

## Workshop Solutions to Sections 5.1 and 5.2

<p>1) The absolute maximum value of <math>f(x) = x^3 - 2x^2</math> in <math>[-1, 2]</math> is at <math>x =</math></p> <p><u>Solution:</u>          Since <math>f(x)</math> is a continuous on <math>[-1, 2]</math>, we can use the Closed Interval Method,</p> $f(x) = x^3 - 2x^2$ $f'(x) = 3x^2 - 4x$ <p>Now, we find the critical numbers of <math>f(x)</math> when</p> $f'(x) = 0 \Rightarrow 3x^2 - 4x = 0 \Rightarrow x(3x - 4) = 0$ $\Rightarrow x = 0 \text{ or } x = \frac{4}{3}$ <p>Thus,</p> $f(-1) = (-1)^3 - 2(-1)^2 = -1 - 2 = -3$ $f(2) = (2)^3 - 2(2)^2 = 8 - 8 = 0$ $f(0) = (0)^3 - 2(0)^2 = 0 - 0 = 0$ $f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 = \frac{64}{27} - \frac{32}{9} = -\frac{32}{27}$ <p>Hence, we see that the absolute maximum value is 0 at <math>x = 0</math> and <math>x = 2</math></p>	<p>2) The absolute minimum value of <math>f(x) = x^3 - 3x^2 + 1</math> in <math>\left[-\frac{1}{2}, 4\right]</math> is</p> <p><u>Solution:</u>          Since <math>f(x)</math> is a continuous on <math>\left[-\frac{1}{2}, 4\right]</math>, we can use the Closed Interval Method,</p> $f(x) = x^3 - 3x^2 + 1$ $f'(x) = 3x^2 - 6x$ <p>Now, we find the critical numbers of <math>f(x)</math> when</p> $f'(x) = 0 \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x - 2) = 0$ $\Rightarrow x = 0 \text{ or } x = 2$ <p>Thus,</p> $f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 1 = -\frac{1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$ $f(4) = (4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$ $f(0) = (0)^3 - 3(0)^2 + 1 = 0 - 0 + 1 = 1$ $f(2) = (2)^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$ <p>Hence, we see that the absolute minimum value is -3 at <math>x = 2</math></p>
<p>3) The absolute maximum point of <math>f(x) = 3x^2 - 12x + 1</math> in <math>[0, 3]</math> is</p> <p><u>Solution:</u>          Since <math>f(x)</math> is a continuous on <math>[0, 3]</math>, we can use the Closed Interval Method,</p> $f(x) = 3x^2 - 12x + 1$ $f'(x) = 6x - 12$ <p>Now, we find the critical numbers of <math>f(x)</math> when</p> $f'(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow 6x = 12$ $\Rightarrow x = 2$ <p>Thus,</p> $f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$ $f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$ $f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$ <p>Hence, we see that the absolute maximum point is <math>(0, 1)</math>.</p>	<p>4) The absolute minimum point of <math>f(x) = 3x^2 - 12x + 1</math> in <math>[0, 3]</math> is</p> <p><u>Solution:</u>          Since <math>f(x)</math> is a continuous on <math>[0, 3]</math>, we can use the Closed Interval Method,</p> $f(x) = 3x^2 - 12x + 1$ $f'(x) = 6x - 12$ <p>Now, we find the critical numbers of <math>f(x)</math> when</p> $f'(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow 6x = 12$ $\Rightarrow x = 2$ <p>Thus,</p> $f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$ $f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$ $f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$ <p>Hence, we see that the absolute minimum point is <math>(2, -11)</math>.</p>
<p>5) The absolute minimum point of <math>f(x) = 3x^2 - 12x + 2</math> in <math>[0, 3]</math> is</p> <p><u>Solution:</u>          Since <math>f(x)</math> is a continuous on <math>[0, 3]</math>, we can use the Closed Interval Method,</p> $f(x) = 3x^2 - 12x + 2$ $f'(x) = 6x - 12$ <p>Now, we find the critical numbers of <math>f(x)</math> when</p> $f'(x) = 0 \Rightarrow 6x - 12 = 0 \Rightarrow 6x = 12$ $\Rightarrow x = 2$ <p>Thus,</p> $f(0) = 3(0)^2 - 12(0) + 2 = 0 - 0 + 2 = 2$ $f(3) = 3(3)^2 - 12(3) + 2 = 27 - 36 + 2 = -7$ $f(2) = 3(2)^2 - 12(2) + 2 = 12 - 24 + 2 = -10$ <p>Hence, we see that the absolute minimum point is <math>(2, -10)</math>.</p>	<p>6) The values in <math>(-3, 3)</math> which make <math>f(x) = x^3 - 9x</math> satisfy Rolle's Theorem on <math>[-3, 3]</math> are</p> <p><u>Solution:</u>  <math>\because f(x)</math> is a polynomial, then</p> <ol style="list-style-type: none"> <li>1- <math>f(x)</math> is a continuous on <math>[-3, 3]</math>.</li> <li>2- <math>f(x)</math> is differentiable on <math>(-3, 3)</math>,  <math>f'(x) = 3x^2 - 9</math></li> <li>3- <math>f(-3) = (-3)^3 - 9(-3) = -27 + 27 = 0 = f(3)</math></li> </ol> <p>Then there is a number <math>c \in (-3, 3)</math> such that</p> $f'(c) = 0 \Rightarrow 3c^2 - 9 = 0 \Rightarrow 3c^2 = 9$ $\Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$ <p>Hence, the values are <math>\pm\sqrt{3} \in (-3, 3)</math>.</p>

7) The values in  $(0,2)$  which make  $f(x) = x^3 - 3x^2 + 2x + 5$  satisfy Rolle's Theorem on  $[0,2]$  are

Solution:

$\because f(x)$  is a polynomial, then

1-  $f(x)$  is a continuous on  $[0,2]$ .

