

Workshop Solutions to Chapter 4

1) If $f(x)$ is a differentiable function, then $f'(x) =$ <u>Solution:</u> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	2) If $f(x) = 4x^2$, then $f'(x) =$ <u>Solution:</u> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{4(x+h)^2 - 4x^2}{h}$
3) If $f(x) = x^2 - 3$, then $f'(x) =$ <u>Solution:</u> $\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{[(x+h)^2 - 3] - [x^2 - 3]}{h} \end{aligned}$	4) If $f(x) = \sqrt{x}$, $x \geq 0$, then $f'(x) =$ <u>Solution:</u> $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$
5) If f is a differentiable function at a, then f is a continuous function at a.	6) If f is a continuous function at a, then f is a differentiable function at a. <u>Solution:</u> False
7) If $y = x^4 + 5x^2 + 3$, then $y' =$ <u>Solution:</u> $y' = 4x^3 + 10x$	8) If $y = x^4 - 5x^2 + 3$, then $y' =$ <u>Solution:</u> $y' = 4x^3 - 10x$
9) If $y = x^{-5/2}$, then $y' =$ <u>Solution:</u> $y' = -\frac{5}{2}x^{-\frac{5}{2}-1} = -\frac{5}{2}x^{-7/2}$	10) If $y = \frac{1}{3x^3} + 2\sqrt{x} = \frac{1}{3}x^{-3} + 2x^{1/2}$, then $y' =$ <u>Solution:</u> $\begin{aligned} y' &= (-3)\left(\frac{1}{3}\right)x^{-3-1} + \left(\frac{1}{2}\right)(2)x^{\frac{1}{2}-1} \\ &= -x^{-4} + x^{-1/2} = -\frac{1}{x^4} + \frac{1}{x^{1/2}} = -\frac{1}{x^4} + \frac{1}{\sqrt{x}} \end{aligned}$
11) If $y = (x-3)(x-2)$, then $y' =$ <u>Solution:</u> $\begin{aligned} y &= (x-3)(x-2) = x^2 - 5x + 6 \\ y' &= 2x - 5 \end{aligned}$	12) If $y = (x^3 + 3)(x^2 - 1)$, then $y' =$ <u>Solution:</u> $\begin{aligned} y &= (x^3 + 3)(x^2 - 1) = x^5 - x^3 + 3x^2 - 3 \\ y' &= 5x^4 - 3x^2 + 6x \end{aligned}$
13) If $y = \sqrt{x}(2x+1)$, then $y' =$ <u>Solution:</u> $\begin{aligned} y &= \sqrt{x}(2x+1) = 2x\sqrt{x} + \sqrt{x} = 2x^{\frac{3}{2}} + x^{\frac{1}{2}} \\ y' &= \left(\frac{3}{2}\right)(2)x^{\frac{3}{2}-1} + \left(\frac{1}{2}\right)x^{\frac{1}{2}-1} = 3x^{\frac{1}{2}} + \frac{1}{2}x^{-\frac{1}{2}} \\ &= 3\sqrt{x} + \frac{1}{2\sqrt{x}} \end{aligned}$ OR Use the rule $(f \cdot g)' = f'g + fg'$ $y' = (2)(\sqrt{x}) + \left(\frac{1}{2\sqrt{x}}\right)(2x+1) = 2\sqrt{x} + \frac{2x+1}{2\sqrt{x}}$	14) If $y = \frac{x+3}{x-2}$, then $y' =$ <u>Solution:</u> Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ $\begin{aligned} y' &= \frac{(1)(x-2) - (x+3)(1)}{(x-2)^2} = \frac{x-2-x-3}{(x-2)^2} = \frac{-5}{(x-2)^2} \\ &= -\frac{5}{(x-2)^2} \end{aligned}$
15) If $y = \frac{x+3}{x-2}$, then $y' _{x=4} =$ <u>Solution:</u> $\begin{aligned} y' &= \frac{(1)(x-2) - (x+3)(1)}{(x-2)^2} = \frac{x-2-x-3}{(x-2)^2} \\ &= \frac{-5}{(x-2)^2} = -\frac{5}{(x-2)^2} \\ y' _{x=4} &= -\frac{5}{(4-2)^2} = -\frac{5}{4} \end{aligned}$	16) If $y = \frac{x-1}{x+2}$, then $y' =$ <u>Solution:</u> Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ $y' = \frac{(1)(x+2) - (x-1)(1)}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$

