## Workshop Solutions to Sections 5.1 and 5.2

1) The absolute maximum value of $f(x) = x^3 - 2x^2$ in	2) The absolute minimum value of $f(x) = x^3 - 3x^2 + 1$ in
[-1,2] is at $x =$	$\left -\frac{1}{2},4\right $ is
Solution:	Solution:
Since $f(x)$ is a continuous on $[-1,2]$ , we can use the Closed	
Interval Method,	Since $f(x)$ is a continuous on $\left[-\frac{1}{2}, 4\right]$ , we can use the
$f(x) = x^3 - 2x^2$	Closed Interval Method,
$f'(x) = 3x^2 - 4x$	$f(x) = x^3 - 3x^2 + 1$
Now, we find the critical numbers of $f(x)$ when	$f'(x) = 3x^2 - 6x$
$f'(x) = 0 \implies 3x^2 - 4x = 0 \implies x(3x - 4) = 0$	Now, we find the critical numbers of $f(x)$ when
4	$f'(x) = 0 \implies 3x^2 - 6x = 0 \implies 3x(x - 2) = 0$
$\Rightarrow x = 0 \text{ or } x = \frac{4}{3}$	$\Rightarrow$ $x = 0$ or $x = 2$
Thus,	Thus,
$f(-1) = (-1)^3 - 2(-1)^2 = -1 - 2 = -3$	$(1)$ $(1)^{3}$ $(1)^{2}$ 1 3 1
$f(2) = (2)^3 - 2(2)^2 = 8 - 8 = 0$	$f\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 3\left(-\frac{1}{2}\right)^2 + 1 = -\frac{1}{8} - \frac{3}{4} + 1 = \frac{1}{8}$
$f(0) = (0)^3 - 2(0)^2 = 0 - 0 = 0$	$f(4) = (4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$
	$f(0) = (0)^3 - 3(0)^2 + 1 = 0 - 0 + 1 = 1$
$f\left(\frac{4}{3}\right) = \left(\frac{4}{3}\right)^3 - 2\left(\frac{4}{3}\right)^2 = \frac{64}{27} - \frac{32}{9} = -\frac{32}{27}$	$f(2) = (2)^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$
Hence, we see that the absolute maximum value is $0$ at	Hence, we see that the absolute minimum value is $-3$ at
x = 0 and $x = 2$	x = 2
3) The absolute maximum point of $f(x) = 3x^2 - 12x + 1$	4) The absolute minimum point of $f(x) = 3x^2 - 12x + 1$
in [0,3] is	
	in [0,3] is
Solution:	Solution:
Since $f(x)$ is a continuous on [0,3], we can use the Closed	Since $f(x)$ is a continuous on [0,3], we can use the Closed
Interval Method,	Interval Method,
$f(x) = 3x^2 - 12x + 1$	$f(x) = 3x^2 - 12x + 1$
f'(x) = 6x - 12	f'(x) = 6x - 12
Now, we find the critical numbers of $f(x)$ when	Now, we find the critical numbers of $f(x)$ when
$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$	$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$
$\Rightarrow x = 2$	$\Rightarrow x = 2$
Thus,	Thus,
$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$	$f(0) = 3(0)^2 - 12(0) + 1 = 0 - 0 + 1 = 1$
$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$	$f(3) = 3(3)^2 - 12(3) + 1 = 27 - 36 + 1 = -8$
$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$	$f(2) = 3(2)^2 - 12(2) + 1 = 12 - 24 + 1 = -11$
Hence, we see that the absolute maximum point is $(0,1)$ .	Hence, we see that the absolute minimum point is $(2, -11)$ .
5) The absolute minimum point of $f(x) = 3x^2 - 12x + 2$	6) The values in (-3,3) which make $f(x) = x^3 - 9x$
in [0,3] is	satisfy Rolle's Theorem on $[-3,3]$ are
Solution:	Solution:
Since $f(x)$ is a continuous on [0,3], we can use the Closed	$\therefore$ $f(x)$ is a polynomial, then
Interval Method,	1- $f(x)$ is a continuous on $[-3,3]$ .
$f(x) = 3x^2 - 12x + 2$	2- $f(x)$ is differentiable on $(-3,3)$ ,
f'(x) = 6x - 12	$f'(x) = 3x^2 - 9$
Now, we find the critical numbers of $f(x)$ when	3- $f(-3) = (-3)^3 - 9(-3) = -27 + 27 = 0 = f(3)$
$f'(x) = 0 \implies 6x - 12 = 0 \implies 6x = 12$	Then there is a number $c \in (-3,3)$ such that
$\Rightarrow x = 2$	$f'(c) = 0 \implies 3c^2 - 9 = 0 \implies 3c^2 = 9$
Thus,	$\Rightarrow c^2 = 3 \Rightarrow c = \pm\sqrt{3}$
$f(0) = 3(0)^2 - 12(0) + 2 = 0 - 0 + 2 = 2$	
$f(3) = 3(3)^2 - 12(3) + 2 = 27 - 36 + 2 = -7$	Hence the values are $\sqrt{2} \in (-2,2)$
$f(2) = 3(2)^2 - 12(2) + 2 = 12 - 24 + 2 = -10$	Hence, the values are $\pm \sqrt{3} \in (-3,3)$ .
Hence, we see that the absolute minimum point is $(2, -10)$ .	

8) The value *c* in (0,5) which makes  $f(x) = x^2 - x - 6$ 7) The values in (0,2) which make  $f(x) = x^3 - 3x^2 + 2x + 5$  satisfy Rolle's Theorem on satisfy the Mean Value Theorem on [0,5] is [0,2] are Solution: Solution:  $\therefore$  f(x) is a polynomial, then  $\therefore$  f(x) is a polynomial, then 1- f(x) is a continuous on [0,5]. 2- f(x) is differentiable on (0,5), 1- f(x) is a continuous on [0,2]. 2- f(x) is differentiable on (0,2), f'(x) = 2x - 1 $f'(x) = 3x^2 - 6x + 2$ 3-  $f(0) = (0)^3 - 3(0)^2 + 2(0) + 5 = 5 = f(2)$ Then there is a number  $c \in (0,5)$  such that  $f'(c) = \frac{f(5) - f(0)}{5 - 0}$  $\begin{array}{l} \Rightarrow & 2c - 1 = \frac{\left[(5)^2 - (5) - 6\right] - \left[(0)^2 - (0) - 6\right]}{5} \\ \Rightarrow & 2c - 1 = \frac{(14) - (-6)}{5} \\ \Rightarrow & 2c - 1 = \frac{14 + 6}{5} \\ \Rightarrow & 2c - 1 = 4 \\ \Rightarrow & 2c = 4 + 1 \\ \Rightarrow & c = \frac{5}{2} \end{array}$ Then there is a number  $c \in (0,2)$  such that  $f'(c) = 0 \implies 3c^2 - 6c + 2 = 0$  $c = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(3)(2)}}{2(3)} = \frac{6 \pm \sqrt{36 - 24}}{6}$  $= \frac{6 \pm \sqrt{12}}{6} = \frac{6 \pm \sqrt{3 \times 4}}{6} = \frac{6 \pm 2\sqrt{3}}{6}$  $= \frac{2(3 \pm \sqrt{3})}{6} = \frac{3 \pm \sqrt{3}}{3} = \frac{3}{3} \pm \frac{\sqrt{3}}{3}$  $=1\pm\frac{\sqrt{3}}{3}$ Hence, the values are  $1 \pm \frac{\sqrt{3}}{3} \in (0,2)$ . Hence, the value *c* is  $\frac{5}{2} \in (0,5)$ . 9) The value c in (0,2) makes  $f(x) = x^3 - x$  satisfied the 10) The value in (0,1) which makes  $f(x) = 3x^2 + 2x + 5$ Mean Value Theorem on [0,2] are satisfy the Mean Value Theorem on [0,1] is Solution: Solution:  $\therefore$  f(x) is a polynomial, then  $\therefore$  f(x) is a polynomial, then 1- f(x) is a continuous on [0,2]. 1- f(x) is a continuous on [0,1]. 2- f(x) is differentiable on (0,2), 2- f(x) is differentiable on (0,1),  $f'(x) = 3x^2 - 1$ f'(x) = 6x + 2Then there is a number  $c \in (0,3)$  such that Then there is a number  $c \in (0,1)$  such that  $f'(c) = \frac{f(1) - f(0)}{1 - 0}$   $\Rightarrow 6c + 2 = \frac{[3(1)^2 + 2(1) + 5] - [3(0)^2 + 2(0) + 5]}{1}$  $f'(c) = \frac{f(2) - f(0)}{2 - 0}$  $\Rightarrow 3c^{2} - 1 = \frac{[(2)^{3} - (2)] - [(0)^{3} - (0)]}{2}$  $\Rightarrow 3c^{2} - 1 = \frac{(6) - (0)}{6}$  $\Rightarrow 6c + 2 = (3 + 2 + 5) - (0 + 0 + 5)$  $\Rightarrow 6c + 2 = 10 - 5$  $\Rightarrow 3c^2 - 1 = \frac{6}{2}$  $\Rightarrow 3c^2 - 1 = 3$  $\Rightarrow 3c^2 = 3 + 1$  $\Rightarrow 6c + 2 = 5$  $\Rightarrow 6c = 5 - 2$  $\Rightarrow 6c = 3$  $\Rightarrow c = \frac{3}{6}$  $\Rightarrow c^2 = \frac{4}{2}$  $\Rightarrow c = \frac{1}{2}$  $\Rightarrow c = \pm \sqrt{\frac{4}{3}}$ Hence, the values are  $\frac{1}{2} \in (0,1)$ .  $\Rightarrow c = \pm \frac{2}{\sqrt{3}}$ Hence, the value c is  $\frac{2}{\sqrt{3}} \in (0,2)$  but  $-\frac{2}{\sqrt{3}} \notin (0,2)$ . 11) The critical numbers of the function  $f(x) = x^3 + 3x^2 - 9x + 1$  are  $\Rightarrow (x+3)(x-1) = 0$  $\Rightarrow x = -3 \text{ or } x = 1$ Solution:  $f'(x) = 3x^2 + 6x - 9$  $f'(x) = 0 \quad \Longrightarrow \quad 3x^2 + 6x - 9 = 0$  $\Rightarrow 3(x^2 + 2x - 3) = 0$  $\Rightarrow x^2 + 2x - 3 = 0$ 

12) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is decreasing	13) The function $f(x) = x^3 + 3x^2 - 9x + 1$ is increasing
on <u>Solution:</u>	on <u>Solution:</u>
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$
$\Rightarrow 3(x^2 + 2x - 3) = 0$	$\Rightarrow 3(x^2 + 2x - 3) = 0$
$\implies x^2 + 2x - 3 = 0$	$\implies x^2 + 2x - 3 = 0$
$\Rightarrow (x+3)(x-1) = 0$ $\Rightarrow x = -3 \text{ or } x = 1$	$\Rightarrow (x+3)(x-1) = 0$ $\Rightarrow x = -3 \text{ or } x = 1$
$ \begin{array}{c}  & x = -3 & \text{of}  x = 1 \\ -3 & 1 \end{array} $	$ \xrightarrow{\longrightarrow} x = -3  \text{or}  x = 1 \\ -3 \qquad 1 $
+ $ +$ Sign of $f'(x)$	+ $ +$ Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ is decreasing on $(-3,1)$	Hence, the function $f(x)$ is increasing on
	$(-\infty, -3) \cup (1, \infty)$
14) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative	15) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has a relative minimum value at the point
maximum value at the point	minimum value at the point
Solution: $f'(x) = 3x^2 + 6x - 9$	Solution: $f'(x) = 3x^2 + 6x - 9$
$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$
$\Rightarrow  3(x^2 + 2x - 3) = 0$	$\Rightarrow  3(x^2 + 2x - 3) = 0$
$\implies x^2 + 2x - 3 = 0$	$\implies x^2 + 2x - 3 = 0$
$\Rightarrow (x+3)(x-1) = 0$	$\implies (x+3)(x-1) = 0$
$\Rightarrow x = -3 \text{ or } x = 1$ -3 1	$\Rightarrow x = -3 \text{ or } x = 1$
$\begin{vmatrix} -3 & 1 \\ + & - & + & \text{Sign of } f'(x) \end{vmatrix}$	$\begin{vmatrix} -3 & 1 \\ + & - & + \\ \end{vmatrix}$ Sign of $f'(x)$
Kind of	$\mathbf{X}$ Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum value at	Hence, the function $f(x)$ has a relative minimum value at
the point $(-3,28)$ .	the point $(1, -4)$ .
$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 1$	$f(1) = (1)^3 + 3(1)^2 - 9(1) + 1$
= -27 + 27 + 27 + 1 = 28 16) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave	$= 1 + 3 - 9 + 1 = -4$ 17) The function $f(x) = x^3 + 3x^2 - 9x + 1$ concave
upward on	downward on
Solution:	Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$f^{\prime\prime}(x) = 6x + 6$	$f^{\prime\prime}(x) = 6x + 6$
$f''(x) = 0 \implies 6x + 6 = 0$	$f''(x) = 0 \implies 6x + 6 = 0$
$\Rightarrow 6x = -6$	$\Rightarrow 6x = -6$
$\implies x = -\frac{6}{6}$	$\Rightarrow x = -\frac{6}{6}$
$\Rightarrow x = -1$	$\Rightarrow x = -1$
-1	-1
- + Sign of $f''(x)$	- + Sign of $f''(x)$
U Kind of concavity	<b>O</b> U Kind of concavity
Hence, the function $f(x)$ is concave upward on $(-1, \infty)$	Hence, the function $f(x)$ is concave downward on
$\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$	$(-\infty, -1)$
	, -/
	3

18) The function $f(x) = x^3 + 3x^2 - 9x + 1$ has an	19) The critical numbers of the function
inflection point at $(x) = x^2 + 5x^2 - 9x + 1$ has an	$f(x) = x^3 - 3x^2 - 9x + 1$ are
Solution:	Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
f''(x) = 6x + 6	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$f''(x) = 0 \implies 6x + 6 = 0$	$\implies 3(x^2 - 2x - 3) = 0$
$\Rightarrow 6x = -6$	$\implies x^2 - 2x - 3 = 0$
$\Rightarrow x = -\frac{6}{6}$	$\Rightarrow (x+1)(x-3) = 0$
$\Rightarrow x = -1$	$\Rightarrow x = -1 \text{ or } x = 3$
x = -1 -1	
- + Sign of $f''(x)$	
Kind of	
O Concavity	
Hence, the function $f(x)$ has an inflection point at	
(-1,12).	
$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 1$	
$= -1 + 3 + 9 + 1 = 12$ 20) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is decreasing	
	21) The function $f(x) = x^3 - 3x^2 - 9x + 1$ is increasing
on Solution:	on Solution:
Solution: $f'(x) = 3x^2 - 6x - 9$	Solution: $f'(x) = 3x^2 - 6x - 9$
$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$\Rightarrow 3(x^2 - 2x - 3) = 0$	$\Rightarrow  3(x^2 - 2x - 3) = 0$
$\Rightarrow x^2 - 2x - 3 = 0$	$\implies x^2 - 2x - 3 = 0$
$\implies (x+1)(x-3) = 0$	$\implies (x+1)(x-3) = 0$
$\Rightarrow$ $x = -1$ or $x = 3$	$\Rightarrow x = -1 \text{ or } x = 3$
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$ Kind of
Kind of monotonicity	monotonicity
Hence, the function $f(x)$ is decreasing on $(-1,3)$	Hence, the function $f(x)$ is increasing on
	$(-\infty, -1) \cup (3, \infty)$
22) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative	23) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has a relative
maximum value at the point	minimum value at the point
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$\Rightarrow 3(x^2 - 2x - 3) = 0$ $\Rightarrow x^2 - 2x - 3 = 0$	$\Rightarrow 3(x^2 - 2x - 3) = 0$ $\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow x^2 - 2x - 3 = 0$ $\Rightarrow (x + 1)(x - 3) = 0$	$\Rightarrow x^2 - 2x - 3 = 0$ $\Rightarrow (x+1)(x-3) = 0$
$\Rightarrow (x+1)(x-3) = 0$ $\Rightarrow x = -1 \text{ or } x = 3$	$\Rightarrow (x + 1)(x - 3) = 0$ $\Rightarrow x = -1 \text{ or } x = 3$
-1 3	-1 3
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum value at	Hence, the function $f(x)$ has a relative minimum value at
the point $(-1,6)$ .	the point $(3, -26)$ .
$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 1$ = -1 - 3 + 9 + 1 = 6.	$f(3) = (3)^3 - 3(3)^2 - 9(3) + 1$ = 27 - 27 - 27 + 1 = -26.
-1 3 + 7 + 1 = 0.	-2i $2i$ $2i$ $1 - 20.$
	1

24) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave	25) The function $f(x) = x^3 - 3x^2 - 9x + 1$ concave
upward on	downward on
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
f''(x) = 6x - 6	f''(x) = 6x - 6 $f''(x) = 0 \implies 6x - 6 = 0$
$f''(x) = 0 \implies 6x - 6 = 0$ $\implies 6x = 6$	$f^{(x)}(x) = 0 \implies 6x - 6 = 0$ $\implies 6x = 6$
$\implies x = \frac{6}{6}$	$\implies x = \frac{6}{6}$
$\Rightarrow x = 1$	$\Rightarrow x = 1$
1	1
- + Sign of $f''(x)$	- + Sign of $f''(x)$
Kind of	Kind of
O U concavity	O U concavity
Hence, the function $f(x)$ is concave upward on $(1, \infty)$	Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$
	Then even the function $f(x)$ is concave downward on $(-\infty, 1)$
26) The function $f(x) = x^3 - 3x^2 - 9x + 1$ has an	27) The critical numbers of the function
inflection point at	$f(x) = x^3 + 3x^2 - 9x + 5$ are
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
f''(x) = 6x - 6 $f''(x) = 0 \implies 6x - 6 = 0$	$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$ $\implies 3(x^2 + 2x - 3) = 0$
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$\Rightarrow 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$
	$ \Rightarrow x + 2x + 3 = 0 \Rightarrow (x + 3)(x - 1) = 0 $
$\implies x = \frac{6}{6}$	$\Rightarrow$ $x = -3$ or $x = 1$
$\Rightarrow x = 1$	
- + Sign of $f''(x)$	
Kind of	
U Concavity	
Hence, the function $f(x)$ has an inflection point at	
(1,-10).	
$f(1) = (1)^3 - 3(1)^2 - 9(1) + 1$	
$= 1 - 3 - 9 + 1 = -10$ 28) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is decreasing	
	29) The function $f(x) = x^3 + 3x^2 - 9x + 5$ is increasing
on <u>Solution:</u>	on <u>Solution:</u>
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$
$\Rightarrow 3(x^2 + 2x - 3) = 0$	$\Rightarrow  3(x^2 + 2x - 3) = 0$
$\implies x^2 + 2x - 3 = 0$	$\implies x^2 + 2x - 3 = 0$
$\Rightarrow (x+3)(x-1) = 0$	$\implies (x+3)(x-1) = 0$
$\Rightarrow x = -3 \text{ or } x = 1$	$\Rightarrow x = -3 \text{ or } x = 1$
$\begin{vmatrix} -3 & 1 \\ + & - & + & \text{Sign of } f'(x) \end{vmatrix}$	-3 1 + - + Sign of $f'(x)$
$+$ $ +$ $\operatorname{Sign}(O) f(x)$	+ $ +$ sign of $f(x)Kind of$
monotonicity	monotonicity
Hence, the function $f(x)$ is decreasing on $(-3,1)$ .	Hence, the function $f(x)$ is increasing on
	$(-\infty, -3) \cup (1, \infty).$

30) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has a relative	
minimum value at the point	maximum value at the point
Solution:	Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 + 6x - 9 = 0$
$\Rightarrow 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$	$\Rightarrow 3(x^2 + 2x - 3) = 0$ $\Rightarrow x^2 + 2x - 3 = 0$
$ \Rightarrow x + 2x - 5 = 0 \Rightarrow (x + 3)(x - 1) = 0 $	
$\Rightarrow (x + 3)(x - 1) = 0$ $\Rightarrow x = -3 \text{ or } x = 1$	$\Rightarrow (x + 3)(x - 1) = 0$ $\Rightarrow x = -3 \text{ or } x = 1$
-3 1	-3 1
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative minimum value at	Hence, the function $f(x)$ has a relative maximum value at
the point $(1,0)$ .	the point $(-3,32)$ .
$f(1) = (1)^3 + 3(1)^2 - 9(1) + 5$	$f(-3) = (-3)^3 + 3(-3)^2 - 9(-3) + 5$
$= 1 + 3 - 9 + 5 = 0$ 32) The function $f(x) = x^3 + 3x^2 - 9x + 5$ has an	$= -27 + 27 + 27 + 5 = 32$ 33) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave
	33) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave downward on
inflection point at <u>Solution:</u>	Solution:
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 + 6x - 9$
f''(x) = 6x + 6	$f''(x) = 5x^{2} + 6x^{2} - 5x^{2}$
$f''(x) = 0 \implies 6x + 6 = 0$	$f''(x) = 0 \implies 6x + 6 = 0$
$\Rightarrow 6x = -6$	$\Rightarrow 6x = -6$
$\implies x = -\frac{6}{6}$	$\rightarrow$ $r = -\frac{6}{6}$
$\rightarrow$ $x = -\frac{6}{6}$	$\Rightarrow x = -\frac{1}{6}$
$\Rightarrow x = -1$	$\Rightarrow x = -1$
$\begin{array}{c c} & -1 \\ \hline & - & + & \text{Sign of } f''(x) \end{array}$	$\begin{array}{c c} & -1 \\ \hline & - & + & \text{Sign of } f''(x) \end{array}$
N         Kind of concavity	Image: Non-approximation     U     Kind of concavity
Hence, the function $f(x)$ has an inflection point at	Hence, the function $f(x)$ is concave downward on
(-1,16).	$(-\infty, -1).$
$f(-1) = (-1)^3 + 3(-1)^2 - 9(-1) + 5$	
$= -1 + 3 + 9 + 5 = 16$ 34) The function $f(x) = x^3 + 3x^2 - 9x + 5$ concave	35) The critical numbers of the function
$y = x^{2} + 5x^{2} - 9x + 5$ concave upward on	$f(x) = x^3 - 3x^2 - 9x + 5$ are
Solution:	$\frac{Solution:}{Solution}$
$f'(x) = 3x^2 + 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
f''(x) = 6x + 6	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$f''(x) = 0 \implies 6x + 6 = 0$	$\implies 3(x^2 - 2x - 3) = 0$
$\Rightarrow 6x = -6$	$\implies x^2 - 2x - 3 = 0$
$\Rightarrow x = -\frac{6}{6}$	$\implies (x+1)(x-3) = 0$
$ \Rightarrow x = -1 $	$\Rightarrow$ $x = -1$ or $x = 3$
$ \Rightarrow x = -1 \\ -1 $	
$- + \operatorname{Sign of} f''(x)$	
Kind of	
<b>N U</b> concavity	
Hence, the function $f(x)$ is concave upward on $(-1, \infty)$ .	

36) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing	37) The function $f(x) = x^3 - 3x^2 - 9x + 5$ is decreasing
on	on
Solution: $f'(x) = 3x^2 - 6x - 9$	Solution: $f'(x) = 3x^2 - 6x - 9$
$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$ \begin{array}{c} y (x) = 0  \implies  3x = 0,  x = y = 0 \\ \implies  3(x^2 - 2x - 3) = 0 \end{array} $	$ \begin{array}{cccc} y & (x) = 0 & \implies & 3x = 0x = y = 0 \\ & \implies & 3(x^2 - 2x - 3) = 0 \end{array} $
$\Rightarrow 3(x - 2x - 3) = 0$ $\Rightarrow x^2 - 2x - 3 = 0$	$\Rightarrow 3(x - 2x - 3) = 0$ $\Rightarrow x^2 - 2x - 3 = 0$
$\Rightarrow (x+1)(x-3) = 0$	$\Rightarrow (x+1)(x-3) = 0$
$\Rightarrow$ $x = -1$ or $x = 3$	$\Rightarrow$ $x = -1$ or $x = 3$
-1 3	-1 3
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on $(-1,3)$ .
$(-\infty, -1) \cup (3, \infty).$ 38) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative	39) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative
(38) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has a relative maximum value at the point	(39) The function $f(x) = x^2 - 3x^2 - 9x + 5$ has a relative minimum value at the point
Solution:	Solution:
$f'(x) = 3x^2 - 6x - 9$	$f'(x) = 3x^2 - 6x - 9$
$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$	$f'(x) = 0 \implies 3x^2 - 6x - 9 = 0$
$\Rightarrow  3(x^2 - 2x - 3) = 0$	$\Rightarrow  3(x^2 - 2x - 3) = 0$
$\implies x^2 - 2x - 3 = 0$	$\implies x^2 - 2x - 3 = 0$
$\Rightarrow (x+1)(x-3) = 0$	$\implies (x+1)(x-3) = 0$
$\Rightarrow$ $x = -1$ or $x = 3$	$\Rightarrow x = -1 \text{ or } x = 3$
$\begin{vmatrix} -1 & 3 \\ + & - & + \\ \end{vmatrix}$ Sign of $f'(x)$	$\begin{vmatrix} -1 & 3 \\ + & - & + \\ \end{vmatrix}$ Sign of $f'(x)$
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$ Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum value at	Hence, the function $f(x)$ has a relative minimum value at
the point $(-1,10)$ .	the point $(3, -22)$ .
$f(-1) = (-1)^3 - 3(-1)^2 - 9(-1) + 5$	$f(3) = (3)^3 - 3(3)^2 - 9(3) + 5$
= -1 - 3 + 9 + 5 = 10.	= 27 - 27 - 27 + 5 = -22.
40) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave	41) The function $f(x) = x^3 - 3x^2 - 9x + 5$ concave
upward on	downward on
Solution: $f'(x) = 3x^2 - 6x - 9$	Solution: $f'(x) = 3x^2 - 6x - 9$
$f''(x) = 5x^2 - 6x - 9$ f''(x) = 6x - 6	$f''(x) = 5x^2 - 6x - 9$ f''(x) = 6x - 6
$f''(x) = 0 \implies 6x - 6 = 0$	$f''(x) = 0 \implies 6x - 6 = 0$
$\Rightarrow 6x = 6$	$\Rightarrow 6x = 6$
$\implies x = \frac{6}{6}$	$\Rightarrow x = \frac{6}{6}$
	0
$\Rightarrow x = 1$	$\Rightarrow x = 1$
$\begin{array}{c c} 1 \\ \hline - & + & \text{Sign of } f''(x) \end{array}$	$\begin{array}{ c c c }\hline & 1 \\ \hline & - & + & \text{Sign of } f''(x) \end{array}$
Kind of	Kind of
O U concavity	O U concavity
Hence, the function $f(x)$ is concave upward on $(1, \infty)$ .	Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$ .
	7
	7

42) The function $f(x) = x^3 - 3x^2 - 9x + 5$ has an	(12) The critical numbers of the function
(42) The function $f(x) = x^2 - 3x^2 - 9x + 5$ has an inflection point at	43) The critical numbers of the function $f(u) = \frac{1}{2}u^3 + \frac{1}{2}u^2 + 2u + 1$
Solution:	$f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ are
$f'(x) = 3x^2 - 6x - 9$	Solution:
f''(x) = 6x - 6	$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$
$f''(x) = 0 \implies 6x - 6 = 0$	$f'(x) = 0 \implies x^2 - x - 2 = 0$
$\Rightarrow 6x = 6$	$ \begin{array}{cccc} y & (x) = 0 & \implies & x & x & 2 = 0 \\ & \implies & (x+1)(x-2) = 0 \end{array} $
$\Rightarrow r - \frac{6}{2}$	$\Rightarrow x = -1 \text{ or } x = 2$
$ \Rightarrow x = \frac{6}{6} \\ \Rightarrow x = 1 $	
$\Rightarrow x = 1$	
- + Sign of $f''(x)$	
Kind of	
O U concavity	
Hence, the function $f(x)$ has an inflection point at $(1, -6)$ .	
$f(1) = (1)^3 - 3(1)^2 - 9(1) + 5$	
= 1 - 3 - 9 + 5 = -6	
$= 1 - 3 - 9 + 5 = -6$ 44) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is	45) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ is
increasing on	decreasing on
Solution:	Solution:
$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$	$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$
$ f'(x) = 0 \implies x^2 - x - 2 = 0 $	
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	$f'(x) = 0 \implies x^2 - x - 2 = 0$ $\implies (x+1)(x-2) = 0$
$\Rightarrow (x + 1)(x - 2) = 0$ $\Rightarrow x = -1 \text{ or } x = 2$	$\Rightarrow (x + 1)(x - 2) = 0$ $\Rightarrow x = -1 \text{ or } x = 2$
-1 2	-1 2
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on $(-1,2)$ .
$(-\infty, -1) \cup (2, \infty).$	
46) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$	47) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$
has a relative maximum point	has a relative minimum point
Solution:	Solution:
$f'(x) = 3\left(\frac{1}{2}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$	$f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$
$ \begin{aligned} f'(x) &= 0  \Rightarrow  x^2 - x - 2 = 0 \\ &\Rightarrow  (x+1)(x-2) = 0 \end{aligned} $	$\Rightarrow (x+1)(x-2) = 0$
$\Rightarrow x = -1 \text{ or } x = 2$	$f'(x) = 0 \implies x^2 - x - 2 = 0$ $\implies (x+1)(x-2) = 0$ $\implies x = -1 \text{ or } x = 2$
-1 2	-1 2
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum point at	Hence, the function $f(x)$ has a relative minimum point at
$\left(-1,\frac{13}{6}\right)$ .	$\left(2,-\frac{7}{3}\right)$ .
$f(-1) = \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1) + 1$	$f(2) = \frac{1}{3}(2)^3 - \frac{1}{2}(2)^2 - 2(2) + 1$
J L 12	0 1
$= -\frac{1}{3} - \frac{1}{2} + 2 + 1 = \frac{13}{6}$	$=\frac{8}{3}-\frac{4}{2}-4+1=-\frac{7}{3}$

48) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ concave upward on Solution: $f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$ $f''(x) = 2x - 1$ $f''(x) = 0 \implies 2x - 1 = 0$ $\implies 2x = 1$ $\implies x = \frac{1}{2}$ $\frac{1}{2}$	49) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ concave downward on Solution: $f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$ $f''(x) = 2x - 1$ $f''(x) = 0 \implies 2x - 1 = 0$ $\implies 2x = 1$ $\implies x = \frac{1}{2}$ $\frac{1}{2}$
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
50) The function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + 1$ has an inflection point at Solution: $f'(x) = 3\left(\frac{1}{3}\right)x^2 - 2\left(\frac{1}{2}\right)x - 2 = x^2 - x - 2$ f''(x) = 2x - 1 $f''(x) = 0 \implies 2x - 1 = 0$ $\implies 2x = 1$ $\implies x = \frac{1}{2}$ $\boxed{-}$ + Sign of $f''(x)$ $\boxed{-}$ U Kind of concavity Hence, the function $f(x)$ has an inflection point at $\left(\frac{1}{2}, -\frac{1}{12}\right)$ . $f\left(\frac{1}{2}\right) = \frac{1}{3}\left(\frac{1}{2}\right)^3 - \frac{1}{2}\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 1$ $= \frac{1}{24} - \frac{1}{8} - 1 + 1 = -\frac{1}{12}$	51) The critical numbers of the function $f(x) = \frac{1}{3}x^{3} + \frac{1}{2}x^{2} - 2x + 1 \text{ are}$ Solution: $f'(x) = 3\left(\frac{1}{3}\right)x^{2} + 2\left(\frac{1}{2}\right)x - 2 = x^{2} + x - 2$ $f'(x) = 0 \implies x^{2} + x - 2 = 0$ $\implies (x + 2)(x - 1) = 0$ $\implies x = -2 \text{ or } x = 1$
52) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ is increasing on Solution: $f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$ $f'(x) = 0 \implies x^2 + x - 2 = 0$ $\implies (x + 2)(x - 1) = 0$ $\implies x = -2 \text{ or } x = 1$ $-2 \qquad 1$ $+ \qquad - \qquad + \qquad \text{Sign of } f'(x)$ $\text{Kind of monotonicity}$ Hence, the function $f(x)$ is increasing on $(-\infty, -2) \cup (1, \infty).$	53) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ is decreasing on Solution: $f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$ $f'(x) = 0 \implies x^2 + x - 2 = 0$ $\implies (x+2)(x-1) = 0$ $\implies x = -2$ or $x = 1$ -2 1 x = -2 1 Hence, the function $f(x)$ is decreasing on $(-2,1)$ .

54) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$	55) The function $f(x) = \frac{1}{2}x^3 + \frac{1}{2}x^2 - 2x + 1$
has a relative maximum point	has a relative minimum point
Solution:	Solution:
$\overline{f'(x)} = 3\left(\frac{1}{2}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$	$\overline{f'(x)} = 3\left(\frac{1}{2}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$
(J) $(L)$	
$f'(x) = 0 \implies x^2 + x - 2 = 0$ $\implies (x+2)(x-1) = 0$	$f'(x) = 0 \implies x^2 + x - 2 = 0$ $\implies (x+2)(x-1) = 0$
$\Rightarrow (x+2)(x-1) = 0$ $\Rightarrow x = -2 \text{ or } x = 1$	$\Rightarrow (x+2)(x-1) = 0$ $\Rightarrow x = -2 \text{ or } x = 1$
-2 1	-2 1
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum point at	Hence, the function $f(x)$ has a relative minimum point at
$\left(-2,\frac{13}{3}\right)$ .	$\left(1,-\frac{1}{6}\right)$ .
$f(-2) = \frac{1}{3}(-2)^3 + \frac{1}{2}(-2)^2 - 2(-2) + 1$	$f(1) = \frac{1}{3}(1)^3 + \frac{1}{2}(1)^2 - 2(1) + 1$
5 2	0 1
$= -\frac{8}{3} + \frac{4}{2} + 4 + 1 = \frac{13}{3}$	$= \frac{1}{3} + \frac{1}{2} - 2 + 1 = -\frac{1}{6}$ 57) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ concave
56) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ concave	57) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ concave
upward on	downward on
Solution:	Solution:
$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$	$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$
$f''(x) = 0 \implies 2x + 1 = 0$ $\implies 2x = -1$	$f''(x) = 0 \implies 2x + 1 = 0$ $\implies 2x = -1$
$\begin{array}{cccc} y & (x) = 0 & \implies & 2x + 1 = 0 \\ & \implies & 2r = -1 \end{array}$	$\begin{array}{cccc} y & (x) = 0 & \implies & 2x + 1 = 0 \\ & \implies & 2r = -1 \end{array}$
	$\Rightarrow x = -\frac{1}{2}$
$\Rightarrow x = -\frac{1}{2}$	$\Rightarrow x = -\frac{1}{2}$
$-\frac{1}{2}$	$-\frac{1}{2}$
- + Sign of $f''(x)$	- + Sign of $f''(x)$
Kind of	∩ U Kind of
	Hence, the function $f(x)$ is concave downward on
Hence, the function $f(x)$ is concave upward on $\left(-\frac{1}{2},\infty\right)$ .	$\left(-\infty,-\frac{1}{2}\right)$ .

1 1	
58) The function $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x + 1$ has an	59) The critical numbers of the function $f(x) = \frac{3}{2} + \frac{1}{2}$
inflection point at	$f(x) = x^3 - 12x + 3$ are
Solution:	Solution:
$\frac{1}{1}$	$f'(x) = 3x^2 - 12$
$f'(x) = 3\left(\frac{1}{3}\right)x^2 + 2\left(\frac{1}{2}\right)x - 2 = x^2 + x - 2$	$f'(x) = 0 \implies 3x^2 - 12 = 0$
$f^{\prime\prime}(x) = 2x + 1$	$ \Rightarrow 3(x^2 - 4) = 0 \Rightarrow x^2 - 4 = 0 $
$f''(x) = 0 \implies 2x + 1 = 0$	$\Rightarrow x^2 - 4 = 0$
$\Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2}$	$\Rightarrow x^2 = 4$
1	$\Rightarrow x = \pm 2$
$\frac{1}{2}$	
- + Sign of $f''(x)$	
Kind of	
<b>II O</b> concavity	
Hence, the function $f(x)$ has an inflection point at	
$\left(-\frac{1}{2},\frac{25}{12}\right)$	
$\int f\left(-\frac{1}{2}\right) = \frac{1}{3}\left(-\frac{1}{2}\right)^3 + \frac{1}{2}\left(-\frac{1}{2}\right)^2 - 2\left(-\frac{1}{2}\right) + 1$	
$=-\frac{1}{24}+\frac{1}{8}+1+1=\frac{1}{12}$	
$= -\frac{1}{24} + \frac{1}{8} + 1 + 1 = \frac{25}{12}$ 60) The function $f(x) = x^3 - 12x + 3$ is increasing	61) The function $f(x) = x^3 - 12x + 3$ is decreasing
on	on
Solution:	Solution:
$f'(x) = 3x^2 - 12$	$f'(x) = 3x^2 - 12$
$f'(x) = 0 \implies 3x^2 - 12 = 0$	$f'(x) = 0 \implies 3x^2 - 12 = 0$
$\Rightarrow 3(x^2 - 4) = 0$	$\Rightarrow 3(x^2 - 4) = 0$ $\Rightarrow x^2 - 4 = 0$
$\Rightarrow x^2 - 4 = 0$	$\Rightarrow x^2 - 4 = 0$
$\implies x^2 = 4$	$\implies x^2 = 4$
$\Rightarrow x = \pm 2$	$\Rightarrow x = \pm 2$
22	22
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ is increasing on	Hence, the function $f(x)$ is decreasing on $(-2,2)$ .
$(-\infty,-2) \cup (2,\infty).$	
62) The function $f(x) = x^3 - 12x + 3$ has a relative	63) The function $f(x) = x^3 - 12x + 3$ has a relative
maximum point at	minimum point at
Solution:	Solution:
$f'(x) = 3x^2 - 12$	$f'(x) = 3x^2 - 12$
$f'(x) = 0 \implies 3x^2 - 12 = 0$	$f'(x) = 0  \Longrightarrow  3x^2 - 12 = 0$
$\implies 3(x^2 - 4) = 0$	$\implies 3(x^2 - 4) = 0$
$\implies x^2 - 4 = 0$	$\Rightarrow x^2 - 4 = 0$
$\Rightarrow x^2 = 4$	$\implies x^2 = 4$
$\Rightarrow x = \pm 2$	$\Rightarrow x = \pm 2$
-2 2	-2 2
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum point at	Hence, the function $f(x)$ has a relative minimum point at
(-2,19).	(2, -13).
((n))	$(0)^{3}$ $(0)^{3}$ $(0)^{3}$ $(0)^{3}$
$f(-2) = (-2)^3 - 12(-2) + 3$	$f(2) = (2)^3 - 12(2) + 3$
$f(-2) = (-2)^3 - 12(-2) + 3$ = -8 + 24 + 3 = 19.	$f(2) = (2)^3 - 12(2) + 3$ = 8 - 24 + 3 = -13

