## Workshop Solutions to Sections 2.3 and 2.4

1) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$ , then $(f+g)(x) =$	2) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$ , then $D_{f+g} =$
Solution:	Solution:
$(f+g)(x) = x^2 + \sqrt{4} - x$	$D_f = \mathbb{R}$
	$g(x)$ is defined when $4 - x \ge 0 \iff x \le 4$ . Thus,
	$D_g = (-\infty, 4]$
	$D_{f+g} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
3) If $f(x) = x^2$ and $g(x) = \sqrt{4} - x$ , then $(f - g)(x) =$	4) If $f(x) = x^2$ and $g(x) = \sqrt{4} - x$ , then $D_{f-g} =$
$\frac{\text{Solution.}}{(f-r)(r)} = r^2 + \sqrt{4-r}$	$\underline{Solution}$
$(f - g)(x) = x - \sqrt{4} - x$	$D_f = \mathbb{R}$ $a(x)$ is defined when $A = x > 0 \iff x < A$ . Thus
	$D_{\alpha} = (-\infty, 4]$
	$D_{f-a} = D_f \cap D_a = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
5) If $f(x) = x^2$ and $g(x) = \sqrt{4 - x}$ , then $(fg)(x) = x^2$	6) If $f(x) = x^2$ and $g(x) = \sqrt{4 - x}$ , then $D_{fg} =$
Solution:	Solution:
$(fg)(x) = x^2\sqrt{4-x}$	$D_f = \mathbb{R}$
	$g(x)$ is defined when $4 - x \ge 0 \iff x \le 4$ . Thus,
	$D_g = (-\infty, 4]$
	$D_{fg} = D_f \cap D_g = \mathbb{R} \cap (-\infty, 4] = (-\infty, 4]$
7) If $f(x) = x^2$ and $g(x) = \sqrt{4 - x}$ , then $(f \circ g)(x) = \frac{1}{2}$	8) If $f(x) = x^2$ and $g(x) = \sqrt{4} - x$ , then $D_{f \circ g} =$
Solution: $(f \circ g)(x) = f(g(x))$	$\frac{\text{Solution:}}{(f \circ g)(g)} = f(g(g))$
$(f \circ g)(x) = f(g(x))$	$(f \circ g)(x) = f(g(x))$
$= f(\sqrt{4} - x) = (\sqrt{4} - x) = 4 - x$	$= f(\sqrt{4} - x) = (\sqrt{4} - x) = 4 - x$
	$D_g = (-\infty, 4]$
	$D_{f(g(x))} = \mathbb{K}$
(2) + (1) + (2)	$D_{f \circ g} = D_g \cap D_{f(g(x))} = (-\infty, 4] \cap \mathbb{R} = (-\infty, 4]$
Solution: (g o f)(x) = $x^2$ and $g(x) = \sqrt{4} - x$ , then $(g \circ f)(x) = x^2$	10) If $f(x) = x^2$ and $g(x) = \sqrt{4} - x$ , then $D_{g \circ f} =$ Solution:
$\frac{d}{d(x)} = a(f(x)) = a(x^2) = \sqrt{4 - x^2}$	$\frac{500000000}{(a \circ f)(x)} = a(f(x)) = a(x^2) = \sqrt{4 - x^2}$
	$D_f = \mathbb{R}$
	$D_{a(f(x))} = [-2,2]$
	$D_{g \circ f} = D_f \cap D_{g(f(x))} = \mathbb{R} \cap [-2,2] = [-2,2]$
11) If $f(x) = x^2$ , then $(f \circ f)(x) =$	12) If $f(x) = x^2$ , then $D_{f \circ f} =$
Solution:	Solution:
$(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$	$(f \circ f)(x) = f(f(x)) = f(x^2) = (x^2)^2 = x^4$
	$D_f = \mathbb{R}$
	$D_{f(f(x))} = \mathbb{R}$
(f)	$D_{f \circ f} = D_f \cap D_{f(f(x))} = \mathbb{R} \cap \mathbb{R} = \mathbb{R}$
13) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$ , then $\left(\frac{j}{g}\right)(x) =$	14) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$ , then $D_{\frac{f}{a}} =$
Solution:	Solution:
$\left(\frac{f}{f}\right)(x) = \frac{x^2}{1-x^2}$	$\left(\frac{f}{2}\right)(x) = \frac{x^2}{2}$
$\langle g \rangle = \sqrt{4-x}$	$(g)^{(x)} = \sqrt{4-x}$
	$D_f = \mathbb{K}$
	$g(x)$ is defined when $4 - x \ge 0 \iff x \le 4$ . Thus, $D_a = (-\infty, 4]$
	$D_{\underline{f}} = \{x \in D_f \cap D_g   g(x) \neq 0\}$
	$g = \mathbb{R} \cap (-\infty \ 4) = (-\infty \ 4)$

15) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$ , then $\left(\frac{g}{t}\right)(x) =$	16) If $f(x) = x^2$ and $g(x) = \sqrt{4-x}$ , then $D_{\frac{g}{f}} =$
Solution:	Solution:
$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$	$\left(\frac{g}{f}\right)(x) = \frac{\sqrt{4-x}}{x^2}$
	$D_f = \mathbb{R}$
	$g(x)$ is defined when $4 - x \ge 0 \iff x \le 4$ . Thus, $D_g = (-\infty, 4]$
