Instructor Problems:

Q1- If $\vec{a} = -\hat{\imath} + 2\hat{\jmath}$ and $\vec{b} = 5\hat{\imath} + 3\hat{\jmath}$ Find: $|a|$, $|b|$, $\vec{a}$+$\vec{b}$, $|\vec{a}$+$\vec{b}|$, θ_{a+b}, $\vec{a}$-$\vec{b}$, $|\vec{a}$-$\vec{b}|$, θ_{a-b} , $3\vec{a}$+$2\vec{b}$, $|3\vec{a}$+$2\vec{b}|$, θ_{3a+2b}

Q2- If $\vec{a} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ and $\vec{b} = 5\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$ Find:

a) The angle between two vectors.

b) A unit vector that in the same direction of $\vec{a}$.

c) A unit vector that in the same direction of $\vec{b}$.

d) the scalar and vector projection of $\vec{b}$ along $\vec{a}$.

e) the scalar and vector projection of $\vec{a}$ along $\vec{b}$.

Q3- If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$, $\vec{b} = \hat{\imath} - 2\hat{\jmath} - 2\hat{k}$ and $\vec{c} = 3\hat{\imath} + 2\hat{\jmath}$, Show that:

a) $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

b) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

c) $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \times \vec{c}$

Chapter (3): Linear Equations; Vectors, Matrices, and Determinants

Section (4) P. (104-105): 9, 10, 12, 13, 15(a), 18, 20

PROBLEMS, SECTION 4

9. Let $\mathbf{A} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{B} = 4\mathbf{i} - 5\mathbf{j}$. Show graphically, and find algebraically, the vectors $-\mathbf{A}$, $3\mathbf{B}$, $\mathbf{A} - \mathbf{B}$, $\mathbf{B} + 2\mathbf{A}$, $\frac{1}{2}(\mathbf{A} + \mathbf{B})$.

10. If $\mathbf{A} + \mathbf{B} = 4\mathbf{j} - \mathbf{i}$ and $\mathbf{A} - \mathbf{B} = \mathbf{i} + 3\mathbf{j}$, find $\mathbf{A}$ and $\mathbf{B}$ algebraically. Show by a diagram how to find $\mathbf{A}$ and $\mathbf{B}$ geometrically.

12. Find the angle between the vectors $\mathbf{A} = -2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$ and $\mathbf{B} = 2\mathbf{i} - 2\mathbf{j}$.

13. If $\mathbf{A} = 4\mathbf{i} - 3\mathbf{k}$ and $\mathbf{B} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, find the scalar projection of $\mathbf{A}$ on $\mathbf{B}$, the scalar projection of $\mathbf{B}$ on $\mathbf{A}$, and the cosine of the angle between $\mathbf{A}$ and $\mathbf{B}$.

15. Let $\mathbf{A} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. (a) Find a *unit* vector in the same direction as $\mathbf{A}$. *Hint:* Divide $\mathbf{A}$ by $|\mathbf{A}|$.

18. Show that $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ and $5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ are orthogonal (perpendicular). Find a third vector perpendicular to both.

20. Find a vector perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} - 2\mathbf{k}$.

Chapter (6): Vector Analysis

Section (3) P. (242): 1

PROBLEMS, SECTION 3

1. If $\mathbf{A} = 2\mathbf{i} - \mathbf{j} - \mathbf{k}$, $\mathbf{B} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{C} = \mathbf{j} + \mathbf{k}$, find $(\mathbf{A} \cdot \mathbf{B})\mathbf{C}$, $\mathbf{A}(\mathbf{B} \cdot \mathbf{C})$, $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$, $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$, $(\mathbf{A} \times \mathbf{B}) \times \mathbf{C}$, $\mathbf{A} \times (\mathbf{B} \times \mathbf{C})$.

Section (4) P. (247): 8

8. In polar coordinates, the position vector of a particle is $\mathbf{r} = r\mathbf{e}_r$. Using (4.13), find the velocity and acceleration of the particle.

