Workshop Solutions to Sections 3.1 and 3.2

1) $$\lim_{x\to -2}(x^3-2x+1) = (-2)^3-2(-2)+1 = -8+4+1 = -3$$

2) $$\lim_{x\to 2}(3x^2+x-4) = 3(2)^2+(2)-4 = 12+2-4 = 10$$

3) $$\lim_{x\to 1}(x^2+3x-5)^3 = ((1)^2+3(1)-5)^3 = (1+3-5)^3 = (-1)^3 = -1$$

4) $$\lim_{x\to -2}(2x^3+3x^2+5) = 2(-2)^3+3(-2)^2+5 = 2(-8)+3(4)+5 = -16+12+5 = 1$$

5) $$\lim_{x\to -2}\frac{x^2-2}{x-2} = \frac{(-2)^2-2}{(-2)-2} = \frac{4-2}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$$

6) $$\lim_{x\to 2}\frac{x^3+5}{x^2+1} = \frac{(2)^3+5}{(2)^2+1} = \frac{8+5}{4+1} = \frac{13}{5}$$

7) $$\lim_{x\to 0}\frac{x^2+3x+5}{x^2-3} = \frac{(0)^2+3(0)+5}{(0)^2-3} = \frac{0+0+5}{0-3} = \frac{5}{-3} = -\frac{5}{3}$$

8) $$\lim_{x\to 1}\frac{x-1}{x^2+x-5} = \frac{(1)-1}{(1)^2+(1)-5} = \frac{1-1}{1+1-5} = \frac{0}{-3} = 0$$

9) $$\lim_{x\to -1}\sqrt{x^3-10x+7} = \sqrt{(-1)^3-10(-1)+7} = \sqrt{-1+10+7} = \sqrt{16} = 4$$

10) $$\lim_{x\to -1}\frac{1-(x+4)^{-2}}{x-2} = \frac{1-\left((-1)+4\right)^{-2}}{(-1)-2} = \frac{1-(-1+4)^{-2}}{-1-2} = \frac{1-(3)^{-2}}{-3} = \frac{1-\frac{1}{3^2}}{-3} = \frac{1-\frac{1}{9}}{-3} = \frac{\frac{8}{9}}{-3} = \frac{8}{9}\times\frac{1}{-3} = \frac{8}{-27} = -\frac{8}{27}$$

11) $$\lim_{x\to -1}\frac{x^3+2x}{8-2x} = \frac{(-1)^3+2(-1)}{8-2(-1)} = \frac{-1-2}{8+2} = \frac{-3}{10} = -\frac{3}{10}$$

12) $$\lim_{x\to 4}\frac{x^2-3x}{5+x} = \frac{(4)^2-3(4)}{5+(4)} = \frac{16-12}{5+4} = \frac{4}{9}$$

13) $$\lim_{x\to 4}\frac{x^2-4x}{5+x} = \frac{(4)^2-4(4)}{5+(4)} = \frac{16-16}{5+4} = \frac{0}{9} = 0$$

15) $$\lim_{x\to 0}\frac{x^3-5x^2}{x^2} = \lim_{x\to 0}\frac{x^2(x-5)}{x^2} = \lim_{x\to 0}(x-5) = (0)-5 = -5$$

14) $$\lim_{x\to 4}\frac{3^{-1}-(2x-5)^{-1}}{4-x} = \lim_{x\to 4}\frac{\frac{1}{3}-\frac{1}{2x-5}}{4-x} = \lim_{x\to 4}\frac{\frac{2x-5-3}{3(2x-5)}}{4-x} = \lim_{x\to 4}\frac{2x-8}{3(2x-5)(4-x)} = \lim_{x\to 4}\frac{2(x-4)}{3(2x-5)(4-x)} = \lim_{x\to 4}\frac{-2(4-x)}{3(2x-5)(4-x)} = \lim_{x\to 4}\frac{-2}{3(2x-5)} = \frac{-2}{3(2(4)-5)} = \frac{-2}{3(8-5)} = \frac{-2}{9} = -\frac{2}{9}$$

16) $$\lim_{x\to 6}\frac{x-6}{x^2-36} = \lim_{x\to 6}\frac{x-6}{(x-6)(x+6)} = \lim_{x\to 6}\frac{1}{x+6} = \frac{1}{(6)+6} = \frac{1}{12}$$