17) If $y = \sqrt{3x^2 + 6x}$, then $y' =$ <u>Solution:</u> Use the rule $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$ $y' = \frac{6x + 6}{2\sqrt{3x^2 + 6x}} = \frac{6(x + 1)}{2\sqrt{3x^2 + 6x}} = \frac{3(x + 1)}{\sqrt{3x^2 + 6x}}$	18) If $y = \sqrt{3x^2 + 6x}$, then $y' _{x=1} =$ <u>Solution:</u> $y' = \frac{6x + 6}{2\sqrt{3x^2 + 6x}} = \frac{6(x + 1)}{2\sqrt{3x^2 + 6x}} = \frac{3(x + 1)}{\sqrt{3x^2 + 6x}}$ $y' _{x=1} = \frac{3((1) + 1)}{\sqrt{3(1)^2 + 6(1)}} = \frac{6}{\sqrt{9}} = \frac{6}{3} = 2$
19) The tangent line equation to the curve $y = x^2 + 2$ at the point (1,3) is <u>Solution:</u> First, we have to find the slope of the curve which is $y' = 2x$ Thus, the slope at $x = 1$ is $y' _{x=1} = 2(1) = 2$ Hence, the tangent line equation passing through the point (1,3) with slope $m = 2$ is $y - 3 = 2(x - 1)$ $y - 3 = 2x - 2$ $y = 2x - 2 + 3$ $y = 2x + 1$	20) The tangent line equation to the curve $y = \frac{2x}{x+1}$ at the point (0,0) is <u>Solution:</u> First, we have to find the slope of the curve which is $y' = \frac{(2)(x + 1) - (2x)(1)}{(x + 1)^2} = \frac{2x + 2 - 2x}{(x + 1)^2} = \frac{2}{(x + 1)^2}$ Thus, the slope at $x = 0$ is $y' _{x=0} = \frac{2}{(0 + 1)^2} = 2$ Hence, the tangent line equation passing through the point (0,0) with slope $m = 2$ is $y - 0 = (2)(x - 0)$ $y = 2x$
21) The tangent line equation to the curve $y = 3x^2 - 13$ at the point (2, -1) is <u>Solution:</u> First, we have to find the slope of the curve which is $y' = 6x$ Thus, the slope at $x = 2$ is $y' _{x=2} = 6(2) = 12$ Hence, the tangent line equation passing through the point (2, -1) with slope $m = 12$ is $y - (-1) = 12(x - 2)$ $y + 1 = 12x - 24$ $y = 12x - 24 - 1$ $y = 12x - 25$	22) The tangent line equation to the curve $y = 3x^2 + 2x + 5$ at the point (0,5) is <u>Solution:</u> First, we have to find the slope of the curve which is $y' = 6x + 2$ Thus, the slope at $x = 2$ is $y' _{x=0} = 6(0) + 2 = 2$ Hence, the tangent line equation passing through the point (0,5) with slope $m = 2$ is $y - 5 = 2(x - 0)$ $y - 5 = 2x$ $y = 2x + 5$
23) If $y = xe^x$, then $y' =$ <u>Solution:</u> Use the rules $(f \cdot g)' = f'g + fg'$ and $(e^u) = e^u \cdot u'$ $y' = (1)(e^x) + (x)(e^x) = e^x + xe^x = e^x(1 + x)$	24) If $y = x - e^x$, then $y'' =$ <u>Solution:</u> Use the rules $(f - g)' = f' - g'$ and $(e^u) = e^u \cdot u'$ $y' = 1 - e^x$ $y'' = -e^x$
