^3 - cx, & x > 2 \end{cases}$ is continuous at 2 is <u>Solution:</u> $\lim_{x \rightarrow 2} f(x)$ exists if $\begin{aligned} \lim_{x \rightarrow 2^+} f(x) &= \lim_{x \rightarrow 2^+} (x^3 - cx) \\ \lim_{x \rightarrow 2^+} (x^3 - cx) &= \lim_{x \rightarrow 2^-} (cx^2 + 2x) \\ (2)^3 - c(2) &= c(2)^2 + 2(2) \\ 8 - 2c &= 4c + 4 \\ -2c - 4c &= 4 - 8 \\ -6c &= -4 \\ c &= \frac{-4}{-6} \\ c &= \frac{2}{3} \end{aligned}$
56) The number c that makes $f(x) = \begin{cases} c^2x^2 - 1, & x \leq 3 \\ x + 5, & x > 3 \end{cases}$ is continuous at 3 is <u>Solution:</u> $\lim_{x \rightarrow 3} f(x)$ exists if $\begin{aligned} \lim_{x \rightarrow 3^+} f(x) &= \lim_{x \rightarrow 3^+} (x + 5) \\ \lim_{x \rightarrow 3^+} (x + 5) &= \lim_{x \rightarrow 3^-} (c^2x^2 - 1) \\ (3) + 5 &= c^2(3)^2 - 1 \\ 8 &= 9c^2 - 1 \\ 9c^2 &= 8 + 1 \\ c^2 &= 1 \\ c &= \pm 1 \end{aligned}$	57) The number c that makes $f(x) = \begin{cases} x - 2, & x > 5 \\ cx - 3, & x \leq 5 \end{cases}$ is continuous at 5 is <u>Solution:</u> $\lim_{x \rightarrow 5} f(x)$ exists if $\begin{aligned} \lim_{x \rightarrow 5^+} f(x) &= \lim_{x \rightarrow 5^+} (x - 2) \\ \lim_{x \rightarrow 5^+} (x - 2) &= \lim_{x \rightarrow 5^-} (cx - 3) \\ (5) - 2 &= c(5) - 3 \\ 3 &= 5c - 3 \\ 5c &= 3 + 3 \\ 5c &= 6 \\ c &= \frac{6}{5} \end{aligned}$

58) The number c that makes $f(x) = \begin{cases} x + 3, & x > -1 \\ 2x - c, & x \leq -1 \end{cases}$ is continuous at -1 is

Solution:

$\lim_{x \rightarrow -1} f(x)$ exists if

$$\begin{aligned}\lim_{x \rightarrow -1^+} f(x) &= \lim_{x \rightarrow -1^-} f(x) \\ \lim_{x \rightarrow -1^+} (x + 3) &= \lim_{x \rightarrow -1^-} (2x - c) \\ (-1) + 3 &= 2(-1) - c \\ 2 &= -2 - c \\ c &= -2 - 2 \\ c &= -4\end{aligned}$$