

Workshop Solutions to Section 3.3

1) If $f(x) = \begin{cases} 2x + 3; & x \geq -2 \\ 2x + 5; & x < -2 \end{cases}$ then $\lim_{x \rightarrow (-2)^-} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow (-2)^-} f(x) = \lim_{x \rightarrow (-2)^-} (2x + 5) = 2(-2) + 5 = -4 + 5 = 1$	2) If $f(x) = \begin{cases} 2x + 3; & x \geq -2 \\ 2x + 5; & x < -2 \end{cases}$ then $\lim_{x \rightarrow (-2)^+} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow (-2)^+} f(x) = \lim_{x \rightarrow (-2)^+} (2x + 3) = 2(-2) + 3 = -4 + 3 = -1$
3) If $f(x) = \begin{cases} 2x + 3; & x \geq -2 \\ 2x + 5; & x < -2 \end{cases}$ then $\lim_{x \rightarrow -2} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow -2} f(x)$ does not exist because $\lim_{x \rightarrow (-2)^-} f(x) \neq \lim_{x \rightarrow (-2)^+} f(x)$	4) If $f(x) = \begin{cases} x^2 - 2x + 3; & x \geq 3 \\ x^3 - 3x - 12; & x < 3 \end{cases}$ then $\lim_{x \rightarrow 3} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (x^3 - 3x - 12) = (3)^3 - 3(3) - 12 = 27 - 9 - 12 = 6$ $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (x^2 - 2x + 3) = (3)^2 - 2(3) + 3 = 9 - 6 + 3 = 6$ $\therefore \lim_{x \rightarrow 3} f(x) = 6$
5) If $f(x) = \begin{cases} x^2 - 7x; & x < 1 \\ 5; & 1 \leq x \leq 3 \\ 3x + 1; & x > 3 \end{cases}$ then $\lim_{x \rightarrow 1^-} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (x^2 - 7x) = (1)^2 - 7(1) = 1 - 7 = -6$	6) If $f(x) = \begin{cases} x^2 - 7x; & x < 1 \\ 5; & 1 \leq x \leq 3 \\ 3x + 1; & x > 3 \end{cases}$ then $\lim_{x \rightarrow 1^+} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (5) = 5$
7) If $f(x) = \begin{cases} x^2 - 7x; & x < 1 \\ 5; & 1 \leq x \leq 3 \\ 3x + 1; & x > 3 \end{cases}$ then $\lim_{x \rightarrow 3^-} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (5) = 5$	8) If $f(x) = \begin{cases} x^2 - 7x; & x < 1 \\ 5; & 1 \leq x \leq 3 \\ 3x + 1; & x > 3 \end{cases}$ then $\lim_{x \rightarrow 3^+} f(x) =$ <u>Solution:</u> $\lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (3x + 1) = 3(3) + 1 = 9 + 1 = 10$
9) If $f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; & x^2 - 4 > 0 \\ \frac{x^2+x-6}{4-x^2}; & x^2 - 4 < 0 \end{cases}$ then $\lim_{x \rightarrow 2^+} f(x) =$ <u>Solution:</u> $f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; & x^2 - 4 > 0 \\ \frac{x^2+x-6}{4-x^2}; & x^2 - 4 < 0 \end{cases}$ $= \begin{cases} \frac{x^2+x-6}{x^2-4}; & x^2 > 4 \\ \frac{x^2+x-6}{-(x^2-4)}; & x^2 < 4 \end{cases}$ $= \begin{cases} \frac{(x+3)(x-2)}{(x-2)(x+2)}; & x > 4 \\ \frac{(x+3)(x-2)}{-(x-2)(x+2)}; & x < 4 \end{cases}$ $= \begin{cases} \frac{x+3}{x+2}; & x > 2 \text{ or } x < -2 \\ -\frac{x+3}{x+2}; & -2 < x < 2 \end{cases}$ then $\therefore \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \left(\frac{x+3}{x+2} \right) = \frac{(2)+3}{(2)+2} = \frac{5}{4}$	10) If $f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; & x^2 - 4 > 0 \\ \frac{x^2+x-6}{4-x^2}; & x^2 - 4 < 0 \end{cases}$ then $\lim_{x \rightarrow 2^-} f(x) =$ <u>Solution:</u> $f(x) = \begin{cases} \frac{x^2+x-6}{x^2-4}; & x^2 - 4 > 0 \\ \frac{x^2+x-6}{4-x^2}; & x^2 - 4 < 0 \end{cases}$ $= \begin{cases} \frac{x^2+x-6}{x^2-4}; & x^2 > 4 \\ \frac{x^2+x-6}{-(x^2-4)}; & x^2 < 4 \end{cases}$ $= \begin{cases} \frac{(x+3)(x-2)}{(x-2)(x+2)}; & x > 4 \\ \frac{(x+3)(x-2)}{-(x-2)(x+2)}; & x < 4 \end{cases}$ $= \begin{cases} \frac{x+3}{x+2}; & x > 2 \text{ or } x < -2 \\ -\frac{x+3}{x+2}; & -2 < x < 2 \end{cases}$ then $\therefore \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} \left(-\frac{x+3}{x+2} \right) = -\frac{(2)+3}{(2)+2} = -\frac{5}{4}$