17) $$\lim_{x\to 6}\frac{x^2-36}{x-6} = \lim_{x\to 6}\frac{(x-6)(x+6)}{x-6} = \lim_{x\to 6}(x+6) = (6)+6 = 12$$

18) $$\lim_{x\to -6}\frac{x+6}{x^2-36} = \lim_{x\to -6}\frac{x+6}{(x-6)(x+6)} = \lim_{x\to -6}\frac{1}{x-6} = \frac{1}{(-6)-6} = \frac{1}{-12} = -\frac{1}{12}$$

19) $$\lim_{x\to 3}\frac{x^3-27}{x-3} = \lim_{x\to 3}\frac{(x-3)(x^2+3x+9)}{x-3} = \lim_{x\to 3}(x^2+3x+9) = (3)^2+3(3)+9 = 9+9+9 = 27$$

20) $$\lim_{x\to 3}\frac{x-3}{x^3-27} = \lim_{x\to 3}\frac{x-3}{(x-3)(x^2+3x+9)} = \lim_{x\to 3}\frac{1}{x^2+3x+9} = \frac{1}{(3)^2+3(3)+9} = \frac{1}{9+9+9} = \frac{1}{27}$$

21)
$$\begin{aligned}\lim_{x\to -2}\frac{x+2}{x^3+8} &= \lim_{x\to -2}\frac{x+2}{(x+2)(x^2-2x+4)}\\ &= \lim_{x\to -2}\frac{1}{x^2-2x+4}\\ &= \frac{1}{(-2)^2-2(-2)+4} = \frac{1}{4+4+4} = \frac{1}{12}\end{aligned}$$

22)
$$\begin{aligned}\lim_{x\to -2}\frac{x^3+8}{x+2} &= \lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{x+2}\\ &= \lim_{x\to -2}(x^2-2x+4) = (-2)^2-2(-2)+4\\ &= 4+4+4 = 12\end{aligned}$$

23)
$$\begin{aligned}\lim_{x\to 4}\frac{x^2-3x-4}{x-4} &= \lim_{x\to 4}\frac{(x-4)(x+1)}{x-4} = \lim_{x\to 4}(x+1)\\ &= (4)+1 = 5\end{aligned}$$

24)
$$\begin{aligned}\lim_{x\to 3}\frac{x^2+4x-21}{x^2-8x+15} &= \lim_{x\to 3}\frac{(x+7)(x-3)}{(x-5)(x-3)} = \lim_{x\to 3}\frac{x+7}{x-5}\\ &= \frac{(3)+7}{(3)-5} = \frac{10}{-2} = -5\end{aligned}$$

25)
$$\begin{aligned}\lim_{x\to 0}\frac{x}{1-(1-x)^2} &= \lim_{x\to 0}\frac{x}{1-(1-2x+x^2)}\\ &= \lim_{x\to 0}\frac{x}{1-1+2x-x^2}\\ &= \lim_{x\to 0}\frac{x}{2x-x^2} = \lim_{x\to 0}\frac{x}{x(2-x)}\\ &= \lim_{x\to 0}\frac{1}{2-x} = \frac{1}{2-(0)} = \frac{1}{2}\end{aligned}$$

26)
$$\lim_{x\to 2}\frac{\sqrt[3]{x+6}-2}{x-2} = \lim_{x\to 2}\frac{\sqrt[3]{x+6}-2}{(x+6)-8} = \lim_{x\to 2}\frac{\sqrt[3]{x+6}-2}{\left(\sqrt[3]{x+6}\right)^3-8}$$
$$= \lim_{x\to 2}\frac{\sqrt[3]{x+6}-2}{\left(\sqrt[3]{x+6}-2\right)\left(\left(\sqrt[3]{x+6}\right)^2+2\sqrt[3]{x+6}+4\right)}$$
$$= \lim_{x\to 2}\frac{1}{\left(\sqrt[3]{x+6}\right)^2+2\sqrt[3]{x+6}+4}$$
$$= \frac{1}{\left(\sqrt[3]{(2)+6}\right)^2+2\sqrt[3]{(2)+6}+4} = \frac{1}{4+4+4} = \frac{1}{12}$$