25) If $x^2 - y^2 = 4$, then $y' =$ <u>Solution:</u> $2x - 2yy' = 0$ $-2yy' = -2x$ $y' = \frac{-2x}{-2y}$ $y' = \frac{x}{y}$	26) If $x^2 + y^2 = 4$, then $y' =$ <u>Solution:</u> $2x + 2yy' = 0$ $2yy' = -2x$ $y' = \frac{-2x}{2y}$ $y' = -\frac{x}{y}$

27) If $y = \frac{x+1}{x+2}$, then $y' =$ <u>Solution:</u> Use the rule $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ $y' = \frac{(1)(x+2) - (x+1)(1)}{(x+2)^2} = \frac{x+2-x-1}{(x+2)^2}$ $= \frac{1}{(x+2)^2}$	28) If $y = \frac{1}{\sqrt[2]{x^5}} + \sec x$, then $y' =$ <u>Solution:</u> Use the rules $(f+g)' = f' + g'$ and $(\sec u)' = \sec u \tan u \cdot u'$ $y = \frac{1}{\sqrt[2]{x^5}} + \sec x = x^{-\frac{5}{2}} + \sec x$ $y' = \left(-\frac{5}{2}\right)x^{-\frac{5}{2}-1} + \sec x \tan x = -\frac{5}{2}x^{-7/2} + \sec x \tan x$
29) If $y = \tan^{-1}(x^3)$, then $y' =$ <u>Solution:</u> Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1+(x^3)^2} \cdot (3x^2) = \frac{3x^2}{1+x^6}$	30) If $y = \tan x - x$, then $y' =$ <u>Solution:</u> Use the rules $(f-g)' = f' - g'$ and $(\tan u)' = \sec^2 u \cdot u'$ $y' = \sec^2 x - 1$
31) If $y = \sec^2 x - 1$, then $y' =$ <u>Solution:</u> Use the rules $(f-g)' = f' - g'$, $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sec u)' = \sec u \tan u \cdot u'$ $y' = 2 \sec x \cdot \sec x \tan x = 2 \sec^2 x \tan x$	32) If $y = x^{\sin x}$, then $y' =$ <u>Solution:</u> Use the rule $(\sin u)' = \cos u \cdot u'$ $y = x^{\sin x}$ $\ln y = \ln x^{\sin x}$ $\ln y = \sin x \cdot \ln x$ $\frac{y'}{y} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x} = \cos x \cdot \ln x + \frac{\sin x}{x}$ $y' = y \left(\cos x \cdot \ln x + \frac{\sin x}{x} \right) = x^{\sin x} \left(\cos x \cdot \ln x + \frac{\sin x}{x} \right)$
33) If $y = x^{\cos x}$, then $y' =$ <u>Solution:</u> Use the rule $(\cos u)' = -\sin u \cdot u'$ $y = x^{\cos x}$ $\ln y = \ln x^{\cos x}$ $\ln y = \cos x \cdot \ln x$ $\frac{y'}{y} = -\sin x \cdot \ln x + \cos x \cdot \frac{1}{x} = -\sin x \cdot \ln x + \frac{\cos x}{x}$ $y' = y \left(-\sin x \cdot \ln x + \frac{\cos x}{x} \right)$ $= x^{\cos x} \left(\frac{\cos x}{x} - \sin x \cdot \ln x \right)$	34) If $y = (2x^2 + \csc x)^9$, then $y' =$ <u>Solution:</u> Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\csc u)' = -\csc u \cot u \cdot u'$ $y' = 9(2x^2 + \csc x)^8 \cdot (4x - \csc x \cot x)$
35) If $y = \frac{5^x}{\cot x}$, then $y' =$ <u>Solution:</u> Use the rules $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}, \quad (a^u)' = a^u \cdot \ln a \cdot u'$ and $(\csc u)' = -\csc u \cot u \cdot u'$ $y' = \frac{(5^x \ln 5)(\cot x) - (5^x)(-\csc^2 x)}{(\cot x)^2}$ $= \frac{5^x(\ln 5 \cot x + \csc^2 x)}{\cot^2 x}$	36) If $y = e^{2x}$, then $y^{(6)} =$ <u>Solution:</u> Use the rule $(e^u)' = e^u \cdot u'$ $y' = 2e^{2x}$ $y'' = 4e^{2x}$ $y''' = 8e^{2x}$ $y^{(4)} = 16e^{2x}$ $y^{(5)} = 32e^{2x}$ $y^{(6)} = 64e^{2x}$