11) $\lim_{x \rightarrow a^-} \frac{ x - a }{x - a} =$ <u>Solution:</u> $f(x) = \frac{ x - a }{x - a} = \begin{cases} \frac{x - a}{x - a} & ; x - a > 0 \\ \frac{-(x - a)}{x - a} & ; x - a < 0 \end{cases} = \begin{cases} 1; & x > a \\ -1; & x < a \end{cases}$ $\therefore \lim_{x \rightarrow a^-} \frac{ x - a }{x - a} = \lim_{x \rightarrow a^-} \frac{-(x - a)}{x - a} = \lim_{x \rightarrow a^-} (-1) = -1$	12) $\lim_{x \rightarrow a^+} \frac{ x - a }{x - a} =$ <u>Solution:</u> $f(x) = \frac{ x - a }{x - a} = \begin{cases} \frac{x - a}{x - a} & ; x - a > 0 \\ \frac{-(x - a)}{x - a} & ; x - a < 0 \end{cases} = \begin{cases} 1; & x > a \\ -1; & x < a \end{cases}$ $\therefore \lim_{x \rightarrow a^+} \frac{ x - a }{x - a} = \lim_{x \rightarrow a^+} \frac{(x - a)}{x - a} = \lim_{x \rightarrow a^+} (1) = 1$
13) $\lim_{x \rightarrow a} \frac{ x - a }{x - a} =$ <u>Solution:</u> $\lim_{x \rightarrow a} \frac{ x - a }{x - a} \text{ does not exist because}$ $\lim_{x \rightarrow a^-} \frac{ x - a }{x - a} \neq \lim_{x \rightarrow a^+} \frac{ x - a }{x - a}$ It is clearly obvious from questions (11) and (12) above.	14) $\lim_{x \rightarrow a^+} \frac{ a - x }{x - a} =$ <u>Solution:</u> $f(x) = \frac{ a - x }{x - a} = \begin{cases} \frac{a - x}{x - a} & ; a - x > 0 \\ \frac{-(a - x)}{x - a} & ; a - x < 0 \end{cases}$ $= \begin{cases} \frac{-(x - a)}{x - a} & ; a > x \\ \frac{(x - a)}{x - a} & ; a < x \end{cases} = \begin{cases} -1; & x < a \\ 1; & x > a \end{cases}$ $\therefore \lim_{x \rightarrow a^+} \frac{ a - x }{x - a} = \lim_{x \rightarrow a^+} (1) = 1$
15) $\lim_{x \rightarrow a^-} \frac{ a - x }{x - a} =$ <u>Solution:</u> $f(x) = \frac{ a - x }{x - a} = \begin{cases} \frac{a - x}{x - a} & ; a - x > 0 \\ \frac{-(a - x)}{x - a} & ; a - x < 0 \end{cases}$ $= \begin{cases} \frac{-(x - a)}{x - a} & ; a > x \\ \frac{(x - a)}{x - a} & ; a < x \end{cases} = \begin{cases} -1; & x < a \\ 1; & x > a \end{cases}$ $\therefore \lim_{x \rightarrow a^-} \frac{ a - x }{x - a} = \lim_{x \rightarrow a^-} (-1) = -1$	16) $\lim_{x \rightarrow a} \frac{ a - x }{x - a} =$ <u>Solution:</u> $\lim_{x \rightarrow a} \frac{ a - x }{x - a} \text{ does not exist because}$ $\lim_{x \rightarrow a^-} \frac{ a - x }{x - a} \neq \lim_{x \rightarrow a^+} \frac{ a - x }{x - a}$ It is clearly obvious from questions (14) and (15) above.
17) $\lim_{x \rightarrow (-a)^-} \frac{ x + a }{x + a} =$ <u>Solution:</u> $f(x) = \frac{ x + a }{x + a} = \begin{cases} \frac{x + a}{x + a} & ; x + a > 0 \\ \frac{-(x + a)}{x + a} & ; x + a < 0 \end{cases} = \begin{cases} 1; & x > -a \\ -1; & x < -a \end{cases}$ $\therefore \lim_{x \rightarrow (-a)^-} \frac{ x + a }{x + a} = \lim_{x \rightarrow (-a)^-} (-1) = -1$	18) $\lim_{x \rightarrow (-a)^+} \frac{ x + a }{x + a} =$ <u>Solution:</u> $f(x) = \frac{ x + a }{x + a} = \begin{cases} \frac{x + a}{x + a} & ; x + a > 0 \\ \frac{-(x + a)}{x + a} & ; x + a < 0 \end{cases} = \begin{cases} 1; & x > -a \\ -1; & x < -a \end{cases}$ $\therefore \lim_{x \rightarrow (-a)^+} \frac{ x + a }{x + a} = \lim_{x \rightarrow (-a)^+} (1) = 1$
19) $\lim_{x \rightarrow -a} \frac{ x + a }{x + a} =$ <u>Solution:</u> $\lim_{x \rightarrow -a} \frac{ x + a }{x + a} \text{ does not exist because}$ $\lim_{x \rightarrow (-a)^-} \frac{ x + a }{x + a} \neq \lim_{x \rightarrow (-a)^+} \frac{ x + a }{x + a}$ It is clearly obvious from questions (17) and (18) above.	