$(A) = f_{ab} = f_{ab} = f_{ab} = \frac{1}{2} + \frac{1}{2}$		$(c)$ The function $f(x) = x^3 + 2x + 2$ concerns
64) The function $f(x) = x^3 - 12x + 3$	concave	65) The function $f(x) = x^3 - 12x + 3$ concave
upward on Solution:		downward on
Solution: $f'(x) = 3x^2 - 12$		Solution: $f'(x) = 3x^2 - 12$
f''(x) = 5x - 12    f''(x) = 6x		$f''(x) = 5x^{-12}$
$f''(x) = 0 \implies 6x = 0$		$f''(x) = 0 \implies 6x = 0$
0		0
$\Rightarrow x = \frac{0}{6}$		$\Rightarrow x = \frac{0}{6}$
$\Rightarrow x = 0$		$\Rightarrow x = 0$
0		0
- +	Sign of $f''(x)$	- + Sign of $f''(x)$
$  \cap   \cup  $	Kind of	Kind of
	concavity	<b>II O</b> concavity
Hence, the function $f(x)$ is concave upw	ard on $(0,\infty)$ .	Hence, the function $f(x)$ is concave downward on $(-\infty, 0)$ .
66) The function $f(x) = x^3 - 12x + 3$	has an	67) The critical numbers of the function
(b) The function $f(x) = x^2 - 12x + 3$ inflection point at	11d5 d11	$f(x) = x^3 - 3x^2 + 1$ are
Solution:		Solution: $f(x) = x^2 - 5x^2 + 1$ are
$f'(x) = 3x^2 - 12$		$f'(x) = 3x^2 - 6x$
f''(x) = 6x		$f'(x) = 0 \implies 3x^2 - 6x = 0$
$f''(x) = 0 \implies 6x = 0$		$\Rightarrow 3(x^2 - 2x) = 0$
$\rightarrow x = 0$		$\Rightarrow x^2 - 2x = 0$
$\implies x = \frac{3}{6}$		$\implies x(x-2) = 0$
$\Rightarrow x = 0$		$\Rightarrow x = 0 \text{ or } x = 2$
	$C_{int} = f_{int} f_{int}(u)$	
- +	Sign of $f''(x)$	
	Kind of	
$  \cap   \cup  $	concavity	
Hence, the function $f(x)$ has an inflection	-	
$f(0) = (0)^3 - 12(0)^2 + 3$	on point at (0,5).	
= 0 - 0 + 3 = 3		
68) The function $f(x) = x^3 - 3x^2 + 1$	is increasing	69) The function $f(x) = x^3 - 3x^2 + 1$ is decreasing
on		on
Solution:		Solution:
$f'(x) = 3x^2 - 6x$		$f'(x) = 3x^2 - 6x$
$f'(x) = 0 \implies 3x^2 - 6x = 0$		$f'(x) = 0 \implies 3x^2 - 6x = 0$
$\implies 3(x^2 - 2x) = 0$		$\implies 3(x^2 - 2x) = 0$
$\implies x^2 - 2x = 0$		$\Rightarrow x^2 - 2x = 0$
$\implies x(x-2) = 0$		$\Rightarrow x(x-2) = 0$
$\Rightarrow x = 0 \text{ or } x = 2$		$\Rightarrow x = 0 \text{ or } x = 2$
	Sign of $f'(x)$	$\begin{array}{ c c c c c } 0 & 2 \\ \hline \\ \hline \\ + & - & + & \text{Sign of } f'(x) \\ \hline \end{array}$
	Sign of $f'(x)$ Kind of	+ - + Sign of $f'(x)$ Kind of
	monotonicity	Mind Of monotonicity
	monotonicity	
Hence, the function $f(x)$ is increasing of	 n	Hence, the function $f(x)$ is decreasing on (0,2).
$(-\infty, 0) \cup (2, \infty).$	-	

70) The function $f(x) = x^3 - 3x^2 + 1$ has a relative	71) The function $f(x) = x^3 - 3x^2 + 1$ has a relative
maximum point at	minimum point at
Solution:	Solution:
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$
$f'(x) = 0 \implies 3x^2 - 6x = 0$	$f'(x) = 0 \implies 3x^2 - 6x = 0$
$\Rightarrow 3(x^2 - 2x) = 0$ $\Rightarrow x^2 - 2x = 0$	$ \Rightarrow 3(x^2 - 2x) = 0 \Rightarrow x^2 - 2x = 0 $
$ \Rightarrow x^2 - 2x = 0 \Rightarrow x(x - 2) = 0 $	$ \Rightarrow x^{2} - 2x = 0 \Rightarrow x(x - 2) = 0 $
$ \Rightarrow x(x-2) = 0 \Rightarrow x = 0 \text{ or } x = 2 $	$\Rightarrow x(x-2) = 0$ $\Rightarrow x = 0 \text{ or } x = 2$
$\begin{array}{cccc} &  & x = 0 & \text{of } x = 2 \\ & 0 & 2 \end{array}$	$\begin{array}{ccc}$
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$
Kind of	Kind of
monotonicity	monotonicity
Hence, the function $f(x)$ has a relative maximum point at	Hence, the function $f(x)$ has a relative minimum point at
(0,1).	(2, -3).
$f(0) = (0)^3 - 3(0)^2 + 1$	$f(2) = (2)^3 - 3(2)^2 + 1$
= 0 - 0 + 1 = 1.	= 8 - 12 + 1 = -3.
72) The function $f(x) = x^3 - 3x^2 + 1$ concave	73) The function $f(x) = x^3 - 3x^2 + 1$ concave
upward on	downward on
Solution: $f'(x) = 2x^2 - 6x$	Solution: $f'(x) = 2x^2 - 6x$
$     f'(x) = 3x^2 - 6x      f''(x) = 6x - 6 $	$f'(x) = 3x^2 - 6x$ f''(x) = 6x - 6
$f''(x) = 0 \implies 6x - 6 = 0$	$f''(x) = 0 \implies 6x - 6 = 0$
$ \begin{array}{c} y  (x) = 0  \implies  0x  0 = 0 \\ \implies  6x = 6 \end{array} $	$\begin{array}{c} f(x) = 0  \implies  6x = 0 \\ \implies  6x = 6 \end{array}$
6	6
$\Rightarrow x = \frac{1}{6}$	$\Rightarrow x = \frac{6}{6}$
$\Rightarrow x = 1$	$\Rightarrow x = 1$
1	1
- + Sign of $f''(x)$	- + Sign of $f''(x)$
Kind of	Kind of
Upped the function f(u) is concerned on (1, co)	$\bigcup_{i=1}^{n} \bigcup_{i=1}^{n} \bigcup_{i$
Hence, the function $f(x)$ is concave upward on $(1, \infty)$ .	Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$ .
74) The function $f(x) = x^3 - 3x^2 + 1$ has an	75) The critical numbers of the function
inflection point at	$f(x) = x^3 - 3x^2 + 2$ are
Solution:	Solution:
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$
f''(x) = 6x - 6	$f'(x) = 0 \implies 3x^2 - 6x = 0$ $\implies 2(x^2 - 3x) = 0$
$ \begin{aligned} f''(x) &= 0 \implies 6x - 6 = 0 \\ &\implies 6x = 6 \end{aligned} $	$ \Rightarrow 3(x^2 - 2x) = 0 \Rightarrow x^2 - 2x = 0 $
	$ \Rightarrow x^2 - 2x = 0 \Rightarrow x(x - 2) = 0 $
$\implies x = \frac{6}{6}$	$\Rightarrow x(x-2) = 0$ $\Rightarrow x = 0 \text{ or } x = 2$
$\Rightarrow x = 1$	$\rightarrow \lambda = 0  01  \lambda = 2$
1	
- + Sign of $f''(x)$	
Kind of	
II U concavity	
Hence, the function $f(x)$ has an inflection point at $(1, -1)$ .	
$f(1) = (1)^3 - 3(1)^2 + 1$	
= 1 - 3 + 1 = -1	