37) If $y = x^{-2}e^{\sin x}$, then $y' =$ <u>Solution:</u> Use the rules $(f \cdot g)' = f'g + fg'$, $(e^u)' = e^u \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $ \begin{aligned} y' &= (-2x^{-3})(e^{\sin x}) + (x^{-2})(e^{\sin x} \cdot \cos x) \\ &= -2x^{-3}e^{\sin x} + x^{-2} \cos x e^{\sin x} \\ &= x^{-3}e^{\sin x}(-2 + x \cos x) \\ &= x^{-3}e^{\sin x}(x \cos x - 2) \end{aligned} $	38) If $y = 5^{\tan x}$, then $y' =$ <u>Solution:</u> Use the rules $(a^u)' = a^u \cdot \ln a \cdot u'$ and $(\tan u)' = \sec^2 u \cdot u'$ $y' = 5^{\tan x} \cdot \ln 5 \cdot \sec^2 x$
39) If $x^2 + y^2 = 3xy + 7$, then $y' =$ <u>Solution:</u> $ \begin{aligned} 2x + 2yy' &= 3y + 3xy' \\ 2yy' - 3xy' &= 3y - 2x \\ y'(2y - 3x) &= 3y - 2x \\ y' &= \frac{3y - 2x}{2y - 3x} \end{aligned} $	40) If $y = \sin^3(4x)$, then $y^{(6)} =$ <u>Solution:</u> Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $ \begin{aligned} y' &= 3 \sin^2(4x) \cdot \cos(4x) \cdot (4) \\ &= 12 \sin^2(4x) \cdot \cos(4x) \end{aligned} $
41) If $y = 3^x \cot x$, then $y' =$ <u>Solution:</u> Use the rules $(f \cdot g)' = f'g + fg'$, $(a^u)' = a^u \cdot \ln a \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $ \begin{aligned} y' &= (3^x \cdot \ln 3)(\cot x) + (3^x)(-\csc^2 x) \\ &= 3^x \ln 3 \cot x - 3^x \csc^2 x \\ &= 3^x(\ln 3 \cot x - \csc^2 x) \end{aligned} $	42) If $y = (2x^2 + \sec x)^7$, then $y' =$ <u>Solution:</u> Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sec u)' = \sec u \tan u \cdot u'$ $y' = 7(2x^2 + \sec x)^6 \cdot (4x + \sec x \tan x)$
43) If $f(x) = \cos x$, then $f^{(45)}(x) =$ <u>Solution:</u> $ \begin{aligned} f'(x) &= -\sin x \\ f''(x) &= -\cos x \\ f'''(x) &= \sin x \\ f^{(4)}(x) &= \cos x \end{aligned} $ Note: $f^{(n)}(x) = \cos x$ whenever n is a multiple of 4. Hence, $ \begin{aligned} f^{(44)}(x) &= \cos x \\ f^{(45)}(x) &= -\sin x \end{aligned} $	44) If $D^{47}(\sin x) =$ <u>Solution:</u> $ \begin{aligned} D(\sin x) &= \cos x \\ D^2(\sin x) &= -\sin x \\ D^3(\sin x) &= -\cos x \\ D^4(\sin x) &= \sin x \end{aligned} $ Note: $D^n(\sin x) = \sin x$ whenever n is a multiple of 4. Hence, $ \begin{aligned} D^{44}(\sin x) &= \sin x \\ D^{45}(\sin x) &= \cos x \\ D^{46}(\sin x) &= -\sin x \\ D^{47}(\sin x) &= -\cos x \end{aligned} $