20) $\lim_{x \rightarrow 0^+} \frac{2x - x }{x^2 + x } =$ <u>Solution:</u> $f(x) = \frac{2x - x }{x^2 + x } = \begin{cases} \frac{2x - (x)}{x^2 + (x)} & ; x > 0 \\ \frac{2x - (-x)}{x^2 + (-x)} & ; x < 0 \end{cases}$ $= \begin{cases} \frac{2x - x}{x^2 + x} & ; x > 0 \\ \frac{2x + x}{x^2 - x} & ; x < 0 \end{cases} = \begin{cases} \frac{x}{x^2 + x} & ; x > 0 \\ \frac{3x}{x^2 - x} & ; x < 0 \end{cases}$ $= \begin{cases} \frac{x(x+1)}{3x} & ; x > 0 \\ \frac{x(x-1)}{3x} & ; x < 0 \end{cases}$ $= \begin{cases} \frac{1}{x+1} & ; x > 0 \\ \frac{3}{x-1} & ; x < 0 \end{cases}$ $\therefore \lim_{x \rightarrow 0^+} \frac{2x - x }{x^2 + x } = \lim_{x \rightarrow 0^+} \frac{1}{x+1} = \frac{1}{0+1} = 1$	21) $\lim_{x \rightarrow 0^-} \frac{2x - x }{x^2 + x } =$ <u>Solution:</u> $\therefore \lim_{x \rightarrow 0^-} \frac{2x - x }{x^2 + x } = \lim_{x \rightarrow 0^-} \frac{3}{x-1} = \frac{3}{0-1} = -3$
22) $\lim_{x \rightarrow 0} \frac{2x - x }{x^2 + x } =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{2x - x }{x^2 + x } \text{ does not exist because}$ $\lim_{x \rightarrow 0^-} \frac{2x - x }{x^2 + x } \neq \lim_{x \rightarrow 0^+} \frac{2x - x }{x^2 + x }$ It is clearly obvious from questions (20) and (21) above.	23) $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x} =$ <u>Solution:</u> $\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{\cos^2 x - \sin^2 x} = \lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x - \sin x}{(\cos x - \sin x)(\cos x + \sin x)}$ $= \lim_{x \rightarrow \frac{\pi}{4}} \frac{1}{\cos x + \sin x} = \frac{1}{\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{4}\right)}$ $= \frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}} = \frac{1}{\frac{2}{\sqrt{2}}} = \frac{\sqrt{2}}{2}$
24) $\lim_{x \rightarrow 0} \frac{\cos^2 x + 2 \cos x - 3}{2 \cos^2 x - \cos x - 1} =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{\cos^2 x + 2 \cos x - 3}{2 \cos^2 x - \cos x - 1} = \lim_{x \rightarrow 0} \frac{(\cos x + 3)(\cos x - 1)}{(2 \cos x + 1)(\cos x - 1)}$ $= \lim_{x \rightarrow 0} \frac{\cos x + 3}{2 \cos x + 1} = \frac{\cos(0) + 3}{2 \cos(0) + 1}$ $= \frac{1 + 3}{2(1) + 1} = \frac{4}{3}$	25) $\lim_{x \rightarrow 0} (\sin^2 x + 3 \tan x - 4) =$ <u>Solution:</u> $\lim_{x \rightarrow 0} (\sin^2 x + 3 \tan x - 4) = \sin^2(0) + 3 \tan(0) - 4$ $= 0 + 3(0) - 4 = -4$
26) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{\sin(nx)}{mx} =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{\sin(nx)}{mx} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{\sin(nx)}{nx} = \frac{n}{m} (1) = \frac{n}{m}$	27) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{\tan(nx)}{mx} =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{\tan(nx)}{mx} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{\tan(nx)}{nx} = \frac{n}{m} (1) = \frac{n}{m}$

28) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{nx}{\sin(mx)} =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{nx}{\sin(mx)} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{mx}{\sin(mx)} = \frac{n}{m} (1) = \frac{n}{m}$	29) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{nx}{\tan(mx)} =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{nx}{\tan(mx)} = \frac{n}{m} \lim_{x \rightarrow 0} \frac{mx}{\tan(mx)} = \frac{n}{m} (1) = \frac{n}{m}$
30) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{\sin(nx)}{\sin(mx)} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(nx)}{\sin(mx)} &= \frac{n}{m} \left(\lim_{x \rightarrow 0} \frac{\sin(nx)}{nx} \right) \left(\lim_{x \rightarrow 0} \frac{mx}{\sin(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$	31) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{\sin(nx)}{\tan(mx)} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(nx)}{\tan(mx)} &= \frac{n}{m} \left(\lim_{x \rightarrow 0} \frac{\sin(nx)}{nx} \right) \left(\lim_{x \rightarrow 0} \frac{mx}{\tan(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$
32) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{\tan(nx)}{\tan(mx)} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan(nx)}{\tan(mx)} &= \frac{n}{m} \left(\lim_{x \rightarrow 0} \frac{\tan(nx)}{nx} \right) \left(\lim_{x \rightarrow 0} \frac{mx}{\tan(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$	33) If $m \neq 0$, then $\lim_{x \rightarrow 0} \frac{\tan(nx)}{\sin(mx)} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow 0} \frac{\tan(nx)}{\sin(mx)} &= \frac{n}{m} \left(\lim_{x \rightarrow 0} \frac{\tan(nx)}{nx} \right) \left(\lim_{x \rightarrow 0} \frac{mx}{\sin(mx)} \right) \\ &= \frac{n}{m} (1)(1) = \frac{n}{m} \end{aligned}$
34) $\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{1 - \cos x} =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{\sin(1 - \cos x)}{1 - \cos x} = 1$	35) $\lim_{x \rightarrow 0} \frac{\sin(\sin(2x))}{\sin(2x)} =$ <u>Solution:</u> $\lim_{x \rightarrow 0} \frac{\sin(\sin(2x))}{\sin(2x)} = 1$
36) $\lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x^2} &= \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x^2} = 2 \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right)^2 \\ &= 2 \left(\lim_{x \rightarrow 0} \frac{\sin x}{x} \right)^2 = 2(1)^2 = 2 \end{aligned}$	37) $\lim_{x \rightarrow \infty} \sqrt{\frac{1}{x^2} - \frac{3}{x} + 4} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow \infty} \sqrt{\frac{1}{x^2} - \frac{3}{x} + 4} &= \sqrt{\lim_{x \rightarrow \infty} \left(\frac{1}{x^2} - \frac{3}{x} + 4 \right)} = \sqrt{0 - 0 + 4} \\ &= 2 \end{aligned}$
38) $\lim_{x \rightarrow \infty} \left(\frac{1}{x^{2/5}} + 2 \right) =$ <u>Solution:</u> $\lim_{x \rightarrow -\infty} \left(\frac{1}{x^{2/5}} + 2 \right) = 0 + 2 = 2$	39) $\lim_{x \rightarrow \infty} \frac{3x + 15}{9x^2 + 4x - 13} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow \infty} \frac{\frac{3x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\frac{3}{x} + \frac{15}{x^2}}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{0 + 0}{9 + 0 + 0} = 0 \end{aligned}$