2-  $f(x)$  is differentiable on  $(0,2)$ ,

$$f'(x) = 3x^2 - 6x + 2$$

$$3- f(0) = (0)^3 - 3(0)^2 + 2(0) + 5 = 5 = f(2)$$

Then there is a number  $c \in (0,2)$  such that

$$f'(c) = 0 \Rightarrow 3c^2 - 6c + 2 = 0$$

$$\begin{aligned} \Rightarrow c &= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)} = \frac{6 \pm \sqrt{36 - 24}}{6} \\ &= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm \sqrt{3 \times 4}}{6} = \frac{6 \pm 2\sqrt{3}}{6} \\ &= \frac{2(3 \pm \sqrt{3})}{6} = \frac{3 \pm \sqrt{3}}{3} = \frac{3}{3} \pm \frac{\sqrt{3}}{3} \\ &= 1 \pm \frac{\sqrt{3}}{3} \end{aligned}$$

Hence, the values are  $1 \pm \frac{\sqrt{3}}{3} \in (0,2)$ .

8) The value  $c$  in  $(0,5)$  which makes  $f(x) = x^2 - x - 6$  satisfy the Mean Value Theorem on  $[0,5]$  is

Solution:

$\because f(x)$  is a polynomial, then

1-  $f(x)$  is a continuous on  $[0,5]$ .

2-  $f(x)$  is differentiable on  $(0,5)$ ,

$$f'(x) = 2x - 1$$

Then there is a number  $c \in (0,5)$  such that

$$\begin{aligned} f'(c) &= \frac{f(5) - f(0)}{5 - 0} \\ \Rightarrow 2c - 1 &= \frac{[(5)^2 - (5) - 6] - [(0)^2 - (0) - 6]}{5} \\ \Rightarrow 2c - 1 &= \frac{(14) - (-6)}{5} \\ \Rightarrow 2c - 1 &= \frac{14 + 6}{5} \\ \Rightarrow 2c - 1 &= 4 \\ \Rightarrow 2c &= 4 + 1 \\ \Rightarrow c &= \frac{5}{2} \end{aligned}$$

Hence, the value  $c$  is  $\frac{5}{2} \in (0,5)$ .

9) The value  $c$  in  $(0,2)$  makes  $f(x) = x^3 - x$  satisfied the Mean Value Theorem on  $[0,2]$  are

Solution:

$\because f(x)$  is a polynomial, then

1-  $f(x)$  is a continuous on  $[0,2]$ .

2-  $f(x)$  is differentiable on  $(0,2)$ ,

$$f'(x) = 3x^2 - 1$$

Then there is a number  $c \in (0,2)$  such that

$$\begin{aligned} f'(c) &= \frac{f(2) - f(0)}{2 - 0} \\ \Rightarrow 3c^2 - 1 &= \frac{[(2)^3 - (2)] - [(0)^3 - (0)]}{2} \\ \Rightarrow 3c^2 - 1 &= \frac{(6) - (0)}{2} \\ \Rightarrow 3c^2 - 1 &= \frac{6}{2} \\ \Rightarrow 3c^2 - 1 &= 3 \\ \Rightarrow 3c^2 &= 3 + 1 \\ \Rightarrow c^2 &= \frac{4}{3} \\ \Rightarrow c &= \pm \sqrt{\frac{4}{3}} \\ \Rightarrow c &= \pm \frac{2}{\sqrt{3}} \end{aligned}$$

Hence, the value  $c$  is  $\frac{2}{\sqrt{3}} \in (0,2)$  but  $-\frac{2}{\sqrt{3}} \notin (0,2)$ .

10) The value in  $(0,1)$  which makes  $f(x) = 3x^2 + 2x + 5$  satisfy the Mean Value Theorem on  $[0,1]$  is

Solution:

$\because f(x)$  is a polynomial, then

1-  $f(x)$  is a continuous on  $[0,1]$ .

2-  $f(x)$  is differentiable on  $(0,1)$ ,

$$f'(x) = 6x + 2$$

Then there is a number  $c \in (0,1)$  such that

$$\begin{aligned} f'(c) &= \frac{f(1) - f(0)}{1 - 0} \\ \Rightarrow 6c + 2 &= \frac{[3(1)^2 + 2(1) + 5] - [3(0)^2 + 2(0) + 5]}{1} \\ \Rightarrow 6c + 2 &= (3 + 2 + 5) - (0 + 0 + 5) \\ \Rightarrow 6c + 2 &= 10 - 5 \\ \Rightarrow 6c + 2 &= 5 \\ \Rightarrow 6c &= 5 - 2 \\ \Rightarrow 6c &= 3 \\ \Rightarrow c &= \frac{3}{6} \\ \Rightarrow c &= \frac{1}{2} \end{aligned}$$

Hence, the values are  $\frac{1}{2} \in (0,1)$ .

11) The critical numbers of the function

$$f(x) = x^3 + 3x^2 - 9x + 1$$

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$\Rightarrow$

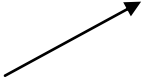
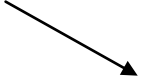

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

12) The function  $f(x) = x^3 + 3x^2 - 9x + 1$  is decreasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \end{aligned}$$


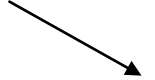

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-3, 1)$

13) The function  $f(x) = x^3 + 3x^2 - 9x + 1$  is increasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \end{aligned}$$

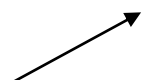
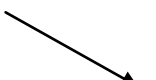

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -3) \cup (1, \infty)$

14) The function  $f(x) = x^3 + 3x^2 - 9x + 1$  has a relative maximum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity


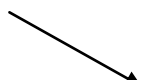

Hence, the function  $f(x)$  has a relative maximum value at the point  $(-3, 28)$ .

$$\begin{aligned} f(-3) &= (-3)^3 + 3(-3)^2 - 9(-3) + 1 \\ &= -27 + 27 + 27 + 1 = 28 \end{aligned}$$

15) The function  $f(x) = x^3 + 3x^2 - 9x + 1$  has a relative minimum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity



Hence, the function  $f(x)$  has a relative minimum value at the point  $(1, -4)$ .

