Workshop Solutions to Sections 3.4 and 3.5

1) $\lim\limits_{x\to 3^+} \dfrac{2}{x-3} =$

<u>Solution:</u>

If $x \to 3^+$, then $x > 3 \implies x - 3 > 0$

$$\therefore \lim_{x\to 3^+} \frac{2}{x-3} = \infty$$

2) $\lim\limits_{x\to 3^-} \dfrac{2}{x-3} =$

<u>Solution:</u>

If $x \to 3^-$, then $x < 3 \implies x - 3 < 0$

$$\therefore \lim_{x\to 3^-} \frac{2}{x-3} = -\infty$$

3) $\lim\limits_{x\to 3^+} \dfrac{-2}{x-3} =$

<u>Solution:</u>

If $x \to 3^+$, then $x > 3 \implies x - 3 > 0$

$$\therefore \lim_{x\to 3^+} \frac{-2}{x-3} = -\infty$$

4) $\lim\limits_{x\to 3^-} \dfrac{-2}{x-3} =$

<u>Solution:</u>

If $x \to 3^-$, then $x < 3 \implies x - 3 < 0$

$$\therefore \lim_{x\to 3^-} \frac{2}{x-3} = \infty$$

5) $\lim\limits_{x\to -3^+} \dfrac{2}{x+3} =$

<u>Solution:</u>

If $x \to -3^+$, then $x > -3 \implies x + 3 > 0$

$$\therefore \lim_{x\to -3^+} \frac{2}{x+3} = \infty$$

6) $\lim\limits_{x\to -3^-} \dfrac{2}{x+3} =$

<u>Solution:</u>

If $x \to -3^-$, then $x < -3 \implies x + 3 < 0$

$$\therefore \lim_{x\to -3^-} \frac{2}{x+3} = -\infty$$

7) $\lim\limits_{x\to 2^+} \dfrac{3x-1}{x-2} =$

<u>Solution:</u>

If $x \to 2^+$, then $x > 2 \implies x - 2 > 0$ and $3x - 1 > 0$

$$\therefore \lim_{x\to 2^+} \frac{3x-1}{x-2} = \infty$$

8) $\lim\limits_{x\to 2^-} \dfrac{3x-1}{x-2} =$

<u>Solution:</u>

If $x \to 2^-$, then $x < 2 \implies x - 2 < 0$ and $3x - 1 > 0$

$$\therefore \lim_{x\to 2^-} \frac{3x-1}{x-2} = -\infty$$

9) $\lim\limits_{x\to -2^+} \dfrac{1-x}{(x+2)^2} =$

<u>Solution:</u>

If $x \to -2^+$, then $x > -2$

$$\implies 1 - x > 0 \text{ and } (x+2)^2 > 0$$

$$\therefore \lim_{x\to -2^+} \frac{1-x}{(x+2)^2} = \infty$$

10) $\lim\limits_{x\to -2^-} \dfrac{1-x}{(x+2)^2} =$

<u>Solution:</u>

If $x \to -2^-$, then $x < -2$

$$\implies 1 - x > 0 \text{ and } (x+2)^2 > 0$$

$$\therefore \lim_{x\to -2^+} \frac{1-x}{(x+2)^2} = \infty$$

11) $\lim\limits_{x\to -2^+} \dfrac{x-1}{(x+2)^2} =$

<u>Solution:</u>

If $x \to -2^+$, then $x > -2$

$$\implies x - 1 < 0 \text{ and } (x+2)^2 > 0$$

$$\therefore \lim_{x\to -2^+} \frac{x-1}{(x+2)^2} = -\infty$$

12) $\lim\limits_{x\to -2^-} \dfrac{x-1}{(x+2)^2} =$

<u>Solution:</u>

If $x \to -2^-$, then $x < -2$

$$\implies x - 1 < 0 \text{ and } (x+2)^2 > 0$$

$$\therefore \lim_{x\to -2^-} \frac{x-1}{(x+2)^2} = -\infty$$

13) $\lim\limits_{x\to 2^+} \dfrac{6x-1}{x^2-4} =$

<u>Solution:</u>

If $x \to 2^+$, then $x^2 > 4$

$$\implies x^2 - 4 > 0 \text{ and } 6x - 1 > 0$$

$$\therefore \lim_{x\to 2^+} \frac{6x-1}{x^2-4} = \infty$$

14) $\lim\limits_{x\to 2^-} \dfrac{6x-1}{x^2-4} =$

<u>Solution:</u>

If $x \to 2^-$, then $x^2 < 4$