40) $\lim_{x \rightarrow \infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow \infty} \frac{\frac{3x^2}{x^2} - \frac{8x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{3 - \frac{8}{x} + \frac{15}{x^2}}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{3 - 0 + 0}{9 + 0 + 0} = \frac{1}{3} \end{aligned}$	41) $\lim_{x \rightarrow -\infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow -\infty} \frac{3x^2 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow -\infty} \frac{\frac{3x^2}{x^2} - \frac{8x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{-3 + \frac{8}{x} - \frac{15}{x^2}}{-9 - \frac{4}{x} + \frac{13}{x^2}} = \frac{-3 + 0 - 0}{-9 - 0 + 0} = \frac{1}{3} \end{aligned}$
42) $\lim_{x \rightarrow \infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow \infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow \infty} \frac{\frac{3x^5}{x^2} - \frac{8x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{3x^3 - \frac{8}{x} + \frac{15}{x^2}}{9 + \frac{4}{x} - \frac{13}{x^2}} = \frac{3(\infty) - 0 + 0}{9 + 0 + 0} = \infty \end{aligned}$	43) $\lim_{x \rightarrow -\infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow -\infty} \frac{3x^5 - 8x + 15}{9x^2 + 4x - 13} &= \lim_{x \rightarrow -\infty} \frac{\frac{3x^5}{x^2} - \frac{8x}{x^2} + \frac{15}{x^2}}{\frac{9x^2}{x^2} + \frac{4x}{x^2} - \frac{13}{x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{-3x^3 + \frac{8}{x} - \frac{15}{x^2}}{-9 - \frac{4}{x} + \frac{13}{x^2}} = \frac{-3(-\infty) + 0 - 0}{-9 - 0 + 0} = -\infty \end{aligned}$
44) $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x + 7} - x) =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 - 3x + 7} - x) &= \lim_{x \rightarrow \infty} \left[(\sqrt{x^2 - 3x + 7} - x) \times \frac{(\sqrt{x^2 - 3x + 7} + x)}{(\sqrt{x^2 - 3x + 7} + x)} \right] \\ &= \lim_{x \rightarrow \infty} \left(\frac{(x^2 - 3x + 7) - x^2}{\sqrt{x^2 - 3x + 7} + x} \right) = \lim_{x \rightarrow \infty} \left(\frac{-3x + 7}{\sqrt{x^2 - 3x + 7} + x} \right) \\ &= \lim_{x \rightarrow \infty} \frac{\frac{-3x}{x} + \frac{7}{x}}{\frac{\sqrt{x^2 - 3x + 7}}{x} + \frac{x}{x}} + \frac{x}{x} \\ &= \lim_{x \rightarrow \infty} \frac{-3 + \frac{7}{x}}{\sqrt{\frac{x^2}{x^2} - \frac{3x}{x^2} + \frac{7}{x^2}} + 1} \\ &= \lim_{x \rightarrow \infty} \frac{-3 + \frac{7}{x}}{\sqrt{1 - \frac{3}{x} + \frac{7}{x^2}} + 1} \\ &= \frac{-3 + 0}{\sqrt{1 - 0 + 0} + 1} = \frac{-3}{1 + 1} = -\frac{3}{2} \end{aligned}$	45) $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow \infty} (\sqrt{x^2 + x} - x) &= \lim_{x \rightarrow \infty} \left[(\sqrt{x^2 + x} - x) \times \frac{\sqrt{x^2 + x} + x}{\sqrt{x^2 + x} + x} \right] \\ &= \lim_{x \rightarrow \infty} \left(\frac{(x^2 + x) - x^2}{\sqrt{x^2 + x} + x} \right) \\ &= \lim_{x \rightarrow \infty} \left(\frac{x}{\sqrt{x^2 + x} + x} \right) \\ &= \lim_{x \rightarrow \infty} \frac{\frac{x}{x}}{\frac{\sqrt{x^2 + x}}{x} + \frac{x}{x}} = \lim_{x \rightarrow \infty} \frac{1}{\sqrt{\frac{x^2}{x^2} + \frac{x}{x^2}} + 1} \\ &= \lim_{x \rightarrow \infty} \frac{1}{\sqrt{1 + \frac{1}{x}} + 1} = \frac{1}{\sqrt{1 + 0} + 1} = \frac{1}{1 + 1} \\ &= \frac{1}{2} \end{aligned}$
46) $\lim_{x \rightarrow \infty} (x^2 - 5x + 4) =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow \infty} (x^2 - 5x + 4) &= \lim_{x \rightarrow \infty} x^2 \left(\frac{x^2}{x^2} - \frac{5x}{x^2} + \frac{4}{x^2} \right) \\ &= \lim_{x \rightarrow \infty} x^2 \left(1 - \frac{5}{x} + \frac{4}{x^2} \right) = (\infty)^2 (1 - 0 + 0) = \infty \end{aligned}$ OR $\lim_{x \rightarrow \infty} (x^2 - 5x + 4) = \lim_{x \rightarrow \infty} (x^2) = \infty$	47) $\lim_{x \rightarrow -\infty} (x^4 - 2x^3 + 9) =$ <u>Solution:</u> $\begin{aligned} \lim_{x \rightarrow -\infty} (x^4 - 2x^3 + 9) &= \lim_{x \rightarrow -\infty} x^4 \left(\frac{x^4}{x^4} - \frac{2x^3}{x^4} + \frac{9}{x^4} \right) \\ &= \lim_{x \rightarrow -\infty} x^4 \left(1 - \frac{2}{x} + \frac{9}{x^4} \right) = (-\infty)^4 (1 - 0 + 0) = \infty \end{aligned}$ OR $\lim_{x \rightarrow -\infty} (x^4 - 2x^3 + 9) = \lim_{x \rightarrow -\infty} (x^4) = \infty$