$$\begin{aligned} f(1) &= (1)^3 + 3(1)^2 - 9(1) + 1 \\ &= 1 + 3 - 9 + 1 = -4 \end{aligned}$$

16) The function  $f(x) = x^3 + 3x^2 - 9x + 1$  concave upward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f''(x) &= 6x + 6 \\ f''(x) = 0 &\Rightarrow 6x + 6 = 0 \\ &\Rightarrow 6x = -6 \\ &\Rightarrow x = -1 \end{aligned}$$



-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(-1, \infty)$

17) The function  $f(x) = x^3 + 3x^2 - 9x + 1$  concave downward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f''(x) &= 6x + 6 \\ f''(x) = 0 &\Rightarrow 6x + 6 = 0 \\ &\Rightarrow 6x = -6 \\ &\Rightarrow x = -1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, -1)$

18) The function  $f(x) = x^3 + 3x^2 - 9x + 1$  has an inflection point at

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f''(x) = 6x + 6$$

$$f''(x) = 0 \Rightarrow 6x + 6 = 0$$

$$\Rightarrow 6x = -6$$

$$\Rightarrow x = -\frac{6}{6}$$

$$\Rightarrow x = -1$$

$$-1$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at  $(-1, 12)$ .

$$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 1$$

$$= -1 + 3 + 9 + 1 = 12$$

20) The function  $f(x) = x^3 - 3x^2 - 9x + 1$  is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

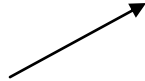
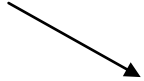

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-1, 3)$

22) The function  $f(x) = x^3 - 3x^2 - 9x + 1$  has a relative maximum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

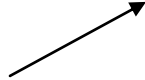
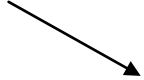

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative maximum value at the point  $(-1, 6)$ .

$$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1$$

$$= -1 - 3 + 9 + 1 = 6.$$

19) The critical numbers of the function

$$f(x) = x^3 - 3x^2 - 9x + 1 \text{ are}$$

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

21) The function  $f(x) = x^3 - 3x^2 - 9x + 1$  is increasing on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$


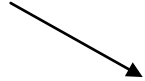

$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -1) \cup (3, \infty)$

23) The function  $f(x) = x^3 - 3x^2 - 9x + 1$  has a relative minimum value at the point

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x - 9 = 0$$

$$\Rightarrow 3(x^2 - 2x - 3) = 0$$


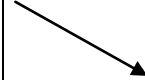
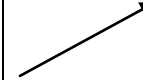
$$\Rightarrow x^2 - 2x - 3 = 0$$

$$\Rightarrow (x + 1)(x - 3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 3$$

$$-1$$

$$3$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative minimum value at the point  $(3, -26)$ .

$$f(3) = (3)^3 - 3(3)^2 - 9(3) + 1$$

$$= 27 - 27 - 27 + 1 = -26.$$

24) The function  $f(x) = x^3 - 3x^2 - 9x + 1$  concave upward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(1, \infty)$

25) The function  $f(x) = x^3 - 3x^2 - 9x + 1$  concave downward on

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, 1)$

26) The function  $f(x) = x^3 - 3x^2 - 9x + 1$  has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at  $(1, -10)$ .

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 1$$

$$= 1 - 3 - 9 + 1 = -10$$

27) The critical numbers of the function  $f(x) = x^3 + 3x^2 - 9x + 5$  are

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

28) The function  $f(x) = x^3 + 3x^2 - 9x + 5$  is decreasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

$$-3 \quad 1$$

+	-	+	Sign of $f'(x)$
$\nearrow$	$\searrow$	$\nearrow$	Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-3, 1)$ .

29) The function  $f(x) = x^3 + 3x^2 - 9x + 5$  is increasing on

Solution:

$$f'(x) = 3x^2 + 6x - 9$$

$$f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0$$

$$\Rightarrow 3(x^2 + 2x - 3) = 0$$

$$\Rightarrow x^2 + 2x - 3 = 0$$

$$\Rightarrow (x + 3)(x - 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = 1$$

$$-3 \quad 1$$

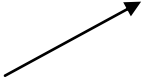
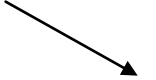

+	-	+	Sign of $f'(x)$
$\nearrow$	$\searrow$	$\nearrow$	Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -3) \cup (1, \infty)$ .

30) The function  $f(x) = x^3 + 3x^2 - 9x + 5$  has a relative minimum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity



Hence, the function  $f(x)$  has a relative minimum value at the point  $(1,0)$ .

$$\begin{aligned} f(1) &= (1)^3 + 3(1)^2 - 9(1) + 5 \\ &= 1 + 3 - 9 + 5 = 0 \end{aligned}$$

32) The function  $f(x) = x^3 + 3x^2 - 9x + 5$  has an inflection point at

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f''(x) &= 6x + 6 \\ f''(x) = 0 &\Rightarrow 6x + 6 = 0 \\ &\Rightarrow 6x = -6 \\ &\Rightarrow x = -\frac{6}{6} \\ &\Rightarrow x = -1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity



Hence, the function  $f(x)$  has an inflection point at  $(-1,16)$ .

$$\begin{aligned} f(-1) &= (-1)^3 + 3(-1)^2 - 9(-1) + 5 \\ &= -1 + 3 + 9 + 5 = 16 \end{aligned}$$

34) The function  $f(x) = x^3 + 3x^2 - 9x + 5$  concave upward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f''(x) &= 6x + 6 \\ f''(x) = 0 &\Rightarrow 6x + 6 = 0 \\ &\Rightarrow 6x = -6 \\ &\Rightarrow x = -\frac{6}{6} \\ &\Rightarrow x = -1 \end{aligned}$$

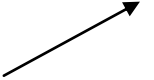
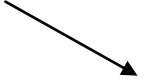

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(-1, \infty)$ .