	$D_{\underline{g}} = \left\{ x \in D_f \cap D_g   f(x) \neq 0 \right\}$
	$f' = \mathbb{R} \setminus \{0\} \cap (-\infty, 4] = (-\infty, 0) \cup (0, 4]$
17) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then	18) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then
(f + g)(x) = Solution:	(j - g)(x) = Solution:
$\overline{(f+g)(x)} = (9-x^2) + (10) = 9-x^2 + 10$ $= 19-x^2$	$\overline{(f-g)(x)} = (9-x^2) - (10) = 9 - x^2 - 10$ $= -x^2 - 1$
19) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then (g - f)(x) =	20) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then $(fg)(x) =$
Solution: $(q - f)(x) = (10) - (9 - x^2) = 10 - 9 + x^2$	Solution: $(f_{\alpha})(x) = (9 - x^2)(10) = 90 - 10x^2$
(g - f)(x) = (10) - (9 - x) = 10 - 9 + x = 1 + x <sup>2</sup>	(fg)(x) = (9 - x)(10) = 90 - 10x
21) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then ( $f \circ g$ )( $x$ ) =	22) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then $(g \circ f)(x) =$
Solution:	Solution:
$(f \circ g)(x) = f(g(x)) = f(10)$ = 9 - 10 <sup>2</sup> = 9 - 100 = -91	$(g \circ f)(x) = g(f(x)) = g(9 - x^2) = 10$
23) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then	24) If $f(x) = 9 - x^2$ and $g(x) = 10$ , then
$(f \circ f)(x) =$	$(g \circ g)(x) =$
$(f \circ f)(x) = f(f(x)) = f(9 - x^2)$	$(g \circ g)(x) = g(g(x)) = g(10) = 10$
$= 9 - (9 - x^2)^2$	
25) If $f(x) = 9 - x^2$ , $g(x) = \sin x$ and $h(x) = 3x + 2$ , then $(f \circ g \circ h)(x) =$	26) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$ , then $(f + g)(x) = x^3$
Solution:	Solution:
$(f \circ g \circ h)(x) = f\left(g(h(x))\right)$	$(f+g)(x) = \sqrt{25 + x^2} + x^3$
= f(g(3x+2))	
$= f(\sin(3x + 2)) = 9 - (\sin(3x + 2))^{2}$	
$=9-\sin^2(3x+2)$	
27) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$ , then	28) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$ , then
Solution:	Solution:
$(f-g)(x) = \sqrt{25 + x^2} - x^3$	$(fg)(x) = x^3\sqrt{25 + x^2}$
29) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$ , then	30) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$ , then
$\left(\frac{f}{g}\right)(x) =$	$(f \circ g)(x) =$ Solution:
Solution: $\sqrt{25 + x^2}$	$\overline{(f \circ g)(x)} = f(g(x)) = f(x^3) = \sqrt{25 + (x^3)^2}$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{23 + x^2}}{x^3}$	$=\sqrt{25+x^{6}}$
31) If $f(x) = \sqrt{25 + x^2}$ and $g(x) = x^3$ , then $(g \circ f)(x) =$	32) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$ , then $(f \circ g)(x) = $ <u>Solution:</u>
$\frac{\text{Solution:}}{(1-c)(1-c)}$	$(f \circ g)(x) = f(g(x)) = f(x-2) = \sqrt{x-2}$
$(g \circ f)(x) = g(f(x)) = g(\sqrt{25 + x^2})$	
$= \left(\sqrt{25 + x^2}\right)$	
$=\sqrt{(25+x^2)^3}$	

33) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$ , then $(g \circ f)(x) =$	34) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$ , then $(g \circ g)(x) =$
$\frac{\text{solution:}}{(g \circ f)(x)} = g(f(x)) = g(\sqrt{x}) = \sqrt{x} - 2$	$\frac{\text{solution:}}{(g \circ g)(x)} = g(g(x)) = g(x-2) = (x-2) - 2$ $= x - 2 - 2 = x - 4$
35) If $f(x) = \sqrt{x}$ and $g(x) = x - 2$ , then $(fg)(x) =$	36) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$ , then $(f \circ a)(x) = -$
$(fg)(x) = (\sqrt{x})(x-2) = (x-2)\sqrt{x}$	$\frac{(f \circ g)(x) =}{Solution:}$ (f \otimes a)(x) = f(a(x)) = f(x^2 + 3) = sin 5(x^2 + 3)
37) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$ , then	38) If $f(x) = \sin 5x$ and $g(x) = x^2 + 3$ , then (f g)(x) =