$$\implies x^2 - 4 < 0 \text{ and } 6x - 1 > 0$$

$$\therefore \lim_{x\to 2^+} \frac{6x-1}{x^2-4} = -\infty$$

15) $\displaystyle\lim_{x\to-2^+}\frac{6x-1}{x^2-4}=$

<u>Solution:</u>

If $x\to-2^+$, then $x^2<4$

$$\Longrightarrow\ x^2-4<0 \text{ and } 6x-1<0$$

$$\therefore\quad \lim_{x\to2^+}\frac{6x-1}{x^2-4}=\infty$$

16) $\displaystyle\lim_{x\to-2^-}\frac{6x-1}{x^2-4}=$

<u>Solution:</u>

If $x\to-2^-$, then $x^2>4$

$$\Longrightarrow\ x^2-4>0 \text{ and } 6x-1<0$$

$$\therefore\quad \lim_{x\to2^+}\frac{6x-1}{x^2-4}=-\infty$$

17) $\displaystyle\lim_{x\to-2^-}\frac{6x-1}{x^2-x-6}=$

<u>Solution:</u>

$$f(x)=\frac{6x-1}{x^2-x-6}=\frac{6x-1}{(x-3)(x+2)}$$

If $x\to-2^-$, then $x<-2$

$$\Longrightarrow\ x-3<0,\ x+2<0 \text{ and } 6x-1<0$$

$$\therefore\quad \lim_{x\to-2^-}\frac{6x-1}{x^2-x-6}=-\infty$$

18) $\displaystyle\lim_{x\to-2^+}\frac{6x-1}{x^2-x-6}=$

<u>Solution:</u>

$$f(x)=\frac{6x-1}{x^2-x-6}=\frac{6x-1}{(x-3)(x+2)}$$

If $x\to-2^+$, then $x>-2$

$$\Longrightarrow\ x-3<0,\ x+2>0 \text{ and } 6x-1<0$$

$$\therefore\quad \lim_{x\to-2^+}\frac{6x-1}{x^2-x-6}=\infty$$

19) $\displaystyle\lim_{x\to3^+}\frac{-1}{x^2-x-6}=$

<u>Solution:</u>

$$f(x)=\frac{-1}{x^2-x-6}=\frac{-1}{(x-3)(x+2)}$$

If $x\to3^+$, then $x>3$

$$\Longrightarrow\ x-3>0,\ x+2>0 \text{ and } -1<0$$

$$\therefore\quad \lim_{x\to3^+}\frac{-1}{x^2-x-6}=-\infty$$

20) $\displaystyle\lim_{x\to3^-}\frac{-1}{x^2-x-6}=$

<u>Solution:</u>

$$f(x)=\frac{-1}{x^2-x-6}=\frac{-1}{(x-3)(x+2)}$$

If $x\to3^-$, then $x<3$

$$\Longrightarrow\ x-3<0,\ x+2>0 \text{ and } -1<0$$

$$\therefore\quad \lim_{x\to3^-}\frac{-1}{x^2-x-6}=\infty$$

21) $\displaystyle\lim_{x\to(\pi/2)^+}\tan x=$

<u>Solution:</u>

$$\lim_{x\to(\pi/2)^+}\tan x=-\infty$$

22) $\displaystyle\lim_{x\to(\pi/2)^-}\tan x=$

<u>Solution:</u>

$$\lim_{x\to(\pi/2)^-}\tan x=\infty$$

23) The vertical asymptote of $f(x)=\frac{1-x}{2x+1}$ is

<u>Solution:</u>

We see that the function $f(x)$ is not defined when $2x+1=0\ \Longrightarrow\ x=-\frac{1}{2}$. Since

$$\lim_{x\to\left(-\frac{1}{2}\right)^+}\frac{1-x}{2x+1}=\infty$$

and

$$\lim_{x\to\left(-\frac{1}{2}\right)^-}\frac{1-x}{2x+1}=-\infty$$

then, $x=-\frac{1}{2}$ is a vertical asymptote.

24) The vertical asymptote of $f(x)=\frac{3-x}{x^2-4}$ is

<u>Solution:</u>

We see that the function $f(x)$ is not defined when $x^2-4=0\ \Longrightarrow\ x=\pm2$. Since

$$\lim_{x\to2^+}\frac{3-x}{x^2-4}=\infty,\qquad \lim_{x\to2^-}\frac{3-x}{x^2-4}=-\infty$$

and

$$\lim_{x\to-2^+}\frac{3-x}{x^2-4}=-\infty,\qquad \lim_{x\to-2^-}\frac{3-x}{x^2-4}=\infty$$

then, $x=\pm2$ are vertical asymptotes.