31) The function  $f(x) = x^3 + 3x^2 - 9x + 5$  has a relative maximum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 + 6x - 9 = 0 \\ &\Rightarrow 3(x^2 + 2x - 3) = 0 \\ &\Rightarrow x^2 + 2x - 3 = 0 \\ &\Rightarrow (x + 3)(x - 1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity



Hence, the function  $f(x)$  has a relative maximum value at the point  $(-3,32)$ .

$$\begin{aligned} f(-3) &= (-3)^3 + 3(-3)^2 - 9(-3) + 5 \\ &= -27 + 27 + 27 + 5 = 32 \end{aligned}$$

33) The function  $f(x) = x^3 + 3x^2 - 9x + 5$  concave downward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 + 6x - 9 \\ f''(x) &= 6x + 6 \\ f''(x) = 0 &\Rightarrow 6x + 6 = 0 \\ &\Rightarrow 6x = -6 \\ &\Rightarrow x = -\frac{6}{6} \\ &\Rightarrow x = -1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, -1)$ .

35) The critical numbers of the function  $f(x) = x^3 - 3x^2 - 9x + 5$  are


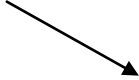

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x - 9 = 0 \\ &\Rightarrow 3(x^2 - 2x - 3) = 0 \\ &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x + 1)(x - 3) = 0 \\ &\Rightarrow x = -1 \text{ or } x = 3 \end{aligned}$$

36) The function  $f(x) = x^3 - 3x^2 - 9x + 5$  is increasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x - 9 = 0 \\ &\Rightarrow 3(x^2 - 2x - 3) = 0 \\ &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x+1)(x-3) = 0 \\ &\Rightarrow x = -1 \text{ or } x = 3 \end{aligned}$$

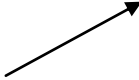
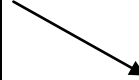
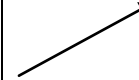
+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -1) \cup (3, \infty)$ .

37) The function  $f(x) = x^3 - 3x^2 - 9x + 5$  is decreasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x - 9 = 0 \\ &\Rightarrow 3(x^2 - 2x - 3) = 0 \\ &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x+1)(x-3) = 0 \\ &\Rightarrow x = -1 \text{ or } x = 3 \end{aligned}$$

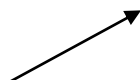
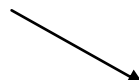

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-1, 3)$ .

38) The function  $f(x) = x^3 - 3x^2 - 9x + 5$  has a relative maximum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x - 9 = 0 \\ &\Rightarrow 3(x^2 - 2x - 3) = 0 \\ &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x+1)(x-3) = 0 \\ &\Rightarrow x = -1 \text{ or } x = 3 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity


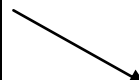

Hence, the function  $f(x)$  has a relative maximum value at the point  $(-1, 10)$ .

$$\begin{aligned} f(-1) &= (-1)^3 - 3(-1)^2 - 9(-1) + 5 \\ &= -1 - 3 + 9 + 5 = 10. \end{aligned}$$

39) The function  $f(x) = x^3 - 3x^2 - 9x + 5$  has a relative minimum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x - 9 = 0 \\ &\Rightarrow 3(x^2 - 2x - 3) = 0 \\ &\Rightarrow x^2 - 2x - 3 = 0 \\ &\Rightarrow (x+1)(x-3) = 0 \\ &\Rightarrow x = -1 \text{ or } x = 3 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity



Hence, the function  $f(x)$  has a relative minimum value at the point  $(3, -22)$ .

$$\begin{aligned} f(3) &= (3)^3 - 3(3)^2 - 9(3) + 5 \\ &= 27 - 27 - 27 + 5 = -22. \end{aligned}$$

40) The function  $f(x) = x^3 - 3x^2 - 9x + 5$  concave upward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$



-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(1, \infty)$ .

41) The function  $f(x) = x^3 - 3x^2 - 9x + 5$  concave downward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, 1)$ .

42) The function  $f(x) = x^3 - 3x^2 - 9x + 5$  has an inflection point at

Solution:

$$f'(x) = 3x^2 - 6x - 9$$

$$f''(x) = 6x - 6$$

$$f''(x) = 0 \Rightarrow 6x - 6 = 0$$

$$\Rightarrow 6x = 6$$

$$\Rightarrow x = \frac{6}{6}$$

$$\Rightarrow x = 1$$

1

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at  $(1, -6)$ .

$$f(1) = (1)^3 - 3(1)^2 - 9(1) + 5$$

$$= 1 - 3 - 9 + 5 = -6$$

44) The function  $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$  is increasing on

Solution:


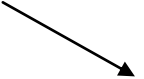
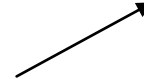
$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -1) \cup (2, \infty)$ .

46) The function  $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$  has a relative maximum point

Solution:

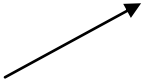
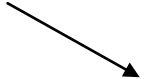

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative maximum point at  $(-1, \frac{13}{6})$ .

$$f(-1) = \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1) + 1$$

$$= -\frac{1}{3} - \frac{1}{2} + 2 + 1 = \frac{13}{6}$$

43) The critical numbers of the function

$$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1 \text{ are}$$

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

45) The function  $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$  is decreasing on

Solution:


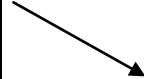
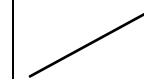
$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-1, 2)$ .

47) The function  $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$  has a relative minimum point

Solution:


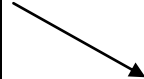

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f'(x) = 0 \Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow (x+1)(x-2) = 0$$

$$\Rightarrow x = -1 \text{ or } x = 2$$

$$\begin{matrix} -1 & & 2 \end{matrix}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative minimum point at  $(2, -\frac{7}{3})$ .

$$f(2) = \frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 - 2(2) + 1$$

$$= \frac{8}{3} - \frac{4}{2} - 4 + 1 = -\frac{7}{3}$$



48) The function  $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$  concave upward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f''(x) = 2x - 1$$

$$f''(x) = 0 \Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$\frac{1}{2}$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $\left(\frac{1}{2}, \infty\right)$ .