Supporting Materials:

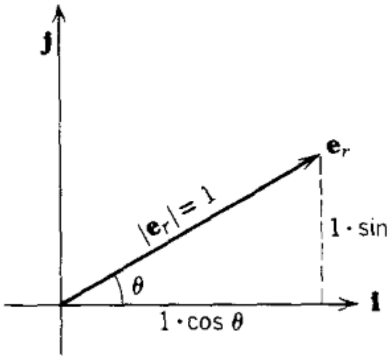

FIGURE 4.3

One straightforward way to do this is to express the unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$ in terms of i and j. From Figure 4.3, we see that the x and y components of $\mathbf{e}_r$ are $\cos\theta$ and $\sin\theta$. Thus we have

(4.11) $$\mathbf{e}_r = \mathbf{i}\cos\theta + \mathbf{j}\sin\theta.$$

Similarly (Problem 7) we find

(4.12) $$\mathbf{e}_\theta = -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta.$$

Differentiating $\mathbf{e}_r$ and $\mathbf{e}_\theta$ with respect to t, we get

(4.13)
$$\frac{d\mathbf{e}_r}{dt} = -\mathbf{i}\sin\theta\frac{d\theta}{dt} + \mathbf{j}\cos\theta\frac{d\theta}{dt} = \mathbf{e}_\theta\frac{d\theta}{dt},$$
$$\frac{d\mathbf{e}_\theta}{dt} = -\mathbf{i}\cos\theta\frac{d\theta}{dt} - \mathbf{j}\sin\theta\frac{d\theta}{dt} = -\mathbf{e}_r\frac{d\theta}{dt}.$$

Section (6) P. (252): 1, 3, 5, 9 (a &b), 20

PROBLEMS, SECTION 6

1. Find the gradient of $w = x^2y^3z$ at $(1, 2, -1)$.

3. Find the derivative of $xy^2 + yz$ at $(1, 1, 2)$ in the direction of the vector $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$.

5. Find the gradient of $\phi = z \sin y - xz$ at the point $(2, \pi/2, -1)$. Starting at this point, in what direction is ϕ *decreasing* most rapidly? Find the derivative of ϕ in the direction $2\mathbf{i} + 3\mathbf{j}$.

9. (a) Given $\phi = x^2 - y^2z$, find $\nabla\phi$ at $(1, 1, 1)$.
   (b) Find the directional derivative of ϕ at $(1, 1, 1)$ in the direction $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

find the following gradients in two ways and show that your answers are equivalent.

20. $\nabla(r^2)$

where $r = \sqrt{x^2 + y^2}$, using (6.7) and also using (6.3). Show that your results are the same by using (4.11) and (4.12).

Supporting Materials:

$$(6.7) \qquad \nabla\phi = \mathbf{e}_r \frac{\partial \phi}{\partial r} + \mathbf{e}_\theta \frac{1}{r} \frac{\partial \phi}{\partial \theta}.$$

$$(6.3) \quad \nabla\phi = \text{grad } \phi = \mathbf{i}\frac{\partial \phi}{\partial x} + \mathbf{j}\frac{\partial \phi}{\partial y} + \mathbf{k}\frac{\partial \phi}{\partial z}.$$

And

$$(4.11) \qquad \mathbf{e}_r = \mathbf{i} \cos\theta + \mathbf{j} \sin\theta.$$

$$(4.12) \qquad \mathbf{e}_\theta = -\mathbf{i} \sin\theta + \mathbf{j} \cos\theta.$$

Section (7) P. (256): 1, 2, 3, 6, 7, 9-12, 19, 20, 21 $(\nabla^2 r^3)$

PROBLEMS, SECTION 7

Compute the divergence and the curl of each of the following vector fields.

1. $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
2. $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$
3. $\mathbf{V} = z\mathbf{i} + y\mathbf{j} + x\mathbf{k}$
6. $\mathbf{V} = x^2y\mathbf{i} + y^2x\mathbf{j} + xyz\mathbf{k}$
7. $\mathbf{V} = x \sin y\ \mathbf{i} + \cos y\ \mathbf{j} + xy\mathbf{k}$

Calculate the Laplacian ∇^2 of each of the following scalar fields.

9. $x^3 - 3xy^2 + y^3$
10. $\ln (x^2 + y^2)$
11. $\sqrt{x^2 - y^2}$
12. $(x + y)^{-1}$

For $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, evaluate

19. $\nabla \cdot \left(\frac{\mathbf{r}}{|\mathbf{r}|}\right)$
20. $\nabla \times \left(\frac{\mathbf{r}}{|\mathbf{r}|}\right)$
21. $(\nabla^2 r^3)$

Disclaimer:

All the problems and excerpts above have been borrowed from the book of "Mathematical Methods in the Physical Sciences" 2^{nd} Edition by Mary L. Boas, and it was done only for the purpose of outlining the students' assignments.