25) The vertical asymptote of $f(x) = \frac{3-x}{x^2-x-6}$ is

Solution:

$$f(x) = \frac{3-x}{x^2-x-6} = \frac{3-x}{(x-3)(x+2)} = \frac{-(x-3)}{(x-3)(x+2)}$$
$$= -\frac{1}{x+2}$$

We see that the function $f(x)$ is not defined when $x^2 - x - 6 = 0 \implies (x-3)(x+2) = 0$ $\implies$ $x = 3$ or $x = -2$. Since

$$\lim_{x\to3} \frac{3-x}{x^2-x-6} = \lim_{x\to3} \frac{3-x}{(x-3)(x+2)}$$
$$= \lim_{x\to3} \frac{-(x-3)}{(x-3)(x+2)} = \lim_{x\to3} \frac{-1}{x+2} = -\frac{1}{5}$$

then, $x = 3$ is a removable discontinuity.

$$\lim_{x\to-2^+} \frac{3-x}{x^2-x-6} = \lim_{x\to-2^+} \frac{3-x}{(x-3)(x+2)} = \infty$$

and

$$\lim_{x\to-2^-} \frac{3-x}{x^2-x-6} = \lim_{x\to-2^-} \frac{3-x}{(x-3)(x+2)} = -\infty$$

then, $x = -2$ is a vertical asymptote only.

26) The vertical asymptote of $f(x) = \frac{7-x}{x^2-5x+6}$ is

Solution:

$$f(x) = \frac{7-x}{x^2-5x+6} = \frac{7-x}{(x-3)(x-2)}$$

We see that the function $f(x)$ is not defined when $-3 = 0$ or $x - 2 = 0 \implies x = 3$ or $x = 2$. Since

$$\lim_{x\to3^+} \frac{7-x}{x^2-5x+6} = \lim_{x\to3^+} \frac{7-x}{(x-3)(x-2)} = \infty$$
$$\lim_{x\to3^-} \frac{7-x}{x^2-5x+6} = \lim_{x\to3^-} \frac{7-x}{(x-3)(x-2)} = -\infty$$

and

$$\lim_{x\to2^+} \frac{7-x}{x^2-5x+6} = \lim_{x\to2^+} \frac{7-x}{(x-3)(x-2)} = -\infty$$
$$\lim_{x\to2^-} \frac{7-x}{x^2-5x+6} = \lim_{x\to2^-} \frac{7-x}{(x-3)(x-2)} = \infty$$

then, $x = 3$ and $x = 2$ are vertical asymptotes.

27) The vertical asymptote of $f(x) = \frac{x-7}{x^2+5x+6}$ is

Solution:

$$f(x) = \frac{x-7}{x^2+5x+6} = \frac{x-7}{(x+3)(x+2)}$$

We see that the function $f(x)$ is not defined when $+3 = 0$ or $x + 2 = 0 \implies x = -3$ or $x = -2$. Since

$$\lim_{x\to-3^+} \frac{x-7}{x^2+5x+6} = \lim_{x\to-3^+} \frac{x-7}{(x+3)(x+2)} = \infty$$
$$\lim_{x\to-3^-} \frac{x-7}{x^2+5x+6} = \lim_{x\to-3^-} \frac{x-7}{(x+3)(x+2)} = -\infty$$

and

$$\lim_{x\to-2^+} \frac{x-7}{x^2+5x+6} = \lim_{x\to-2^+} \frac{x-7}{(x+3)(x+2)} = -\infty$$
$$\lim_{x\to-2^-} \frac{x-7}{x^2+5x+6} = \lim_{x\to-2^-} \frac{x-7}{(x+3)(x+2)} = \infty$$

then, $x = -3$ and $x = -2$ are vertical asymptotes.