49) The function  $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$  concave downward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f''(x) = 2x - 1$$

$$f''(x) = 0 \Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$\frac{1}{2}$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $\left(-\infty, \frac{1}{2}\right)$ .

50) The function  $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$  has an inflection point at

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$$

$$f''(x) = 2x - 1$$

$$f''(x) = 0 \Rightarrow 2x - 1 = 0$$

$$\Rightarrow 2x = 1$$

$$\Rightarrow x = \frac{1}{2}$$

$$\frac{1}{2}$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at

$$\left(\frac{1}{2}, -\frac{1}{12}\right).$$

$$f\left(\frac{1}{2}\right) = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$$

$$= \frac{1}{24} - \frac{1}{8} - 1 + 1 = -\frac{1}{12}$$

51) The critical numbers of the function

$$f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1 \text{ are}$$

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f'(x) = 0 \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

52) The function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$  is increasing on

Solution:

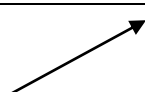
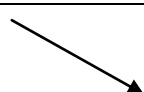

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f'(x) = 0 \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$-2 \quad 1$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -2) \cup (1, \infty)$ .

53) The function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$  is decreasing on

Solution:

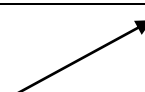
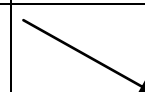

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f'(x) = 0 \Rightarrow x^2 + x - 2 = 0$$

$$\Rightarrow (x+2)(x-1) = 0$$

$$\Rightarrow x = -2 \text{ or } x = 1$$

$$-2 \quad 1$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity


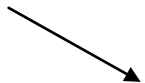
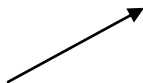
Hence, the function  $f(x)$  is decreasing on  $(-2, 1)$ .

54) The function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$   
has a relative maximum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$\begin{aligned} f'(x) = 0 &\Rightarrow x^2 + x - 2 = 0 \\ &\Rightarrow (x+2)(x-1) = 0 \\ &\Rightarrow x = -2 \quad \text{or} \quad x = 1 \\ &\quad -2 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative maximum point at  $\left(-2, \frac{13}{3}\right)$ .



$$\begin{aligned} f(-2) &= \frac{1}{3}(-2)^3 + \frac{1}{2}(-2)^2 - 2(-2) + 1 \\ &= -\frac{8}{3} + \frac{4}{2} + 4 + 1 = \frac{13}{3} \end{aligned}$$

56) The function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$  concave upward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$\begin{aligned} f''(x) &= 2x + 1 \\ f''(x) = 0 &\Rightarrow 2x + 1 = 0 \\ &\Rightarrow 2x = -1 \\ &\Rightarrow x = -\frac{1}{2} \\ &\quad -\frac{1}{2} \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity


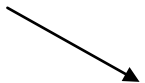

Hence, the function  $f(x)$  is concave upward on  $\left(-\frac{1}{2}, \infty\right)$ .

55) The function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$   
has a relative minimum point

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$\begin{aligned} f'(x) = 0 &\Rightarrow x^2 + x - 2 = 0 \\ &\Rightarrow (x+2)(x-1) = 0 \\ &\Rightarrow x = -2 \quad \text{or} \quad x = 1 \\ &\quad -2 \qquad \qquad 1 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative minimum point at  $\left(1, -\frac{1}{6}\right)$ .



$$\begin{aligned} f(1) &= \frac{1}{3}(1)^3 + \frac{1}{2}(1)^2 - 2(1) + 1 \\ &= \frac{1}{3} + \frac{1}{2} - 2 + 1 = -\frac{1}{6} \end{aligned}$$

57) The function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$  concave downward on

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$\begin{aligned} f''(x) &= 2x + 1 \\ f''(x) = 0 &\Rightarrow 2x + 1 = 0 \\ &\Rightarrow 2x = -1 \\ &\Rightarrow x = -\frac{1}{2} \\ &\quad -\frac{1}{2} \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $\left(-\infty, -\frac{1}{2}\right)$ .

58) The function  $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$  has an inflection point at

Solution:

$$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$$

$$f''(x) = 2x + 1$$

$$f''(x) = 0 \Rightarrow 2x + 1 = 0$$

$$\Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$$

$-$	$+$	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at

$$\left(-\frac{1}{2}, \frac{25}{12}\right).$$

$$f\left(-\frac{1}{2}\right) = \frac{1}{3}\left(-\frac{1}{2}\right)^3 + \frac{1}{2}\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1$$

$$= -\frac{1}{24} + \frac{1}{8} + 1 + 1 = \frac{25}{12}$$

60) The function  $f(x) = x^3 - 12x + 3$  is increasing on

Solution:

$$f'(x) = 3x^2 - 12$$


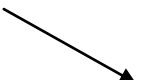
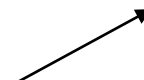
$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$+$	$-$	$+$	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -2) \cup (2, \infty)$ .

62) The function  $f(x) = x^3 - 12x + 3$  has a relative maximum point at

Solution:

$$f'(x) = 3x^2 - 12$$


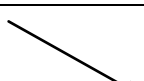

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$+$	$-$	$+$	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative maximum point at  $(-2, 19)$ .

$$f(-2) = (-2)^3 - 12(-2) + 3$$

$$= -8 + 24 + 3 = 19.$$

59) The critical numbers of the function  $f(x) = x^3 - 12x + 3$  are

Solution:

$$f'(x) = 3x^2 - 12$$

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

61) The function  $f(x) = x^3 - 12x + 3$  is decreasing on

Solution:

$$f'(x) = 3x^2 - 12$$

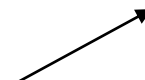
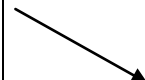

$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$+$	$-$	$+$	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-2, 2)$ .