76) The function $f(x) = x^3 - 3x^2 + 2$ is increasing on Solution: $f'(x) = 3x^2 - 6x$ $f'(x) = 0 \implies 3x^2 - 6x = 0$ $\implies 3(x^2 - 2x) = 0$ $\implies x^2 - 2x = 0$ $\implies x(x - 2) = 0$ $\implies x = 0 \text{ or } x = 2$ 0 2 f'(x) Hence, the function $f(x)$ is increasing on $(-\infty, 0) \cup (2, \infty)$ .	77) The function $f(x) = x^3 - 3x^2 + 2$ is decreasing on Solution: $f'(x) = 3x^2 - 6x$ $f'(x) = 0 \implies 3x^2 - 6x = 0$ $\implies 3(x^2 - 2x) = 0$ $\implies x^2 - 2x = 0$ $\implies x(x - 2) = 0$ $\implies x = 0 \text{ or } x = 2$ 0 2 Hence, the function $f(x)$ is decreasing on (0,2).		
78) The function $f(x) = x^3 - 3x^2 + 2$ has a relative	79) The function $f(x) = x^3 - 3x^2 + 2$ has a relative		
minimum point at	maximum point at $3x + 2$ has a relative		
Solution:	Solution:		
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 6x$		
$f'(x) = 0 \implies 3x^2 - 6x = 0$	$f'(x) = 0  \Longrightarrow  3x^2 - 6x = 0$		
$\implies 3(x^2 - 2x) = 0$	$\implies 3(x^2 - 2x) = 0$		
$\implies x^2 - 2x = 0$	$\implies x^2 - 2x = 0$		
$\implies x(x-2) = 0$	$\implies x(x-2) = 0$		
$\Rightarrow x = 0 \text{ or } x = 2$	$\Rightarrow x = 0 \text{ or } x = 2$		
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$		
Kind of	Kind of		
monotonicity	monotonicity		
Hence, the function $f(x)$ has a relative minimum point at	Hence, the function $f(x)$ has a relative maximum point at		
(2, -2).	(0,2). $f(0) = (0)^3 - 2(0)^2 + 2$		
$f(2) = (2)^3 - 3(2)^2 + 2$	$f(0) = (0)^3 - 3(0)^2 + 2$		
$= 8 - 12 + 2 = -2.$ (80) The function $f(x) = x^3 - 2x^2 + 2$ conceive	$= 0 - 0 + 2 = 2.$ (21) The function $f(x) = x^3 - 2x^2 + 2$ conceive		
80) The function $f(x) = x^3 - 3x^2 + 2$ concave downward on	81) The function $f(x) = x^3 - 3x^2 + 2$ concave		
Solution:	upward on Solution:		
$f'(x) = 3x^2 - 6x$	Solution: $f'(x) = 3x^2 - 6x$		
f''(x) = 5x - 6x - 6	$f'(x) = 3x^2 - 6x$ f''(x) = 6x - 6		
$f''(x) = 0 \implies 6x - 6 = 0$	$f''(x) = 0 \implies 6x - 6 = 0$		
$\begin{array}{c} f (x) = 0  \implies  0x  0 = 0 \\ \implies  6x = 6 \end{array}$	$\begin{array}{c} f(x) = 0  \implies  6x = 0 \\ \implies  6x = 6 \end{array}$		
	6		
$\implies x = \frac{6}{6}$	$\Rightarrow x = \frac{3}{6}$		
$\Rightarrow x = 1$	$\Rightarrow x = 1$		
1	1		
- + Sign of $f''(x)$	- + Sign of $f''(x)$		
Kind of	Kind of		
Concavity	<b>II U</b> concavity		
Hence, the function $f(x)$ is concave downward on $(-\infty, 1)$ .	Hence, the function $f(x)$ is concave upward on $(1, \infty)$ .		

(22) The function $f(x) = x^3 - 2x^2 + 2$ has an	(2) The evitical purphers of the function		
82) The function $f(x) = x^3 - 3x^2 + 2$ has an infloction point at	83) The critical numbers of the function $f(x) = x^3 - 6x^2 - 36x$ are		
inflection point at <u>Solution:</u>	$\int (x) = x^2 - 6x^2 - 36x^2 \text{ are}$ Solution:		
$f'(x) = 3x^2 - 6x$	$f'(x) = 3x^2 - 12x - 36$		
f''(x) = 5x - 6x - 6	$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$		
$f''(x) = 0 \implies 6x - 6 = 0$	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		
$\begin{array}{c} y  (x) = 0  \implies  0x  0 = 0 \\ \implies  6x = 6 \end{array}$	$\Rightarrow 3(x + x - 12) = 0$ $\Rightarrow x^2 - 4x - 12 = 0$		
	$ \Rightarrow x + 2)(x - 6) = 0 $		
$\implies x = \frac{6}{6}$	$\Rightarrow x = -2 \text{ or } x = 6$		
$\Rightarrow x = 1$			
1			
- + Sign of $f''(x)$			
Kind of			
II U concavity			
Hence, the function $f(x)$ has an inflection point at (1,0).			
$f(1) = (1)^3 - 3(1)^2 + 2$			
= 1 - 3 + 2 = 0			
84) The function $f(x) = x^3 - 6x^2 - 36x$ is decreasing on	85) The function $f(x) = x^3 - 6x^2 - 36x$ is increasing on		
Solution:	Solution:		
$f'(x) = 3x^2 - 12x - 36$	$f'(x) = 3x^2 - 12x - 36$		
$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$	$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$		
$\implies 3(x^2 - 4x - 12) = 0$	$\implies 3(x^2 - 4x - 12) = 0$		
$\implies x^2 - 4x - 12 = 0$ $\implies (x + 2)(x - 0) = 0$	$\Rightarrow x^2 - 4x - 12 = 0$ $\Rightarrow (x + 2)(x - 6) = 0$		
$\Rightarrow (x+2)(x-6) = 0$ $\Rightarrow x = -2 \text{ or } x = 6$	$\Rightarrow (x+2)(x-6) = 0$ $\Rightarrow x = -2 \text{ or } x = 6$		
$ \begin{array}{c}  & x = -2 & \text{of}  x = 0 \\ -2 & 6 \end{array} $	$ \xrightarrow{\longrightarrow} x = -2  \text{or}  x = 0 \\ -2 \qquad 6 $		
$\begin{vmatrix} -2 & 0 \\ + & - & + \\ \end{vmatrix}$ Sign of $f'(x)$	$\begin{array}{ c c c c c }\hline - & - & - & - & - & - & - & - & - & - $		
$\mathbf{X} \mathbf{X} \mathbf{X} \mathbf{X} \mathbf{X}$ Kind of	$\mathbf{x}$ Kind of		
monotonicity	monotonicity		
Hence, the function $f(x)$ is decreasing on $(-2,6)$ .	Hence, the function $f(x)$ is increasing on		
	$(-\infty, -2) \cup (6, \infty).$		
86) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative	87) The function $f(x) = x^3 - 6x^2 - 36x$ has a relative		
minimum value at the point	maximum value at the point		
Solution:	Solution:		
$f'(x) = 3x^2 - 12x - 36$	$f'(x) = 3x^2 - 12x - 36$		
$f'(x) = 0 \implies 3x^2 - 12x - 36 = 0$	$f'(x) = 0  \Longrightarrow  3x^2 - 12x - 36 = 0$		
$\implies 3(x^2 - 4x - 12) = 0$	$\Rightarrow  3(x^2 - 4x - 12) = 0$		
$\implies x^2 - 4x - 12 = 0$	$\implies x^2 - 4x - 12 = 0$		
$\implies (x+2)(x-6) = 0$	$\implies (x+2)(x-6) = 0$		
$\Rightarrow x = -2 \text{ or } x = 6$	$\Rightarrow$ $x = -2$ or $x = 6$		
-2 6	-2 6		
+ - + Sign of $f'(x)$	+ - + Sign of $f'(x)$		
Kind of	Kind of		
monotonicity	monotonicity		
Hence, the function $f(x)$ has a relative minimum value at	Hence, the function $f(x)$ has a relative maximum value at		
the point $(6, -216)$ .	the point $(-2,40)$ .		
$f(6) = (6)^3 - 6(6)^2 - 36(6)$ = 216 - 216 - 216 = -216	$f(-2) = (-2)^3 - 6(-2)^2 - 36(-2)$ = -8 - 24 + 72 = 40		
- 210 - 210 - 210210	$0-24 \pm 72 - 40$		
	1		