48)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} &= \lim_{x \rightarrow -\infty} \frac{\frac{\sqrt{3x^2 - 8}}{-x} + \frac{2}{-x}}{\frac{x}{-x} + \frac{5}{-x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{3x^2 - 8}{x^2}} - \frac{2}{x}}{-1 - \frac{5}{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{3x^2}{x^2} - \frac{8}{x^2}} - \frac{2}{x}}{-1 - \frac{5}{x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{3 - \frac{8}{x^2}} - \frac{2}{x}}{-1 - \frac{5}{x}} = \frac{\sqrt{3 - 0} - 0}{-1 - 0} = -\sqrt{3} \end{aligned}$$

49)

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} &= \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{3x^2 - 8}}{x} + \frac{2}{x}}{\frac{x}{x} + \frac{5}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{3x^2 - 8}{x^2}} + \frac{2}{x}}{1 + \frac{5}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{3x^2}{x^2} - \frac{8}{x^2}} + \frac{2}{x}}{1 + \frac{5}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{3 - \frac{8}{x^2}} + \frac{2}{x}}{1 + \frac{5}{x}} = \frac{\sqrt{3 - 0} + 0}{1 + 0} = \sqrt{3} \end{aligned}$$

50) The horizontal asymptotes of

$$f(x) = \frac{\sqrt{3x^2 - 8} + 2}{x + 5}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5}$$

It is clear from the previous questions (48) and (49) that

$$\lim_{x \rightarrow \infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = \sqrt{3}$$

and

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{3x^2 - 8} + 2}{x + 5} = -\sqrt{3}$$

Thus, the horizontal asymptotes are

$$y = \pm\sqrt{3}$$

51) The horizontal asymptote of

$$f(x) = \frac{1 - x}{2x + 1}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{1 - x}{2x + 1}$$

$$\lim_{x \rightarrow \infty} \frac{1 - x}{2x + 1} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{x}{x}}{\frac{2x}{x} + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x} - 1}{2 + \frac{1}{x}} = \frac{0 - 1}{2 + 0} = -\frac{1}{2}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{1 - x}{2x + 1} &= \lim_{x \rightarrow -\infty} \frac{\frac{1}{-x} - \frac{-x}{-x}}{\frac{2x}{-x} + \frac{1}{-x}} = \lim_{x \rightarrow -\infty} \frac{\frac{1}{-x} + 1}{-2 - \frac{1}{x}} = \frac{0 + 1}{-2 - 0} \\ &= -\frac{1}{2} \end{aligned}$$

Thus, the horizontal asymptote is $y = -\frac{1}{2}$

52) The horizontal asymptote of

$$f(x) = \frac{7x^2 + 5}{3x^2 + 2}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{7x^2 + 5}{3x^2 + 2}$$

$$\lim_{x \rightarrow \infty} \frac{7x^2 + 5}{3x^2 + 2} = \lim_{x \rightarrow \infty} \frac{\frac{7x^2}{x^2} + \frac{5}{x^2}}{\frac{3x^2}{x^2} + \frac{2}{x^2}} = \lim_{x \rightarrow \infty} \frac{7 + \frac{5}{x^2}}{3 + \frac{2}{x^2}} = \frac{7 + 0}{3 + 0} = \frac{7}{3}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{7x^2 + 5}{3x^2 + 2} &= \lim_{x \rightarrow -\infty} \frac{\frac{7x^2}{-x^2} + \frac{5}{-x^2}}{\frac{3x^2}{-x^2} + \frac{2}{-x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{-7 - \frac{5}{x^2}}{-3 - \frac{2}{x^2}} = \frac{-7 - 0}{-3 - 0} = \frac{7}{3} \end{aligned}$$

Thus, the horizontal asymptote is $y = \frac{7}{3}$

53) The horizontal asymptote of

$$f(x) = \frac{\sqrt{x^2 + 2x - 3}}{2x + 7}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{\sqrt{x^2 + 2x - 3}}{2x + 7}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} &= \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{x^2 + 2x - 3}}{x}}{\frac{2x}{x} + \frac{7}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{x^2 + 2x - 3}{x^2}}}{2 + \frac{7}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}}}{2 + \frac{7}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \frac{2}{x} - \frac{3}{x^2}}}{2 + \frac{7}{x}} = \frac{\sqrt{1 + 0 - 0}}{2 + 0} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 2x - 3}}{2x + 7} &= \lim_{x \rightarrow -\infty} \frac{\frac{\sqrt{x^2 + 2x - 3}}{-x}}{\frac{2x}{-x} + \frac{7}{-x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{x^2 + 2x - 3}{x^2}}}{-2 - \frac{7}{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{x^2}{x^2} + \frac{2x}{x^2} - \frac{3}{x^2}}}{-2 - \frac{7}{x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{1 + \frac{2}{x} - \frac{3}{x^2}}}{-2 - \frac{7}{x}} = \frac{\sqrt{1 + 0 - 0}}{-2 - 0} = -\frac{1}{2} \end{aligned}$$