63) The function  $f(x) = x^3 - 12x + 3$  has a relative minimum point at

Solution:

$$f'(x) = 3x^2 - 12$$

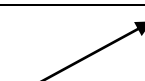
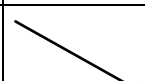
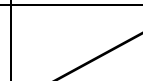
$$f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$$

$$\Rightarrow 3(x^2 - 4) = 0$$

$$\Rightarrow x^2 - 4 = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$+$	$-$	$+$	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative minimum point at  $(2, -13)$ .

$$f(2) = (2)^3 - 12(2) + 3$$

$$= 8 - 24 + 3 = -13$$

64) The function  $f(x) = x^3 - 12x + 3$  concave upward on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$$f''(x) = 0 \Rightarrow 6x = 0$$

$$\Rightarrow x = \frac{0}{6}$$

$$\Rightarrow x = 0$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(0, \infty)$ .

65) The function  $f(x) = x^3 - 12x + 3$  concave downward on

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$$f''(x) = 0 \Rightarrow 6x = 0$$

$$\Rightarrow x = \frac{0}{6}$$

$$\Rightarrow x = 0$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, 0)$ .

66) The function  $f(x) = x^3 - 12x + 3$  has an inflection point at

Solution:

$$f'(x) = 3x^2 - 12$$

$$f''(x) = 6x$$

$$f''(x) = 0 \Rightarrow 6x = 0$$

$$\Rightarrow x = \frac{0}{6}$$

$$\Rightarrow x = 0$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at  $(0, 3)$ .

$$f(0) = (0)^3 - 12(0)^2 + 3$$

$$= 0 - 0 + 3 = 3$$

67) The critical numbers of the function  $f(x) = x^3 - 3x^2 + 1$  are

Solution:

$$f'(x) = 3x^2 - 6x$$

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

68) The function  $f(x) = x^3 - 3x^2 + 1$  is increasing on

Solution:

$$f'(x) = 3x^2 - 6x$$

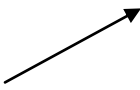
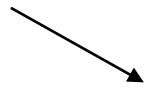

$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, 0) \cup (2, \infty)$ .

69) The function  $f(x) = x^3 - 3x^2 + 1$  is decreasing on

Solution:

$$f'(x) = 3x^2 - 6x$$


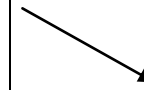
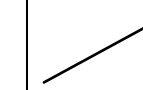
$$f'(x) = 0 \Rightarrow 3x^2 - 6x = 0$$

$$\Rightarrow 3(x^2 - 2x) = 0$$

$$\Rightarrow x^2 - 2x = 0$$

$$\Rightarrow x(x - 2) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$


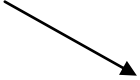
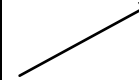
+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(0, 2)$ .

70) The function  $f(x) = x^3 - 3x^2 + 1$  has a relative maximum point at

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x = 0 \\ &\Rightarrow 3(x^2 - 2x) = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \text{ or } x = 2 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity



Hence, the function  $f(x)$  has a relative maximum point at  $(0, 1)$ .

$$\begin{aligned} f(0) &= (0)^3 - 3(0)^2 + 1 \\ &= 0 - 0 + 1 = 1. \end{aligned}$$

72) The function  $f(x) = x^3 - 3x^2 + 1$  concave upward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$



-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(1, \infty)$ .

74) The function  $f(x) = x^3 - 3x^2 + 1$  has an inflection point at

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

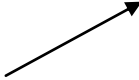
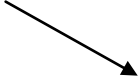
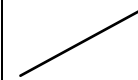
Hence, the function  $f(x)$  has an inflection point at  $(1, -1)$ .

$$\begin{aligned} f(1) &= (1)^3 - 3(1)^2 + 1 \\ &= 1 - 3 + 1 = -1 \end{aligned}$$

71) The function  $f(x) = x^3 - 3x^2 + 1$  has a relative minimum point at

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x = 0 \\ &\Rightarrow 3(x^2 - 2x) = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \text{ or } x = 2 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity



Hence, the function  $f(x)$  has a relative minimum point at  $(2, -3)$ .

$$\begin{aligned} f(2) &= (2)^3 - 3(2)^2 + 1 \\ &= 8 - 12 + 1 = -3. \end{aligned}$$

73) The function  $f(x) = x^3 - 3x^2 + 1$  concave downward on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, 1)$ .


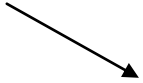

75) The critical numbers of the function  $f(x) = x^3 - 3x^2 + 2$  are

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x = 0 \\ &\Rightarrow 3(x^2 - 2x) = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \text{ or } x = 2 \end{aligned}$$

76) The function  $f(x) = x^3 - 3x^2 + 2$  is increasing on  
Solution:


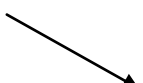
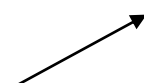
$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x = 0 \\ &\Rightarrow 3(x^2 - 2x) = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \quad \text{or} \quad x = 2 \\ &\quad \quad \quad 0 \qquad \qquad \quad 2 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, 0) \cup (2, \infty)$ .

78) The function  $f(x) = x^3 - 3x^2 + 2$  has a relative minimum point at  
Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x = 0 \\ &\Rightarrow 3(x^2 - 2x) = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \quad \text{or} \quad x = 2 \\ &\quad \quad \quad 0 \qquad \qquad \quad 2 \end{aligned}$$



+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative minimum point at  $(2, -2)$ .

$$\begin{aligned} f(2) &= (2)^3 - 3(2)^2 + 2 \\ &= 8 - 12 + 2 = -2. \end{aligned}$$

80) The function  $f(x) = x^3 - 3x^2 + 2$  concave downward on  
Solution:


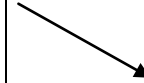

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, 1)$ .

77) The function  $f(x) = x^3 - 3x^2 + 2$  is decreasing on  
Solution:

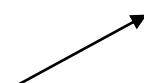
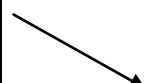

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x = 0 \\ &\Rightarrow 3(x^2 - 2x) = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \quad \text{or} \quad x = 2 \\ &\quad \quad \quad 0 \qquad \qquad \quad 2 \end{aligned}$$

+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(0, 2)$ .