27)
$$\begin{aligned}\lim_{x\to 0}\frac{\sqrt{x+25}-5}{x} &\\ &= \lim_{x\to 0}\left[\frac{\sqrt{x+25}-5}{x}\times\frac{\sqrt{x+25}+5}{\sqrt{x+25}+5}\right]\\ &= \lim_{x\to 0}\frac{(x+25)-25}{x\left(\sqrt{x+25}+5\right)}\\ &= \lim_{x\to 0}\frac{x}{x\left(\sqrt{x+25}+5\right)}\\ &= \lim_{x\to 0}\frac{1}{\sqrt{x+25}+5} = \frac{1}{\sqrt{(0)+25}+5}\\ &= \frac{1}{5+5} = \frac{1}{10}\end{aligned}$$

28)
$$\begin{aligned}\lim_{x\to 0}\frac{x}{\sqrt{x+25}-5} &= \lim_{x\to 0}\left[\frac{x}{\sqrt{x+25}-5}\times\frac{\sqrt{x+25}+5}{\sqrt{x+25}+5}\right]\\ &= \lim_{x\to 0}\frac{x\left(\sqrt{x+25}+5\right)}{(x+25)-25}\\ &= \lim_{x\to 0}\frac{x\left(\sqrt{x+25}+5\right)}{x}\\ &= \lim_{x\to 0}\left(\sqrt{x+25}+5\right) = \sqrt{(0)+25}+5\\ &= 5+5 = 10\end{aligned}$$

29)
$$\begin{aligned}\lim_{x\to 2}\frac{x-2}{2-\sqrt{6-x}} &= \lim_{x\to 2}\left[\frac{x-2}{2-\sqrt{6-x}}\times\frac{2+\sqrt{6-x}}{2+\sqrt{6-x}}\right]\\ &= \lim_{x\to 2}\frac{(x-2)\left(2+\sqrt{6-x}\right)}{4-(6-x)}\\ &= \lim_{x\to 2}\frac{(x-2)\left(2+\sqrt{6-x}\right)}{4-6+x}\\ &= \lim_{x\to 2}\frac{(x-2)\left(2+\sqrt{6-x}\right)}{x-2}\\ &= \lim_{x\to 2}\left(2+\sqrt{6-x}\right) = 2+\sqrt{6-(2)}\\ &= 2+2 = 4\end{aligned}$$

30)
$$\lim_{x\to 2}\frac{2-\sqrt{6-x}}{x+2} = \frac{2-\sqrt{6-(2)}}{(2)+2} = \frac{2-2}{4} = 0$$

31)
$$\begin{aligned}\lim_{x\to 3}\frac{1-\sqrt{x-2}}{2-\sqrt{x+1}} &\\ &= \lim_{x\to 3}\left[\frac{1-\sqrt{x-2}}{2-\sqrt{x+1}}\times\frac{1+\sqrt{x-2}}{1+\sqrt{x-2}}\right.\\ &\quad\left.\times\frac{2+\sqrt{x+1}}{2+\sqrt{x+1}}\right]\\ &= \lim_{x\to 3}\left[\frac{1-(x-2)}{4-(x+1)}\times\frac{2+\sqrt{x+1}}{1+\sqrt{x-2}}\right]\\ &= \lim_{x\to 3}\left[\frac{3-x}{3-x}\times\frac{2+\sqrt{x+1}}{1+\sqrt{x-2}}\right]\\ &= \lim_{x\to 3}\frac{2+\sqrt{x+1}}{1+\sqrt{x-2}} = \frac{2+\sqrt{(3)+1}}{1+\sqrt{(3)-2}} = \frac{2+2}{1+1}\\ &= \frac{4}{2} = 2\end{aligned}$$