28) The vertical asymptote of $f(x) = \frac{x-7}{x^2+3x}$ is

Solution:

$$f(x) = \frac{x-7}{x^2+3x} = \frac{x-7}{x(x+3)}$$

We see that the function $f(x)$ is not defined when $= 0$ or $x + 3 = 0 \implies x = 0$ or $x = -3$. Since

$$\lim_{x\to-3^+} \frac{x-7}{x^2+3x} = \lim_{x\to-3^+} \frac{x-7}{x(x+3)} = \infty$$
$$\lim_{x\to-3^-} \frac{x-7}{x^2+3x} = \lim_{x\to-3^-} \frac{x-7}{x(x+3)} = -\infty$$

and

$$\lim_{x\to0^+} \frac{x-7}{x^2+3x} = \lim_{x\to0^+} \frac{x-7}{x(x+3)} = -\infty$$
$$\lim_{x\to0^-} \frac{x-7}{x^2+3x} = \lim_{x\to0^-} \frac{x-7}{x(x+3)} = \infty$$

then, $x = -3$ and $x = 0$ are vertical asymptotes.

29) The vertical asymptote of $f(x)=\frac{x-7}{x^2-3x}$ is

Solution:

$$f(x)=\frac{x-7}{x^2-3x}=\frac{x-7}{x(x-3)}$$

We see that the function $f(x)$ is not defined when $=0$ or $x-3=0 \implies x=0$ or $x=3$. Since

$$\lim_{x\to 3^+}\frac{x-7}{x^2-3x}=\lim_{x\to 3^+}\frac{x-7}{x(x-3)}=-\infty$$
$$\lim_{x\to 3^-}\frac{x-7}{x^2-3x}=\lim_{x\to 3^-}\frac{x-7}{x(x-3)}=\infty$$

and

$$\lim_{x\to 0^+}\frac{x-7}{x^2-3x}=\lim_{x\to 0^+}\frac{x-7}{x(x-3)}=\infty$$
$$\lim_{x\to 0^-}\frac{x-7}{x^2-3x}=\lim_{x\to 0^-}\frac{x-7}{x(x-3)}=-\infty$$

then, $x=3$ and $x=0$ are vertical asymptotes.

30) The vertical asymptotes of $f(x)=\frac{2x^2+1}{x^2-9}$ are

Solution:

$$f(x)=\frac{2x^2+1}{x^2-9}=\frac{2x^2+1}{(x+3)(x-3)}$$

We see that the function $f(x)$ is not defined when $x^2-9=0 \implies x=\pm 3$. Since

$$\lim_{x\to 3^+}\frac{2x^2+1}{x^2-9}=\lim_{x\to 3^+}\frac{2x^2+1}{(x+3)(x-3)}=\infty$$
$$\lim_{x\to 3^-}\frac{2x^2+1}{x^2-9}=\lim_{x\to 3^-}\frac{2x^2+1}{(x+3)(x-3)}=-\infty$$

and

$$\lim_{x\to -3^+}\frac{2x^2+1}{x^2-9}=\lim_{x\to -3^+}\frac{2x^2+1}{(x+3)(x-3)}=-\infty$$
$$\lim_{x\to -3^-}\frac{2x^2+1}{x^2-9}=\lim_{x\to -3^-}\frac{2x^2+1}{(x+3)(x-3)}=\infty$$

then, $x=\pm 3$ are vertical asymptotes.

31) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous at $a=2$ because

1 − $f(2)=\frac{(2)+1}{(2)^2-9}=\frac{3}{-5}=-\frac{3}{5}$

2 − $\lim_{x\to 3^-}\frac{x+1}{x^2-9}=\lim_{x\to 2}\frac{(2)+1}{(2)^2-9}=\frac{3}{-5}=-\frac{3}{5}$

3 − $\lim_{x\to 2}\frac{x+1}{x^2-9}=f(2)$

OR

We know that $D_f=\mathbb{R}\setminus\{\pm 3\}$, so $\{2\}\in D_f$.

Note: Any function is continuous on its domain.

32) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at $a=\pm 3$ because we know that $D_f=\mathbb{R}\setminus\{\pm 3\}$, so $\{\pm 3\}\notin D_f$.

33) The function $f(x)=\frac{x+1}{x^2-9}$ is discontinuous at ± 3 because $\{\pm 3\}\notin D_f$.

34) The function $f(x)=\frac{x+1}{x^2-9}$ is continuous on its domain which is $D_f=\mathbb{R}\setminus\{\pm 3\}$.

35) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 3, & x=0\end{cases}$ is continuous at $a=0$ because

1- $f(0)=3$

2- $\lim_{x\to 0}\frac{\sin 3x}{x}=3\lim_{x\to 0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim_{x\to 0}f(x)=f(0)$

36) The function $f(x)=\begin{cases}\frac{\sin 3x}{x}, & x\neq 0\\ 5, & x=0\end{cases}$ is discontinuous at $a=0$ because

1- $f(0)=5$

2- $\lim_{x\to 0}\frac{\sin 3x}{x}=3\lim_{x\to 0}\frac{\sin 3x}{3x}=3(1)=3$

3- $\lim_{x\to 0}f(x)\neq f(0)$

37) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 7, & x=1\end{cases}$ is discontinuous at $a=1$ because

1- $f(1)=7$

2- $\lim_{x\to 1}\frac{2x^2-3x+1}{x-1}=\lim_{x\to 1}\frac{(2x-1)(x-1)}{x-1}=\lim_{x\to 1}(2x-1)=1$

3- $\lim_{x\to 1}f(x)\neq f(1)$

38) The function $f(x)=\begin{cases}\frac{2x^2-3x+1}{x-1}, & x\neq 1\\ 1, & x=1\end{cases}$ is continuous at $a=1$ because

1- $f(1)=1$

2- $\lim_{x\to 1}\frac{2x^2-3x+1}{x-1}=\lim_{x\to 1}\frac{(2x-1)(x-1)}{x-1}=\lim_{x\to 1}(2x-1)=1$

3- $\lim_{x\to 1}f(x)=f(1)$

39) The function $f(x)=\frac{x^2-x-2}{x-2}$ is discontinuous at $a=2$ because $\{2\}\notin D_f$.

40) The function $f(x) = \begin{cases} 2x+3, & x > 2 \\ 3x+1, & x \le 2 \end{cases}$ is continuous at $a = 2$ because

1- $f(2) = 3(2) + 1 = 7$

2- $\lim_{x \to 2^+} (2x+3) = 2(2) + 3 = 7$

$\lim_{x \to 2^-} (3x+1) = 3(2) + 1 = 7$

$\therefore \lim_{x \to 2} f(x) = 7$

3- $\lim_{x \to 2} f(x) = f(2)$

41) The function $f(x) = \frac{x+3}{\sqrt{x^2-4}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2 - 4 > 0 \implies x^2 > 4 \implies \sqrt{x^2} > \sqrt{4}$$
$$\implies |x| > 2 \iff x > 2 \text{ or } x < -2$$

Hence,

$D_f = (-\infty, -2) \cup (2, \infty)$.

42) The function $f(x) = \sqrt{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2 - 4 \ge 0 \implies x^2 \ge 4 \implies \sqrt{x^2} \ge \sqrt{4}$$
$$\implies |x| \ge 2 \iff x \ge 2 \text{ or } x \le -2$$

Hence,

$D_f = (-\infty, -2] \cup [2, \infty)$.

43) The function $f(x) = \sqrt{4-x^2}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4 - x^2 \ge 0 \implies -x^2 \ge -4 \implies x^2 \le 4$$
$$\implies \sqrt{x^2} \le \sqrt{4} \implies |x| \le 2 \iff -2 \le x \le 2$$

Hence,

$D_f = [-2,2]$.

44) The function $f(x) = \frac{x+3}{\sqrt{4-x^2}}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$4 - x^2 > 0 \implies -x^2 > -4 \implies x^2 < 4$$
$$\implies \sqrt{x^2} < \sqrt{4} \implies |x| < 2 \iff -2 < x < 2$$

Hence,

$D_f = (-2,2)$.

45) The function $f(x) = \frac{x+1}{x^2-4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x^2 - 4 \ne 0 \implies x^2 \ne 4 \implies x \ne \pm 2$$

Hence,

$D_f = \mathbb{R} \setminus \{\pm 2\}$

$= (-\infty, -2) \cup (-2,2) \cup (2, \infty) = \{x \in \mathbb{R} : x \ne \pm 2\}$.

46) The function $f(x) = \log_2(x+2)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x + 2 > 0 \implies x > -2$$

Hence,

$D_f = (-2, \infty)$.

47) The function $f(x) = \sqrt{x-1} + \sqrt{x+4}$ is continuous on its domain where $f(x)$ is defined, we mean that

$$x - 1 \ge 0 \text{ and } x + 4 \ge 0 \implies x \ge 1 \cap x \ge -4$$

Hence,

$D_f = [1, \infty)$.

48) The function $f(x) = 5^x$ is continuous on its domain .

Hence,

$D_f = \mathbb{R} = (-\infty, \infty)$.

49) The function $f(x) = e^x$ is continuous on its domain .

Hence,

$D_f = \mathbb{R} = (-\infty, \infty)$.