Thus, the horizontal asymptotes are

$$y = \pm \frac{1}{2}$$

54) The horizontal asymptote of

$$f(x) = \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1}$$

Solution:

First, we have to find

$$\lim_{x \rightarrow \pm\infty} \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1}$$

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1} &= \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{2x - 3}}{x^2}}{\frac{2x^2}{x^2} + \frac{7x}{x^2} - \frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{2x - 3}{x^4}}}{2 + \frac{7}{x} - \frac{1}{x^2}} = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{2x}{x^4} - \frac{3}{x^4}}}{2 + \frac{7}{x} - \frac{1}{x^2}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{2}{x^3} - \frac{3}{x^4}}}{2 + \frac{7}{x} - \frac{1}{x^2}} = \frac{\sqrt{0 - 0}}{2 + 0 - 0} = \frac{0}{2} = 0 \end{aligned}$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{2x - 3}}{2x^2 + 7x - 1} &= \lim_{x \rightarrow -\infty} \frac{\frac{\sqrt{2x - 3}}{-x^2}}{\frac{2x^2}{-x^2} + \frac{7x}{-x^2} - \frac{1}{-x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{2x - 3}{x^4}}}{-2 - \frac{7}{x} + \frac{1}{x^2}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{2x}{x^4} - \frac{3}{x^4}}}{-2 - \frac{7}{x} + \frac{1}{x^2}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{2}{x^3} - \frac{3}{x^4}}}{-2 - \frac{7}{x} + \frac{1}{x^2}} = \frac{\sqrt{0 - 0}}{-2 - 0 + 0} = \frac{0}{-2} = 0 \end{aligned}$$

Thus, the horizontal asymptote is

$$y = 0$$

55)

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow -\infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} &= \lim_{x \rightarrow -\infty} \frac{\frac{\sqrt{4x^2 - 8}}{-x} + \frac{3}{-x}}{\frac{x}{-x} + \frac{1}{-x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{4x^2 - 8}{x^2}} - \frac{3}{x}}{-1 - \frac{1}{x}} = \lim_{x \rightarrow -\infty} \frac{\sqrt{\frac{4x^2}{x^2} - \frac{8}{x^2}} - \frac{3}{x}}{-1 - \frac{1}{x}} \\ &= \lim_{x \rightarrow -\infty} \frac{\sqrt{4 - \frac{8}{x^2}} - \frac{3}{x}}{-1 - \frac{1}{x}} = \frac{\sqrt{4 - 0} - 0}{-1 - 0} = -2 \end{aligned}$$

56)

$$\lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} =$$

Solution:

$$\begin{aligned} \lim_{x \rightarrow \infty} \frac{\sqrt{4x^2 - 8} + 3}{x + 1} &= \lim_{x \rightarrow \infty} \frac{\frac{\sqrt{4x^2 - 8}}{x} + \frac{3}{x}}{\frac{x}{x} + \frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{4x^2 - 8}{x^2}} + \frac{3}{x}}{1 + \frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\sqrt{\frac{4x^2}{x^2} - \frac{8}{x^2}} + \frac{3}{x}}{1 + \frac{1}{x}} \\ &= \lim_{x \rightarrow \infty} \frac{\sqrt{4 - \frac{8}{x^2}} + \frac{3}{x}}{1 + \frac{1}{x}} = \frac{\sqrt{4 - 0} + 0}{1 + 0} = 2 \end{aligned}$$