32) If $2x \le f(x) \le 3x^2 - 8$, then $\lim_{x\to 2} f(x) =$ Solution: $\lim_{x\to 2} 2x = 2(2) = 4$ and $\lim_{x\to 2}(3x^2 - 8) = 3(2)^2 - 8 = 12 - 8 = 4$ It follows from the Sandwich Theorem that $\lim_{x\to 2} f(x) = 4$	33) $\lim_{x\to 0}\left[x\cos\left(x+\frac{1}{x}\right)\right] =$ We know that the cosine of any angle is between -1 and 1. So, $-1 \le \cos\left(x+\frac{1}{x}\right) \le 1$ Now, multiply throughout by x, we get $-x \le x\cos\left(x+\frac{1}{x}\right) \le x$ But $\lim_{x\to 0} x = 0$ and $\lim_{x\to 0}(-x) = 0$. It follows from the Sandwich Theorem that $\lim_{x\to 0}\left[x\cos\left(x+\frac{1}{x}\right)\right] = 0$
34) $\lim_{x\to 0}\left[x\sin\left(\frac{1}{x}\right)\right] =$ We know that the sine of any angle is between -1 and 1. So, $-1 \le \sin\left(\frac{1}{x}\right) \le 1$ Now, multiply throughout by x, we get $-x \le x\sin\left(\frac{1}{x}\right) \le x$ But $\lim_{x\to 0} x = 0$ and $\lim_{x\to 0}(-x) = 0$. It follows from the Sandwich Theorem that $\lim_{x\to 0}\left[x\sin\left(\frac{1}{x}\right)\right] = 0$	35) If $\frac{x^2+1}{x-1} \le f(x) \le x - 1$, then $\lim_{x\to 0} f(x) =$ Solution: $\lim_{x\to 0}\frac{x^2+1}{x-1} = \frac{(0)^2+1}{(0)-1} = \frac{1}{-1} = -1$ and $\lim_{x\to 0}(x-1) = (0) - 1 = -1$ It follows from the Sandwich Theorem that $\lim_{x\to 0} f(x) = -1$
36) If $4(x-1) \le f(x) \le x^3 + x - 2$, then $\lim_{x\to 1} f(x) =$ Solution: $\lim_{x\to 1}\big(4(x-1)\big) = 4\big((1)-1\big) = 4 \times 0 = 0$ and $\lim_{x\to 1}(x^3 + x - 2) = (1)^3 + (1) - 2 = 1 + 1 - 2 = 0$ It follows from the Sandwich Theorem that $\lim_{x\to 1} f(x) = 0$	37) If $\lim_{x\to 3}\frac{f(x)+4}{x-1} = 3,$ then $\lim_{x\to 3} f(x) =$ Solution: $\lim_{x\to 3}\frac{f(x)+4}{x-1} = \frac{\lim_{x\to 3}(f(x)+4)}{\lim_{x\to 3}(x-1)} = \frac{\lim_{x\to 3} f(x) + \lim_{x\to 3}(4)}{\lim_{x\to 3}(x) - \lim_{x\to 3}(1)}$ $= \frac{\lim_{x\to 3} f(x) + 4}{3-1} = \frac{\lim_{x\to 3} f(x) + 4}{2}$ Now $\frac{\lim_{x\to 3} f(x) + 4}{2} = 3$ $\lim_{x\to 3} f(x) + 4 = 6 \quad \Leftrightarrow \quad \lim_{x\to 3} f(x) = 2$

38) $\lim_{x\to 2}\frac{2^{-1}-(3x-4)^{-1}}{2-x}$

$$= \lim_{x\to 2}\frac{\frac{1}{2}-\frac{1}{3x-4}}{2-x}$$

$$= \lim_{x\to 2}\frac{\frac{3x-4-2}{2(3x-4)}}{2-x}$$

$$= \lim_{x\to 2}\frac{\frac{3x-6}{2(3x-4)}}{2-x}$$

$$= \lim_{x\to 2}\frac{\frac{3(x-2)}{2(3x-4)}}{2-x}$$

$$= \lim_{x\to 2}\frac{3(x-2)}{2(3x-4)(2-x)}$$

$$= \lim_{x\to 2}\frac{-3(2-x)}{2(3x-4)(2-x)} = \lim_{x\to 2}\frac{-3}{2(3x-4)}$$

$$= \frac{-3}{2(3(2)-4)} = \frac{-3}{2\times 2} = -\frac{3}{4}$$

39) $\lim_{x\to 0}\frac{(x+1)^3-1}{x} = \lim_{x\to 0}\frac{(x^3+3x^2+3x+1)-1}{x}$