45) If $y = x^x$, then $y' =$ <u>Solution:</u> Use the rule $(\ln u)' = \frac{u'}{u}$ $ \begin{aligned} y &= x^x \\ \ln y &= \ln x^x \\ \ln y &= x \ln x \\ \frac{y'}{y} &= (1)(\ln x) + (x)\left(\frac{1}{x}\right) \\ \frac{y'}{y} &= \ln x + 1 \\ y' &= y(1 + \ln x) = x^x(1 + \ln x) \end{aligned} $	46) If $f(x) = \frac{\ln x}{x^2}$, then $f'(1) =$ <u>Solution:</u> Use the rules $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ and $(\ln u)' = \frac{u'}{u}$ $ \begin{aligned} f'(x) &= \frac{\left(\frac{1}{x}\right)(x^2) - (\ln x)(2x)}{(x^2)^2} = \frac{x - 2x \ln x}{x^4} \\ &= \frac{x(1 - 2 \ln x)}{x^4} = \frac{1 - 2 \ln x}{x^3} \\ \therefore f'(1) &= \frac{1 - 2 \ln(1)}{(1)^3} = \frac{1 - 2(0)}{1} = 1 \end{aligned} $
47) If $y = \cot^{-1}(e^x)$, then $y' =$ <u>Solution:</u> Use the rules $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ and $(e^u)' = e^u \cdot u'$ $y' = -\frac{1}{1 + (e^x)^2} \cdot e^x = -\frac{e^x}{1 + e^{2x}}$	48) If $y = \tan^{-1}(e^x)$, then $y' =$ <u>Solution:</u> Use the rules $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ and $(e^u)' = e^u \cdot u'$ $y' = \frac{1}{1 + (e^x)^2} \cdot e^x = \frac{e^x}{1 + e^{2x}}$

49) If $y = \sin^{-1}(e^x)$, then $y' =$ <u>Solution:</u> Use the rules $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ and $(e^u)' = e^u \cdot u'$ $y' = \frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = \frac{e^x}{\sqrt{1-e^{2x}}}$	50) If $y = \cos^{-1}(e^x)$, then $y' =$ <u>Solution:</u> Use the rules $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ and $(e^u)' = e^u \cdot u'$ $y' = -\frac{1}{\sqrt{1-(e^x)^2}} \cdot e^x = -\frac{e^x}{\sqrt{1-e^{2x}}}$
51) If $y = \cos(2x^3)$, then $y' =$ <u>Solution:</u> Use the rule $(\cos u)' = -\sin u \cdot u'$ $y' = -\sin(2x^3) \cdot (6x^2) = -6x^2 \sin(2x^3)$	52) If $y = \csc x \cot x$, then $y' =$ <u>Solution:</u> Use the rules $(f \cdot g)' = f'g + fg'$, $(\csc u)' = -\csc u \cot u \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = (-\csc x \cot x)(\cot x) + (\csc x)(-\csc^2 x)$ $= -\csc x \cot^2 x - \csc^3 x = -\csc x(\cot^2 x + \csc^2 x)$
53) If $y = \sqrt{x^2 - 2 \sec x}$, then $y' =$ <u>Solution:</u> Use the rules $(\sqrt{u})' = \frac{u'}{2\sqrt{u}} \quad \text{and} \quad (\sec u)' = \sec u \tan u \cdot u'$ $y' = \frac{2x - 2 \sec x \tan x}{2\sqrt{x^2 - 2 \sec x}} = \frac{2(x - \sec x \tan x)}{2\sqrt{x^2 - 2 \sec x}}$ $= \frac{x - \sec x \tan x}{\sqrt{x^2 - 2 \sec x}}$	54) If $y = (3x^2 + 1)^6$, then $y' =$ <u>Solution:</u> Use the rule $(u)^n = n(u)^{n-1} \cdot u'$ $y' = 6(3x^2 + 1)^5 \cdot (6x) = 36x(3x^2 + 1)^5$
55) If $xy + \tan x = 2x^3 + \sin y$, then $y' =$ <u>Solution:</u> $[(1)(y) + (x)(y')]' + \sec^2 x = 6x^2 + \cos y \cdot y'$ $y + xy' + \sec^2 x = 6x^2 + y' \cos y$ $xy' - y' \cos y = 6x^2 - y - \sec^2 x$ $y'(x - \cos y) = 6x^2 - y - \sec^2 x$ $y' = \frac{6x^2 - y - \sec^2 x}{x - \cos y}$	56) If $y = x^{-1} \sec x$, then $y' =$ <u>Solution:</u> Use the rules $(f \cdot g)' = f'g + fg'$ and $(\sec u)' = \sec u \tan u \cdot u'$ $y' = (-x^{-2})(\sec x) + (x^{-1})(\sec x \tan x)$ $= x^{-1} \sec x \tan x - x^{-2} \sec x$ $= x^{-2} \sec x (x \tan x - 1)$