79) The function  $f(x) = x^3 - 3x^2 + 2$  has a relative maximum point at  
Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f'(x) = 0 &\Rightarrow 3x^2 - 6x = 0 \\ &\Rightarrow 3(x^2 - 2x) = 0 \\ &\Rightarrow x^2 - 2x = 0 \\ &\Rightarrow x(x - 2) = 0 \\ &\Rightarrow x = 0 \quad \text{or} \quad x = 2 \\ &\quad \quad \quad 0 \qquad \qquad \quad 2 \end{aligned}$$



+	-	+	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  has a relative maximum point at  $(0, 2)$ .

$$\begin{aligned} f(0) &= (0)^3 - 3(0)^2 + 2 \\ &= 0 - 0 + 2 = 2. \end{aligned}$$

81) The function  $f(x) = x^3 - 3x^2 + 2$  concave upward on  
Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$

-	+	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(1, \infty)$ .

82) The function  $f(x) = x^3 - 3x^2 + 2$  has an inflection point at

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 6x \\ f''(x) &= 6x - 6 \\ f''(x) = 0 &\Rightarrow 6x - 6 = 0 \\ &\Rightarrow 6x = 6 \\ &\Rightarrow x = \frac{6}{6} \\ &\Rightarrow x = 1 \end{aligned}$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at  $(1,0)$ .

$$\begin{aligned} f(1) &= (1)^3 - 3(1)^2 + 2 \\ &= 1 - 3 + 2 = 0 \end{aligned}$$

83) The critical numbers of the function  $f(x) = x^3 - 6x^2 - 36x$  are

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 12x - 36 \\ f'(x) = 0 &\Rightarrow 3x^2 - 12x - 36 = 0 \\ &\Rightarrow 3(x^2 - 4x - 12) = 0 \\ &\Rightarrow x^2 - 4x - 12 = 0 \\ &\Rightarrow (x + 2)(x - 6) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 6 \end{aligned}$$

84) The function  $f(x) = x^3 - 6x^2 - 36x$  is decreasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 12x - 36 \\ f'(x) = 0 &\Rightarrow 3x^2 - 12x - 36 = 0 \\ &\Rightarrow 3(x^2 - 4x - 12) = 0 \\ &\Rightarrow x^2 - 4x - 12 = 0 \\ &\Rightarrow (x + 2)(x - 6) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 6 \end{aligned}$$

+	-	+	Sign of $f'(x)$
$\nearrow$	$\searrow$	$\nearrow$	Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-2,6)$ .

85) The function  $f(x) = x^3 - 6x^2 - 36x$  is increasing on

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 12x - 36 \\ f'(x) = 0 &\Rightarrow 3x^2 - 12x - 36 = 0 \\ &\Rightarrow 3(x^2 - 4x - 12) = 0 \\ &\Rightarrow x^2 - 4x - 12 = 0 \\ &\Rightarrow (x + 2)(x - 6) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 6 \end{aligned}$$

+	-	+	Sign of $f'(x)$
$\nearrow$	$\searrow$	$\nearrow$	Kind of monotonicity

Hence, the function  $f(x)$  is increasing on  $(-\infty, -2) \cup (6, \infty)$ .

86) The function  $f(x) = x^3 - 6x^2 - 36x$  has a relative minimum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 12x - 36 \\ f'(x) = 0 &\Rightarrow 3x^2 - 12x - 36 = 0 \\ &\Rightarrow 3(x^2 - 4x - 12) = 0 \\ &\Rightarrow x^2 - 4x - 12 = 0 \\ &\Rightarrow (x + 2)(x - 6) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 6 \end{aligned}$$

+	-	+	Sign of $f'(x)$
$\nearrow$	$\searrow$	$\nearrow$	Kind of monotonicity

Hence, the function  $f(x)$  has a relative minimum value at the point  $(6, -216)$ .

$$\begin{aligned} f(6) &= (6)^3 - 6(6)^2 - 36(6) \\ &= 216 - 216 - 216 = -216 \end{aligned}$$

87) The function  $f(x) = x^3 - 6x^2 - 36x$  has a relative maximum value at the point

Solution:

$$\begin{aligned} f'(x) &= 3x^2 - 12x - 36 \\ f'(x) = 0 &\Rightarrow 3x^2 - 12x - 36 = 0 \\ &\Rightarrow 3(x^2 - 4x - 12) = 0 \\ &\Rightarrow x^2 - 4x - 12 = 0 \\ &\Rightarrow (x + 2)(x - 6) = 0 \\ &\Rightarrow x = -2 \text{ or } x = 6 \end{aligned}$$

+	-	+	Sign of $f'(x)$
$\nearrow$	$\searrow$	$\nearrow$	Kind of monotonicity

Hence, the function  $f(x)$  has a relative maximum value at the point  $(-2, 40)$ .

$$\begin{aligned} f(-2) &= (-2)^3 - 6(-2)^2 - 36(-2) \\ &= -8 - 24 + 72 = 40 \end{aligned}$$

88) The function  $f(x) = x^3 - 6x^2 - 36x$  has an inflection point at

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$$f''(x) = 0 \Rightarrow 6x - 12 = 0$$

$$\Rightarrow 6x = 12$$

$$\Rightarrow x = \frac{12}{6}$$

$$\Rightarrow x = 2$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  has an inflection point at  $(2, -88)$ .

$$f(2) = (2)^3 - 6(2)^2 - 36(2)$$

$$= 8 - 24 - 72 = -88$$

89) The function  $f(x) = x^3 - 6x^2 - 36x$  concave downward on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$$f''(x) = 0 \Rightarrow 6x - 12 = 0$$

$$\Rightarrow 6x = 12$$

$$\Rightarrow x = \frac{12}{6}$$

$$\Rightarrow x = 2$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-\infty, 2)$ .

90) The function  $f(x) = x^3 - 6x^2 - 36x$  concave upward on

Solution:

$$f'(x) = 3x^2 - 12x - 36$$

$$f''(x) = 6x - 12$$

$$f''(x) = 0 \Rightarrow 6x - 12 = 0$$

$$\Rightarrow 6x = 12$$

$$\Rightarrow x = \frac{12}{6}$$

$$\Rightarrow x = 2$$

-	+	Sign of $f''(x)$
$\cap$	$\cup$	Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(2, \infty)$ .