$$= \lim_{x\to 0}\frac{x^3+3x^2+3x}{x}$$

$$= \lim_{x\to 0}\frac{x(x^2+3x+3)}{x} = \lim_{x\to 0}(x^2+3x+3)$$

$$= (0)^2+3(0)+3 = 3$$

40) If

$$\lim_{x\to 1}\frac{f(x)+3x}{x^2-5f(x)} = 1,$$

then

$$\lim_{x\to 1} f(x) =$$

<u>Solution:</u>

$$\lim_{x\to 1}\frac{f(x)+3x}{x^2-5f(x)} = \frac{\lim_{x\to 1}(f(x)+3x)}{\lim_{x\to 1}(x^2-5f(x))}$$

$$= \frac{\lim_{x\to 1} f(x) + \lim_{x\to 1}(3x)}{\lim_{x\to 1}(x^2) - \lim_{x\to 1}(5f(x))}$$

$$= \frac{\lim_{x\to 1} f(x) + 3(1)}{(1)^2 - 5\lim_{x\to 1} f(x)} = \frac{\lim_{x\to 1} f(x) + 3}{1 - 5\lim_{x\to 1} f(x)}$$

Now

$$\frac{\lim_{x\to 1} f(x) + 3}{1 - 5\lim_{x\to 1} f(x)} = 1$$

$$\lim_{x\to 1} f(x) + 3 = (1)\left(1 - 5\lim_{x\to 1} f(x)\right)$$

$$\Leftrightarrow \quad \lim_{x\to 1} f(x) + 3 = 1 - 5\lim_{x\to 1} f(x)$$

$$\Leftrightarrow \quad \lim_{x\to 1} f(x) + 5\lim_{x\to 1} f(x) = 1 - 3$$

$$\Leftrightarrow \quad 6\lim_{x\to 1} f(x) = -2$$

$$\Leftrightarrow \quad \lim_{x\to 1} f(x) = \frac{-2}{6} = -\frac{1}{3}$$

41) $\lim_{x\to 4}\frac{x^2-6x+8}{x^2+x-20}$

$$= \lim_{x\to 4}\frac{(x-2)(x-4)}{(x-4)(x+5)}$$

$$= \lim_{x\to 4}\frac{x-2}{x+5} = \frac{(4)-2}{(4)+5} = \frac{2}{9}$$

42) $\lim_{x\to -2}\frac{x^3+8}{x^2-x-6}$

$$= \lim_{x\to -2}\frac{(x+2)(x^2-2x+4)}{(x-3)(x+2)}$$

$$= \lim_{x\to -2}\frac{x^2-2x+4}{x-3} = \frac{(-2)^2-2(-2)+4}{(-2)-3}$$

$$= \frac{4+4+4}{-5} = \frac{12}{-5} = -\frac{12}{5}$$

43) $\lim_{x\to 1}\left[\frac{x^2-2}{x+4}+x^2-2x\right] = \frac{(1)^2-2}{(1)+4}+(1)^2-2(1)$

$$= \frac{1-2}{1+4}+1-2 = \frac{-1}{5}-1 = \frac{-1-5}{5} = -\frac{6}{5}$$

44) $\lim\limits_{x\to-2}\dfrac{4x^2+6x-4}{2x^2-8}$

$$
\begin{aligned}
&= \lim_{x\to-2}\frac{2(2x^2+3x-2)}{2(x^2-4)} \\
&= \lim_{x\to-2}\frac{2x^2+3x-2}{x^2-4} \\
&= \lim_{x\to-2}\frac{(2x-1)(x+2)}{(x-2)(x+2)} \\
&= \lim_{x\to-2}\frac{2x-1}{x-2} = \frac{2(-2)-1}{(-2)-2} = \frac{-4-1}{-2-2} \\
&= \frac{-5}{-4} = \frac{5}{4}
\end{aligned}
$$

45) $\lim\limits_{x\to-1}\dfrac{x^2-2x-3}{x^5-x^3}$

$$
\begin{aligned}
&= \lim_{x\to-1}\frac{(x-3)(x+1)}{x^3(x^2-1)} \\
&= \lim_{x\to-1}\frac{(x-3)(x+1)}{x^3(x-1)(x+1)} \\
&= \lim_{x\to-1}\frac{x-3}{x^3(x-1)} = \frac{(-1)-3}{(-1)^3\big((-1)-1\big)} \\
&= \frac{-1-3}{(-1)(-2)} = \frac{-4}{2} = -2
\end{aligned}
$$