57) If $y = \sin^{-1}(x^3)$, then $y' =$ <u>Solution:</u> Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ $y' = \frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2 = \frac{3x^2}{\sqrt{1-x^6}}$	58) If $y = \cos^{-1}(x^3)$, then $y' =$ <u>Solution:</u> Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ $y' = -\frac{1}{\sqrt{1-(x^3)^2}} \cdot 3x^2 = -\frac{3x^2}{\sqrt{1-x^6}}$
59) If $y = \sec^{-1}(x^3)$, then $y' =$ <u>Solution:</u> Use the rule $(\sec^{-1} u)' = \frac{u'}{ u \sqrt{u^2-1}}$ $y' = \frac{1}{x^3 \sqrt{(x^3)^2 - 1}} \cdot 3x^2 = \frac{3x^2}{x^3 \sqrt{x^6 - 1}} = \frac{3}{x \sqrt{x^6 - 1}}$	60) If $y = \csc^{-1}(x^3)$, then $y' =$ <u>Solution:</u> Use the rule $(\csc^{-1} u)' = -\frac{u'}{ u \sqrt{u^2-1}}$ $y' = -\frac{1}{x^3 \sqrt{(x^3)^2 - 1}} \cdot 3x^2 = -\frac{3x^2}{x^3 \sqrt{x^6 - 1}} = -\frac{3}{x \sqrt{x^6 - 1}}$

61) If $y = \ln(x^3 - 2 \sec x)$, then $y' =$ <u>Solution:</u> Use the rules $(\ln u)' = \frac{u'}{u} \quad \text{and} \quad (\sec u)' = \sec u \tan u \cdot u'$ $y' = \frac{1}{x^3 - 2 \sec x} \cdot (3x^2 - 2 \sec x \tan x)$ $= \frac{3x^2 - 2 \sec x \tan x}{x^3 - 2 \sec x}$	62) If $y = \ln(\cos x)$, then $y' =$ <u>Solution:</u> Use the rules $(\ln u)' = \frac{u'}{u} \quad \text{and} \quad (\cos u)' = -\sin u \cdot u'$ $y' = \frac{1}{\cos x} \cdot (-\sin x) = -\frac{\sin x}{\cos x} = -\tan x$
63) If $y = \ln(\sin x)$, then $y' =$ <u>Solution:</u> Use the rules $(\ln u)' = \frac{u'}{u} \quad \text{and} \quad (\sin u)' = \cos u \cdot u'$ $y' = \frac{1}{\sin x} \cdot (\cos x) = \frac{\cos x}{\sin x} = \cot x$	64) If $y = \ln \sqrt{3x^2 + 5x}$, then $y' =$ <u>Solution:</u> Use the rules $(\ln u)' = \frac{u'}{u}$ and $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$ $y' = \frac{1}{\sqrt{3x^2 + 5x}} \cdot \left(\frac{6x + 5}{2\sqrt{3x^2 + 5x}} \right) = \frac{6x + 5}{2(3x^2 + 5x)}$
65) If $y = \log_5(x^3 - 2 \csc x)$, then $y' =$ <u>Solution:</u> Use the rules $(\log_a u)' = \frac{u'}{u \ln a} \quad \text{and} \quad (\csc u)' = -\csc u \cot u \cdot u'$ $y' = \frac{1}{(x^3 - 2 \csc x)(\ln 5)} \cdot [3x^2 - 2(-\csc x \cot x)]$ $= \frac{3x^2 + 2 \csc x \cot x}{(x^3 - 2 \csc x)(\ln 5)}$	66) If $y = \ln \frac{x-1}{\sqrt{x+2}}$, then $y' =$ <u>Solution:</u> Use the rules $(\ln u)' = \frac{u'}{u}$, $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$ and $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$ $y' = \frac{1}{\frac{x-1}{\sqrt{x+2}}} \cdot \left(\frac{(1)(\sqrt{x+2}) - (x-1)\left(\frac{1}{2\sqrt{x+2}}\right)}{(\sqrt{x+2})^2} \right)$ $= \frac{\sqrt{x+2}}{x-1} \cdot \left(\frac{\sqrt{x+2} - \frac{x-1}{2\sqrt{x+2}}}{x+2} \right)$ $= \frac{\sqrt{x+2}}{x-1} \cdot \left(\frac{2(x+2) - (x-1)}{2\sqrt{x+2}(x+2)} \right)$ $= \frac{\sqrt{x+2}}{x-1} \cdot \left(\frac{x+5}{2\sqrt{x+2}(x+2)} \right)$ $= \frac{x+5}{2(x-1)(x+2)}$