Solution:	Solution:
$(g \circ f)(x) = g(f(x)) = g(\sin 5x) = (\sin 5x)^2 + 3$ $= \sin^2 5x + 3$	$(fg)(x) = (\sin 5x)(x^2 + 3) = (x^2 + 3)\sin 5x$
39) If $f(x) = \sqrt{x}$ and $g(x) = \cos x$ , then $(g \circ f)(x) =$ Solution:	40) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$ , then
$(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \cos\sqrt{x}$	$(f \circ g)(x) = $ <u>Solution:</u>
	$(f \circ g)(x) = f(g(x)) = f(1 - x^2) = (1 - x^2) + \frac{1}{1 - x^2}$
41) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$ , then	42) If $f(x) = x + \frac{1}{x}$ and $g(x) = 1 - x^2$ , then
$(g \circ f)(x) =$ <u>Solution:</u>	(fg)(x) = <u>Solution:</u>
$(g \circ f)(x) = g(f(x)) = g\left(x + \frac{1}{x}\right) = 1 - \left(x + \frac{1}{x}\right)^2$	$(fg)(x) = \left(x + \frac{1}{x}\right)(1 - x^2)$
43) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units unwards, then the new graph	44) If the graph of the function $f(x) = x^2$ is shifted a distance 2 units downwards, then the new graph
represented the graph of the function is	represented the graph of the function is
Solution: $r^2 + 2$	Solution: $r^2 - 2$
45) If the graph of the function $f(x) = x^2$ is shifted a	46) If the graph of the function $f(x) = x^2$ is shifted a
distance 2 units to the right , then the new graph represented the graph of the function is	distance 2 units to the left , then the new graph represented the graph of the function is
Solution:	Solution:
$(x-2)^2 = x^2 - 4x + 4$	$(x+2)^2 = x^2 + 4x + 4$
47) If the graph of the function $f(x) = \cos x$ is	48) If the graph of the function $f(x) = \cos x$ is
stretched vertically by a factor of 2, then the new graph	compressed vertically by a factor of $\frac{1}{2}$ , then the new graph
Solution:	represented the graph of the function is Solution:
$2\cos x$	$\frac{1}{-\cos x}$
49) If the graph of the function $f(x) = \cos x$ is	$2^{\cos x}$ 50) If the graph of the function $f(x) = \cos x$ is stretched
compressed horizontally by a factor of 2, then the new	horizontally by a factor of $\frac{1}{2}$ , then the new graph
graph represented the graph of the function is Solution:	represented the graph of the function is
$\cos 2x$	Solution:
[1] The graph of the function $f(x) = \sqrt{x}$ is reflected	$\frac{\cos \frac{1}{2}}{2}$
about the $x - axis$ if	about the $y - axis$ if
Solution: $f(u) = \sqrt{u}$	Solution: $f(u) = \sqrt{-u}$
53) If the graph of the function $f(x) = -\sqrt{x}$ is shifted a	54) If the graph of the function $f(x) = e^x$ is shifted a
distance 2 units upwards , then the new graph	distance 2 units downwards , then the new graph
represented the graph of the function is Solution:	represented the graph of the function is Solution:
$e^x + 2$	$e^x - 2$

55) If the graph of the function $f(x) = e^x$ is shifted a	56) If the graph of the function $f(x) = e^x$ is shifted a
distance 2 units to the right , then the new graph	distance 2 units to the left , then the new graph
represented the graph of the function is	represented the graph of the function is
Solution:	Solution:
$e^{-2\pi}$	$e^{-\tau}$
57) $\frac{2\pi}{3}$ rad $=\frac{2\pi}{3} \times \frac{100}{\pi} = 120^{\circ}$	58) $\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{130}{\pi} = 150^{\circ}$
59) $\frac{7\pi}{6}$ rad $=\frac{7\pi}{6} \times \frac{180^{\circ}}{\pi} = 210^{\circ}$	60) $\frac{3\pi}{2}$ rad $=\frac{3\pi}{2} \times \frac{180^{\circ}}{\pi} = 270^{\circ}$