46) $\lim\limits_{x\to3}\dfrac{\sqrt{2x+1}(x^2-9)}{(2x+3)(x-3)}$

$$
\begin{aligned}
&= \lim_{x\to3}\frac{\sqrt{2x+1}(x-3)(x+3)}{(2x+3)(x-3)} \\
&= \lim_{x\to3}\frac{\sqrt{2x+1}(x+3)}{2x+3} = \frac{\sqrt{2(3)+1}\big((3)+3\big)}{2(3)+3} \\
&= \frac{6\sqrt{7}}{9} = \frac{2\sqrt{7}}{3}
\end{aligned}
$$

47) $\lim\limits_{x\to1}\dfrac{\sqrt{3-2x}-1}{x-1} = \lim\limits_{x\to1}\left[\dfrac{\sqrt{3-2x}-1}{x-1}\times\dfrac{\sqrt{3-2x}+1}{\sqrt{3-2x}+1}\right]$

$$
\begin{aligned}
&= \lim_{x\to1}\frac{(3-2x)-1}{(x-1)\big(\sqrt{3-2x}+1\big)} \\
&= \lim_{x\to1}\frac{2-2x}{(x-1)\big(\sqrt{3-2x}+1\big)} \\
&= \lim_{x\to1}\frac{2(1-x)}{(x-1)\big(\sqrt{3-2x}+1\big)} = \\
&= \lim_{x\to1}\frac{-2(x-1)}{(x-1)\big(\sqrt{3-2x}+1\big)} = \\
&= \lim_{x\to1}\frac{-2}{\sqrt{3-2x}+1} = \frac{-2}{\sqrt{3-2(1)}+1} \\
&= \frac{-2}{\sqrt{3-2}+1} = \frac{-2}{2} = -1
\end{aligned}
$$

48) $\lim\limits_{x\to0}\dfrac{(x+1)^2-1}{x} = \lim\limits_{x\to0}\dfrac{(x^2+2x+1)-1}{x}$

$$
\begin{aligned}
&= \lim_{x\to0}\frac{x^2+2x}{x} = \lim_{x\to0}\frac{x(x+2)}{x} \\
&= \lim_{x\to0}(x+2) = (0)+2 = 2
\end{aligned}
$$

49) $\lim\limits_{x\to1}\dfrac{\sqrt{2x+2}-2}{\sqrt{3x-2}-1}$

$$
\begin{aligned}
&= \lim_{x\to1}\left[\frac{\sqrt{2x+2}-2}{\sqrt{3x-2}-1}\times\frac{\sqrt{2x+2}+2}{\sqrt{2x+2}+2}\times\frac{\sqrt{3x-2}+1}{\sqrt{3x-2}+1}\right] \\
&= \lim_{x\to1}\left[\frac{(2x+2)-4}{(3x-2)-1}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right] \\
&= \lim_{x\to1}\left[\frac{2x-2}{3x-3}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right] \\
&= \lim_{x\to1}\left[\frac{2(x-1)}{3(x-1)}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right] \\
&= \lim_{x\to1}\left[\frac{2}{3}\times\frac{\sqrt{3x-2}+1}{\sqrt{2x+2}+2}\right] = \frac{2}{3}\times\frac{\sqrt{3(1)-2}+1}{\sqrt{2(1)+2}+2} \\
&= \frac{2}{3}\times\frac{\sqrt{1}+1}{\sqrt{4}+2} = \frac{2}{3}\times\frac{2}{4} = \frac{1}{3}
\end{aligned}
$$

50) $\lim_{x\to 2}\frac{3-\sqrt{2x+5}}{x-2}$

$$= \lim_{x\to 2}\left[\frac{3-\sqrt{2x+5}}{x-2}\times\frac{3+\sqrt{2x+5}}{3+\sqrt{2x+5}}\right]$$
$$= \lim_{x\to 2}\frac{9-(2x+5)}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{4-2x}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{2(2-x)}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{-2(x-2)}{(x-2)\left(3+\sqrt{2x+5}\right)}$$
$$= \lim_{x\to 2}\frac{-2}{3+\sqrt{2x+5}} = \frac{-2}{3+\sqrt{2(2)+5}}$$
$$= \frac{-2}{3+\sqrt{9}} = \frac{-2}{6} = -\frac{1}{3}$$