91) The critical numbers of the function  $f(x) = -x^3 - 6x^2 - 9x + 1$  are

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

92) The function  $f(x) = -x^3 - 6x^2 - 9x + 1$  is decreasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

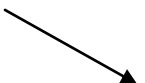

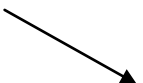
$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

-	+	-	Sign of $f'(x)$
			Kind of monotonicity

Hence, the function  $f(x)$  is decreasing on  $(-\infty, -3) \cup (-1, \infty)$ .

93) The function  $f(x) = -x^3 - 6x^2 - 9x + 1$  is increasing on

Solution:

$$f'(x) = -3x^2 - 12x - 9$$

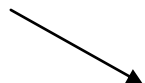
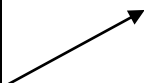
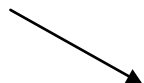
$$f'(x) = 0 \Rightarrow -3x^2 - 12x - 9 = 0$$

$$\Rightarrow -3(x^2 + 4x + 3) = 0$$

$$\Rightarrow x^2 + 4x + 3 = 0$$

$$\Rightarrow (x + 3)(x + 1) = 0$$

$$\Rightarrow x = -3 \text{ or } x = -1$$

-	+	-	Sign of $f'(x)$
			Kind of monotonicity

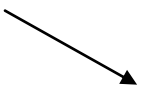
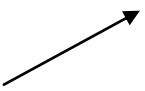
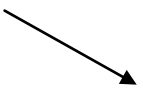
Hence, the function  $f(x)$  is increasing on  $(-3, -1)$ .



94) The function  $f(x) = -x^3 - 6x^2 - 9x + 1$  has a relative minimum value at the point

Solution:

$$\begin{aligned} f'(x) &= -3x^2 - 12x - 9 \\ f'(x) = 0 &\Rightarrow -3x^2 - 12x - 9 = 0 \\ &\Rightarrow -3(x^2 + 4x + 3) = 0 \\ &\Rightarrow x^2 + 4x + 3 = 0 \\ &\Rightarrow (x+3)(x+1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = -1 \\ &\quad -3 \qquad \qquad -1 \end{aligned}$$

-	+	-	Sign of $f'(x)$
			Kind of monotonicity

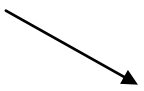
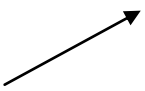
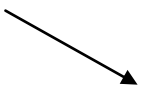
Hence, the function  $f(x)$  has a relative minimum value at the point  $(-3, 1)$ .

$$\begin{aligned} f(-3) &= -(-3)^3 - 6(-3)^2 - 9(-3) + 1 \\ &= 27 - 54 + 27 + 1 = 1. \end{aligned}$$

95) The function  $f(x) = -x^3 - 6x^2 - 9x + 1$  has a relative maximum value at the point

Solution:

$$\begin{aligned} f'(x) &= -3x^2 - 12x - 9 \\ f'(x) = 0 &\Rightarrow -3x^2 - 12x - 9 = 0 \\ &\Rightarrow -3(x^2 + 4x + 3) = 0 \\ &\Rightarrow x^2 + 4x + 3 = 0 \\ &\Rightarrow (x+3)(x+1) = 0 \\ &\Rightarrow x = -3 \text{ or } x = -1 \\ &\quad -3 \qquad \qquad -1 \end{aligned}$$

-	+	-	Sign of $f'(x)$
			Kind of monotonicity



Hence, the function  $f(x)$  has a relative maximum value at the point  $(-1, 5)$ .

$$\begin{aligned} f(-1) &= -(-1)^3 - 6(-1)^2 - 9(-1) + 1 \\ &= 1 - 6 + 9 + 1 = 5. \end{aligned}$$

96) The function  $f(x) = -x^3 - 6x^2 - 9x + 1$  has an inflection point at

Solution:

$$\begin{aligned} f'(x) &= -3x^2 - 12x - 9 \\ f''(x) &= -6x - 12 \\ f''(x) = 0 &\Rightarrow -6x - 12 = 0 \\ &\Rightarrow -6x = 12 \\ &\Rightarrow x = -\frac{12}{6} \\ &\Rightarrow x = -2 \\ &\quad -2 \end{aligned}$$

+	-	Sign of $f''(x)$
		Kind of concavity



Hence, the function  $f(x)$  has an inflection point at  $(-2, 3)$ .

$$\begin{aligned} f(-2) &= -(-2)^3 - 6(-2)^2 - 9(-2) + 1 \\ &= 8 - 24 + 18 + 1 = 3 \end{aligned}$$

97) The function  $f(x) = -x^3 - 6x^2 - 9x + 1$  concave downward on

Solution:

$$\begin{aligned} f'(x) &= -3x^2 - 12x - 9 \\ f''(x) &= -6x - 12 \\ f''(x) = 0 &\Rightarrow -6x - 12 = 0 \\ &\Rightarrow -6x = 12 \\ &\Rightarrow x = -\frac{12}{6} \\ &\Rightarrow x = -2 \\ &\quad -2 \end{aligned}$$



+	-	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave downward on  $(-2, \infty)$ .

98) The function  $f(x) = -x^3 - 6x^2 - 9x + 1$  concave upward on

Solution:

$$\begin{aligned} f'(x) &= -3x^2 - 12x - 9 \\ f''(x) &= -6x - 12 \\ f''(x) = 0 &\Rightarrow -6x - 12 = 0 \\ &\Rightarrow -6x = 12 \\ &\Rightarrow x = -\frac{12}{6} \\ &\Rightarrow x = -2 \\ &\quad -2 \end{aligned}$$

+	-	Sign of $f''(x)$
		Kind of concavity

Hence, the function  $f(x)$  is concave upward on  $(-\infty, -2)$ .