50) The function $f(x) = \sin^{-1}(3x-5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1 \le 3x - 5 \le 1 \iff 4 \le 3x \le 6 \iff \frac{4}{3} \le x \le 2 .$$

Hence,

$D_f = \left[\frac{4}{3}, 2\right]$.

51) The function $f(x) = \cos^{-1}(3x+5)$ is continuous on its domain where $f(x)$ is defined, we mean that

$$-1 \le 3x + 5 \le 1 \iff -6 \le 3x \le -4 \iff -2 \le x \le -\frac{4}{3} .$$

Hence,

$D_f = \left[-2, -\frac{4}{3}\right]$.

52) The number c that makes $f(x) = \begin{cases} c + x, & x > 2 \\ 2x - c, & x \le 2 \end{cases}$ is continuous at $x = 2$ is

<u>Solution:</u>

$\lim_{x \to 2} f(x)$ exists if

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$$
$$\lim_{x \to 2^+} (c + x) = \lim_{x \to 2^-} (2x - c)$$
$$c + 2 = 4 - c$$
$$c + c = 4 - 2$$
$$2c = 2$$
$$c = 1$$

53) The number c that makes

$f(x) = \begin{cases} cx^2 - 2x + 1, & x \le -1 \\ 3x + 2, & x > -1 \end{cases}$ is continuous at -1 is

<u>Solution:</u>

$\lim_{x \to -1} f(x)$ exists if

$$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$$
$$\lim_{x \to -1^+} (3x + 2) = \lim_{x \to -1^-} (cx^2 - 2x + 1)$$
$$3(-1) + 2 = c(-1)^2 - 2(-1) + 1$$
$$-1 = c + 3$$
$$c = -1 - 3$$
$$c = -4$$

54) The number c that makes $f(x) = \begin{cases} \frac{\sin cx}{x} + 2x - 1, & x < 0 \\ 3x + 4 \quad , & x \geq 0 \end{cases}$ is continuous at 0 is Solution: $\lim_{x \to 0} f(x)$ exists if $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$ $\lim_{x \to 0^+} (3x + 4) = \lim_{x \to 0^-} \left(\frac{\sin cx}{x} + 2x - 1 \right)$ $3(0) + 4 = c(1) + 2(0) - 1$ $4 = c - 1$ $c = 4 + 1$ $c = 5$	55) The value c that makes $f(x) = \begin{cases} cx^2 + 2x, & x \leq 2 \\ x^3 - cx \ , & x > 2 \end{cases}$ is continuous at 2 is Solution: $\lim_{x \to 2} f(x)$ exists if $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$ $\lim_{x \to 2^+} (x^3 - cx \ ) = \lim_{x \to 2^-} (cx^2 + 2x)$ $(2)^3 - c(2) = c(2)^2 + 2(2)$ $8 - 2c = 4c + 4$ $-2c - 4c = 4 - 8$ $-6c = -4$ $c = \frac{-4}{-6}$ $c = \frac{2}{3}$
56) The number c that makes $f(x) = \begin{cases} c^2x^2 - 1, & x \leq 3 \\ x + 5 \quad , & x > 3 \end{cases}$ is continuous at 3 is Solution: $\lim_{x \to 3} f(x)$ exists if $\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$ $\lim_{x \to 3^+} (x + 5) = \lim_{x \to 3^-} (c^2x^2 - 1)$ $(3) + 5 = c^2(3)^2 - 1$ $8 = 9c^2 - 1$ $9c^2 = 8 + 1$ $c^2 = 1$ $c = \pm 1$	57) The number c that makes $f(x) = \begin{cases} x - 2 \ , & x > 5 \\ cx - 3, & x \leq 5 \end{cases}$ is continuous at 5 is Solution: $\lim_{x \to 5} f(x)$ exists if $\lim_{x \to 5^+} f(x) = \lim_{x \to 5^-} f(x)$ $\lim_{x \to 5^+} (x - 2) = \lim_{x \to 5^-} (cx - 3)$ $(5) - 2 = c(5) - 3$ $3 = 5c - 3$ $5c = 3 + 3$ $5c = 6$ $c = \frac{6}{5}$
58) The number c that makes $f(x) = \begin{cases} x + 3 \ , & x > -1 \\ 2x - c, & x \leq -1 \end{cases}$ is continuous at -1 is Solution: $\lim_{x \to -1} f(x)$ exists if $\lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$ $\lim_{x \to -1^+} (x + 3) = \lim_{x \to -1^-} (2x - c)$ $(-1) + 3 = 2(-1) - c$ $2 = -2 - c$ $c = -2 - 2$ $c = -4$	