61) $120^{\circ} = 120 \times \frac{\pi}{180^{\circ}} = \frac{2\pi}{3}$ rad	62) $270^{\circ} = 270 \times \frac{\pi}{180^{\circ}} = \frac{3\pi}{2}$ rad
63) $\frac{5\pi}{12}$ rad $=\frac{5\pi}{12} \times \frac{180^{\circ}}{\pi} = 75^{\circ}$	64) $\frac{3\pi}{4}$ rad $=\frac{3\pi}{4} \times \frac{180^{\circ}}{\pi} = 135^{\circ}$
65) $150^{\circ} = 150 \times \frac{\pi}{180^{\circ}} = \frac{5\pi}{6}$ rad	66) $210^{\circ} = 210 \times \frac{\pi}{180^{\circ}} = \frac{7\pi}{6}$ rad
67) $\frac{1}{\sec x} = \cos x$	68) $\frac{1}{\csc x} = \sin x$
69) $\frac{1}{\cot x} = \tan x$	70) $\frac{\sin x}{\cos x} = \tan x$
71) $\frac{\cos x}{\sin x} = \cot x$	
72) If $\cos x = \frac{3}{\pi}$ and $0 < x < \frac{\pi}{2}$ , then $\cot x = 4$	73) If $\cos x = \frac{3}{\pi}$ and $0 < x < \frac{\pi}{2}$ , then $\tan x = \sqrt{2}$
Solution:	Solution:
$\cos r = \frac{3}{2} = \frac{adj}{2}$	$\cos r = \frac{3}{2} = \frac{adj}{2}$
$2 \cos x = 5 = hyp$	203 x = 5 = hyp
3	3
Now, we should find the length of the opposite side using	Now, we should find the length of the opposite side using
the Pythagorean Theorem, so	the Pythagorean Theorem, so
$ opposite  = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$	$ opposite  = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
$\therefore$ cot $r = 1 - adj - 3$	$\therefore \tan x = \frac{1}{2} = \frac{opp}{4} = \frac{4}{2}$
$\frac{1}{\tan x} = \frac{1}{\cos p} = \frac{1}{4}$	$\frac{1}{\cos x} - \frac{1}{\cos x} - \frac{1}$
74) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$ , then $\sin x = \sqrt{1 + \frac{\pi}{2}}$	75) If $\cos x = \frac{3}{5}$ and $0 < x < \frac{\pi}{2}$ , then $\csc x =$
Solution: 5	Solution: 5
$\cos x = \frac{3}{4} = \frac{adj}{4}$	$\cos x = \frac{3}{4} = \frac{adj}{4}$
5 hyp	5 hyp
Now, we should find the length of the opposite side using	Now, we should find the length of the opposite side using
the Pythagorean Theorem, so	the Pythagorean Theorem, so
$ opposite  = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$	$ opposite  = \sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4$
$\sin x - \frac{opp}{4} - \frac{4}{4}$	$\frac{1}{1} - \frac{hyp}{5} = \frac{5}{5}$
$\frac{1}{hyp} = \frac{1}{5}$	$\frac{1}{10000000000000000000000000000000000$
76) $\sin(\frac{5\pi}{6}) =$	77) $\cos(\frac{5\pi}{6}) =$
Solution:	Solution:
$\frac{5\pi}{5}$ rad $=\frac{5\pi}{5} \times \frac{180^{\circ}}{5} = 150^{\circ}$	$\frac{5\pi}{5}$ rad $=\frac{5\pi}{5} \times \frac{180^{\circ}}{5} = 150^{\circ}$
$6$ $6$ $\pi$ So we deduce now that $\sin\left(\frac{5\pi}{2}\right)$ is in the second quarter	$6$ $6$ $\pi$ So we deduce now that $\cos\left(\frac{5\pi}{2}\right)$ is in the second quarter
$\sin\left(\frac{5\pi}{2}\right) = \sin(150^\circ) = \sin(180^\circ - 30^\circ) = \sin(30^\circ)$	$\cos\left(\frac{5\pi}{6}\right) = \cos(150^\circ) = \cos(180^\circ - 30^\circ)$
$(6)$ $(\pi)$ 1	(6)  (100)
$= \sin\left(\frac{\pi}{6}\right) = \frac{\pi}{2}$	$= -\cos(30^{\circ}) = -\cos(\frac{\pi}{6}) = -\frac{\sqrt{3}}{2}$
78) $\tan\left(\frac{3\pi}{6}\right) =$	79) $\cot\left(\frac{3\pi}{6}\right) =$
Solution:	Solution:
$\frac{5\pi}{6}$ rad $=\frac{5\pi}{6} \times \frac{180}{\pi} = 150^{\circ}$ (5 $\pi$ )	$\int \frac{5\pi}{6} \operatorname{rad} = \frac{5\pi}{6} \times \frac{180}{\pi} = 150^{\circ} $ (5 $\pi$ )
So we deduce now that $\tan\left(\frac{3n}{2}\right)$ is in the second	So we deduce now that $\cot\left(\frac{3\pi}{2}\right)$ is in the second quarter