67) If $y = 2x^3 - \sin x$, then $y' =$ <u>Solution:</u> Use the rule $(\sin u)' = \cos u \cdot u'$ $y' = 6x^2 - \cos x$	
68) If $y = x^3 \cos x$, then $y' =$ <u>Solution:</u> Use the rules $(f \cdot g)' = f'g + fg'$ and $(\cos u)' = -\sin u \cdot u'$ $y' = (3x^2)(\cos x) + (x^3)(-\sin x)$ $= 3x^2 \cos x - x^3 \sin x$	

69) If $y = x^{\sqrt{x}}$, then $y' =$ <u>Solution:</u> Use the rule $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$ $y = x^{\sqrt{x}}$ $\ln y = \ln x^{\sqrt{x}}$ $\ln y = \sqrt{x} \ln x$ $\frac{y'}{y} = \left(\frac{1}{2\sqrt{x}}\right)(\ln x) + (\sqrt{x})\left(\frac{1}{x}\right)$ $\frac{y'}{y} = \frac{\ln x}{2\sqrt{x}} + \frac{\sqrt{x}}{x} = \frac{x \ln x + 2x}{2x\sqrt{x}} = \frac{x(\ln x + 2)}{2x\sqrt{x}}$ $= \frac{\ln x + 2}{2\sqrt{x}}$ $y' = y \left(\frac{\ln x + 2}{2\sqrt{x}}\right) = x^{\sqrt{x}} \left(\frac{\ln x + 2}{2\sqrt{x}}\right)$	70) If $y = (\sin x)^x$, then $y' =$ <u>Solution:</u> Use the rule $(\sin u)' = \cos u \cdot u'$ $y = (\sin x)^x$ $\ln y = \ln(\sin x)^x$ $\ln y = x \ln(\sin x)$ $\frac{y'}{y} = (1)(\ln(\sin x)) + (x) \left(\frac{\cos x}{\sin x}\right)$ $\frac{y'}{y} = \ln(\sin x) + \frac{x \cos x}{\sin x} = \ln(\sin x) + x \cot x$ $y' = y(\ln(\sin x) + x \cot x)$ $= (\sin x)^x (\ln(\sin x) + x \cot x)$
71) If $y = \log_7(x^3 - 2)$, then $y' =$ <u>Solution:</u> Use the rule $(\log_a u)' = \frac{u'}{u \ln a}$ $y' = \frac{1}{(x^3 - 2)(\ln 7)} \cdot (3x^2) = \frac{3x^2}{(x^3 - 2)(\ln 7)}$	72) If $y = \cos(x^5)$, then $y' =$ <u>Solution:</u> Use the rule $(\cos u)' = -\sin u \cdot u'$ $y' = -\sin(x^5) \cdot (5x^4) = -5x^4 \sin(x^5)$
73) If $y = \sec x \tan x$, then $y' =$ <u>Solution:</u> $(f \cdot g)' = f'g + fg'$, $(\sec u)' = \sec u \tan u \cdot u'$ and $(\tan u)' = \sec^2 u \cdot u'$ $y' = (\sec x \tan x)(\tan x) + (\sec x)(\sec^2 x)$ $= \sec x \tan^2 x + \sec^3 x = \sec x(\tan^2 x + \sec^2 x)$	74) If $D^{99}(\cos x) =$ <u>Solution:</u> $D(\cos x) = -\sin x$ $D^2(\cos x) = -\cos x$ $D^3(\cos x) = \sin x$ $D^4(\cos x) = \cos x$ Note: $D^n(\cos x) = \cos x$ whenever n is a multiple of 4. Hence, $D^{96}(\cos x) = \cos x$ $D^{97}(\cos x) = -\sin x$ $D^{98}(\cos x) = -\cos x$ $D^{99}(\cos x) = \sin x$