51) $\lim_{x\to -1}\frac{x^2+3x+2}{x^2+1} = \frac{(-1)^2+3(-1)+2}{(-1)^2+1} = \frac{1-3+2}{1+1}$

$$= \frac{0}{2} = 0$$

52) If

$$\lim_{x\to k} f(x) = -\frac{1}{2}$$

and

$$\lim_{x\to k} g(x) = \frac{2}{3}$$

Then

$$\lim_{x\to k}\frac{f(x)}{g(x)} = \frac{-\frac{1}{2}}{\frac{2}{3}} = -\frac{1}{2}\times\frac{3}{2} = -\frac{3}{4}$$

53) $\lim_{x\to 0}\frac{\sqrt{x+4}-2}{x} = \lim_{x\to 0}\left[\frac{\sqrt{x+4}-2}{x}\times\frac{\sqrt{x+4}+2}{\sqrt{x+4}+2}\right]$

$$= \lim_{x\to 0}\frac{(x+4)-4}{x\left(\sqrt{x+4}+2\right)}$$
$$= \lim_{x\to 0}\frac{x}{x\left(\sqrt{x+4}+2\right)}$$
$$= \lim_{x\to 0}\frac{1}{\sqrt{x+4}+2} = \frac{1}{\sqrt{(0)+4}+2}$$
$$= \frac{1}{\sqrt{4}+2} = \frac{1}{4}$$

54) $\lim_{x\to -1}\frac{x^2-5x-6}{x+1} = \lim_{x\to -1}\frac{(x-6)(x+1)}{x+1} = \lim_{x\to -1}(x-6)$

$$= (-1)-6 = -7$$

55) $\lim_{x\to 0}\frac{(x+3)^{-1}-3^{-1}}{x} = \lim_{x\to 0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x} = \lim_{x\to 0}\frac{\frac{3-(x+3)}{3(x+3)}}{x}$

$$= \lim_{x\to 0}\frac{-x}{3x(x+3)} = \lim_{x\to 0}\frac{-1}{3(x+3)}$$
$$= \frac{-1}{3\left((0)+3\right)} = \frac{-1}{9} = -\frac{1}{9}$$

56) If

$$\lim_{x\to 1} f(x) = 3$$

$$\lim_{x\to 1} g(x) = -4$$

and

$$\lim_{x\to 1} h(x) = -1$$

then

$$\lim_{x\to 1}\left[\frac{5f(x)}{2g(x)}+h(x)\right] = \frac{\lim_{x\to 1} 5f(x)}{\lim_{x\to 1} 2g(x)} + \lim_{x\to 1} h(x)$$
$$= \frac{5\lim_{x\to 1} f(x)}{2\lim_{x\to 1} g(x)} + \lim_{x\to 1} h(x)$$
$$= \frac{5(3)}{2(-4)} + (-1) = \frac{15}{-8} - 1 = -\frac{15}{8} - 1$$
$$= \frac{-15-8}{8} = -\frac{23}{8}$$

57) If

$$\lim_{x\to 1} g(x) = -4$$

and

$$\lim_{x\to 1} h(x) = -1$$

then

$$\lim_{x\to 1}\sqrt{g(x)h(x)} = \sqrt{\left[\lim_{x\to 1} g(x)\right]\left[\lim_{x\to 1} h(x)\right]} = \sqrt{(-4)(-1)}$$
$$= \sqrt{4} = 2$$

58) If

$$\lim_{x\to 1} f(x) = 3$$
$$\lim_{x\to 1} g(x) = -4$$

and

$$\lim_{x\to 1} h(x) = -1$$

then

$$\lim_{x\to 1}[2f(x)g(x)h(x)] = 2\left[\lim_{x\to 1} f(x)\right]\left[\lim_{x\to 1} g(x)\right]\left[\lim_{x\to 1} h(x)\right]$$
$$= 2(3)(-4)(-1) = 24$$