88) The function $f(x)$	-	6 <i>x</i> has an	89) The function $f(x) = x^3 - 6x^2 - 36x$ concave			
inflection point a	t		downward on			
Solution:		Solution: $(1/2) = 2 \cdot \frac{3}{2} \cdot \frac{1}{2} = 2 \cdot \frac{1}{2}$				
) 、	$ x) = 3x^2 - 12x - f''(x) = 6x - 12 $	36		$x = 3x^2 - 12x -$	36	
$\int f''(x) = 0 \implies 6x$			f''(x) = 6x - 12 $f''(x) = 0 \implies 6x - 12 = 0$			
$ \begin{vmatrix} y & (x) = 0 \\ \Rightarrow & 6z \\ \Rightarrow & 6z \\ \end{vmatrix} $			$\begin{array}{cccc} y & (x) = 0 & \implies & 6x = 12 = 0 \\ & \implies & 6x = 12 \end{array}$			
$ \Rightarrow x = \frac{12}{6}  \Rightarrow x = 2 $		$ \Rightarrow  x = \frac{12}{6} \\ \Rightarrow  x = 2 $				
				= 2		
		Sign of $f''(x)$	2	1	Sign of $f''(w)$	
_	+	Sign of $f''(x)$	_	+	Sign of $f''(x)$	
		Kind of			Kind of	
$    \cap$	U	concavity		U	concavity	
Hence, the function <i>f</i>	f(x) has an inflecti		Hence, the function $f($	(x) is concave dow	,	
(2, -88).	(,				,_,_,	
$f(2) = (2)^3 - 6(2)^2$	- 36(2)					
= 8 - 24 - 72 90) The function <i>f</i> (2)	= -88					
90) The function $f(x)$	$x) = x^3 - 6x^2 - 3$	6 <i>x</i> concave	91) The critical numbers of the function			
upward on			$f(x) = -x^3 - 6x^2$	$x^{2} - 9x + 1$ are		
Solution:		0.4	Solution:			
	$x) = 3x^2 - 12x - x^2 - 12x $	36		$f'(x) = -3x^2 - 12x - 9$ $f'(x) = 0 \implies -3x^2 - 12x - 9 = 0$		
$f''(x) = 0 \implies 6$	f''(x) = 6x - 12			$x^2 - 12x - 9 \equiv 0$ $x^2 + 4x + 3) = 0$		
$ \begin{vmatrix} f & (x) = 0 \\ \Rightarrow & 6z \\ \Rightarrow & 6z \\ \end{vmatrix} $				$x^{+} + 4x + 3) = 0$ + 4x + 3 = 0		
				(x + 3)(x + 1) = 0		
$\Rightarrow x$	$=\frac{1}{6}$		-	x = -3 or $x = -1$		
$\Rightarrow x$	= 2					
2	2					
-	+	Sign of $f''(x)$				
		Kind of				
$    \cap$	U	concavity				
Hence, the function <i>f</i>	f(x) is concave upy					
92) The function $f(x)$	$x) = -x^3 - 6x^2 - 6x$	9x + 1 is	93) The function $f(x)$	$y = -x^3 - 6x^2 - 6x^2$	9x + 1 is	
decreasing on			increasing on			
Solution:			Solution:			
$f'(x) = -3x^2 - 12x - 9$			$y = -3x^2 - 12x - 12x$	- 9		
$\int f'(x) = 0  \Longrightarrow  -3$			$f'(x) = 0 \implies -3x$			
	$(x^2 + 4x + 3) = 0$	)		$x^2 + 4x + 3) = 0$		
	+4x + 3 = 0 + 2)(x + 1) = 0			+4x + 3 = 0		
	(x+3)(x+1) = 0 = -3 or $x = -2$	1		(x+3)(x+1) = 0 = -3 or $x = -1$		
$\rightarrow$ x -3	-3 or $x = -1$	L	$ \rightarrow x3 $	-3  01  x = -1	-	
	+ -	Sign of $f'(x)$	- +		Sign of $f'(x)$	
	* \	Kind of		× \	Kind of	
$\parallel$ $\backslash$ $\mid$ $\rangle$	$< \mid $	monotonicity	$\parallel$ $\backslash$ $\mid$ $/$		monotonicity	
Hence, the function $f(x)$ is decreasing on		Hence, the function $f($	(x) is increasing o	n (-3, -1).		
$(-\infty, -3) \cup (-1, \infty).$						

94) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has	95) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has		
a relative minimum value at the point	a relative maximum value at the point		
Solution:	Solution:		
$f'(x) = -3x^2 - 12x - 9$	$f'(x) = -3x^2 - 12x - 9$		
$f'(x) = 0 \implies -3x^2 - 12x - 9 = 0$	$f'(x) = 0 \implies -3x^2 - 12x - 9 = 0$		
$\Rightarrow -3(x^2 + 4x + 3) = 0$	$\Rightarrow -3(x^2 + 4x + 3) = 0$		
$\Rightarrow x^2 + 4x + 3 = 0$	$\Rightarrow x^2 + 4x + 3 = 0$		
$\Rightarrow$ $(x+3)(x+1) = 0$	$\Rightarrow$ $(x+3)(x+1) = 0$		
$\Rightarrow$ $x = -3$ or $x = -1$	$\Rightarrow$ $x = -3$ or $x = -1$		
- + $-$ Sign of $f'(x)$	- + $-$ Sign of $f'(x)$		
Kind of	Kind of		
monotonicity	monotonicity		
Hence, the function $f(x)$ has a relative minimum value at	Hence, the function $f(x)$ has a relative maximum value at		
the point $(-3,1)$ .	the point $(-1,5)$ .		
$f(-3) = -(-3)^3 - 6(-3)^2 - 9(-3) + 1$	$f(-1) = -(-1)^3 - 6(-1)^2 - 9(-1) + 1$		
= 27 - 54 + 27 + 1 = 1.	= 1 - 6 + 9 + 1 = 5.		
96) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ has an	97) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave		
inflection point at	downward on		
Solution:	Solution:		
$f'(x) = -3x^2 - 12x - 9$	$f'(x) = -3x^2 - 12x - 9$		
f''(x) = -6x - 12	f''(x) = -6x - 12		
$f''(x) = 0 \implies -6x - 12 = 0$	$f''(x) = 0  \Rightarrow  -6x - 12 = 0$		
$\Rightarrow -6x = 12$	$\Rightarrow -6x = 12$		
$\Rightarrow x = -\frac{12}{6}$	$\implies x = -\frac{12}{6}$		
$\Rightarrow x = -2^{6}$	$\Rightarrow x = -2^{6}$		
-2	-2		
+ - Sign of $f''(x)$	+ - Sign of $f''(x)$		
Kind of	Kind of		
U N concavity	U N concavity		
Hence, the function $f(x)$ has an inflection point at $(-2,3)$ .	Hence, the function $f(x)$ is concave downward on $(-2, \infty)$ .		
$f(-2) = -(-2)^3 - 6(-2)^2 - 9(-2) + 1$			
$= 8 - 24 + 18 + 1 = 3$ 98) The function $f(x) = -x^3 - 6x^2 - 9x + 1$ concave			
upward on			
Solution:			
$f'(x) = -3x^2 - 12x - 9$			
$f^{\prime\prime}(x) = -6x - 12$			
$f''(x) = 0  \Longrightarrow  -6x - 12 = 0$			
$\Rightarrow -6x = 12$			
$\implies x = -\frac{12}{6}$			
$\implies x = -2 \\ -2$			
-2 + - Sign of $f''(x)$			
Kind of			
U Nind Of concavity			
$I$ Hence the function $f(x)$ is conclude unward on $I = \infty$			
Hence, the function $f(x)$ is concave upward on $(-\infty, -2)$ .			