75) If $y = (x + \sec x)^3$, then $y' =$ <u>Solution:</u> Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sec u)' = \sec u \tan u \cdot u'$ $y' = 3(x + \sec x)^2 \cdot (1 + \sec x \tan x)$	76) If $x^2 = 5y^2 + \sin y$, then $y' =$ <u>Solution:</u> $2x = 10yy' + \cos y \cdot y'$ $y'(10y + \cos y) = 2x$ $y' = \frac{2x}{10y + \cos y}$
77) If $x^2 - 5y^2 + \sin y = 0$, then $y' =$ <u>Solution:</u> $2x - 10yy' + \cos y \cdot y' = 0$ $y'(-10y + \cos y) = -2x$ $y' = \frac{-2x}{-10y + \cos y} = \frac{2x}{10y - \cos y}$	78) If $y = \sin x \sec x$, then $y' =$ <u>Solution:</u> $(f \cdot g)' = f'g + fg'$, $(\sin u)' = \cos u \cdot u'$ and $(\sec u)' = \sec u \tan u \cdot u'$ $y' = (\cos x)(\sec x) + (\sin x)(\sec x \tan x)$ $= 1 + \sin x \cdot \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = 1 + \frac{\sin^2 x}{\cos^2 x} = 1 + \tan^2 x$ $= \sec^2 x$

79) If $f(x) = \sin^2(x^3 + 1)$, then $f'(x) =$ <u>Solution:</u> Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\sin u)' = \cos u \cdot u'$ $f'(x) = 2 \sin(x^3 + 1) \cdot (\cos(x^3 + 1)) \cdot (3x^2)$ $= 6x^2 \sin(x^3 + 1) \cos(x^3 + 1)$	80) If $y = (x + \cot x)^3$, then $y' =$ <u>Solution:</u> Use the rules $(u)^n = n(u)^{n-1} \cdot u'$ and $(\cot u)' = -\csc^2 u \cdot u'$ $y' = 3(x + \cot x)^2 \cdot (1 - \csc^2 x)$
81) If $y = \tan^{-1}\left(\frac{x}{2}\right)$, then $y' =$ <u>Solution:</u> Use the rule $(\tan^{-1} u)' = \frac{u'}{1+u^2}$ $y' = \frac{1}{1 + \left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = \frac{1}{2\left(1 + \frac{x^2}{4}\right)} = \frac{1}{2\left(\frac{4+x^2}{4}\right)} = \frac{2}{4+x^2}$	82) If $y = \cot^{-1}\left(\frac{x}{2}\right)$, then $y' =$ <u>Solution:</u> Use the rule $(\cot^{-1} u)' = -\frac{u'}{1+u^2}$ $y' = -\frac{1}{1 + \left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} = -\frac{1}{2\left(1 + \frac{x^2}{4}\right)} = -\frac{1}{2\left(\frac{4+x^2}{4}\right)} = -\frac{2}{4+x^2}$
83) If $y = \sin^{-1}\left(\frac{x}{3}\right)$, then $y' =$ <u>Solution:</u> Use the rule $(\sin^{-1} u)' = \frac{u'}{\sqrt{1-u^2}}$ $y' = \frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} = \frac{1}{3\sqrt{1 - \frac{x^2}{9}}} = \frac{1}{3\sqrt{\frac{9-x^2}{9}}} = \frac{1}{\sqrt{9-x^2}}$	84) If $y = \cos^{-1}\left(\frac{x}{3}\right)$, then $y' =$ <u>Solution:</u> Use the rule $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ $y' = -\frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3} = -\frac{1}{3\sqrt{1 - \frac{x^2}{9}}} = -\frac{1}{3\sqrt{\frac{9-x^2}{9}}} = -\frac{1}{\sqrt{9-x^2}}$
85) If $D^{99}(\sin x) =$ <u>Solution:</u> $D(\sin x) = \cos x$ $D^2(\sin x) = -\sin x$ $D^3(\sin x) = -\cos x$ $D^4(\sin x) = \sin x$ Note: $D^n(\sin x) = \sin x$ whenever n is a multiple of 4. Hence, $D^{96}(\sin x) = \sin x$ $D^{97}(\sin x) = \cos x$ $D^{98}(\sin x) = -\sin x$ $D^{99}(\sin x) = -\cos x$	