Chapter (2): Complex Numbers

Section (4) P. (48): 1, 2, 3, 5, 7, 11, 16, 17 18

PROBLEMS, SECTION 4

Plot the following numbers in the complex plane. For each number, give the numerical value of its real part x, its imaginary part y, its modulus or absolute value r, and one value of the angle θ. Label each plotted point in five ways as in Figure 3.3. Find and plot the complex conjugate of each number.

1. $1 + i$
2. $i - 1$
3. $1 - i\sqrt{3}$
5. $2i$
7. -1
11. $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$
16. $5(\cos 0 + i\sin 0)$
17. $\sqrt{2}\,e^{-i\pi/4}$
18. $3e^{i\pi/2}$

Supporting Materials:

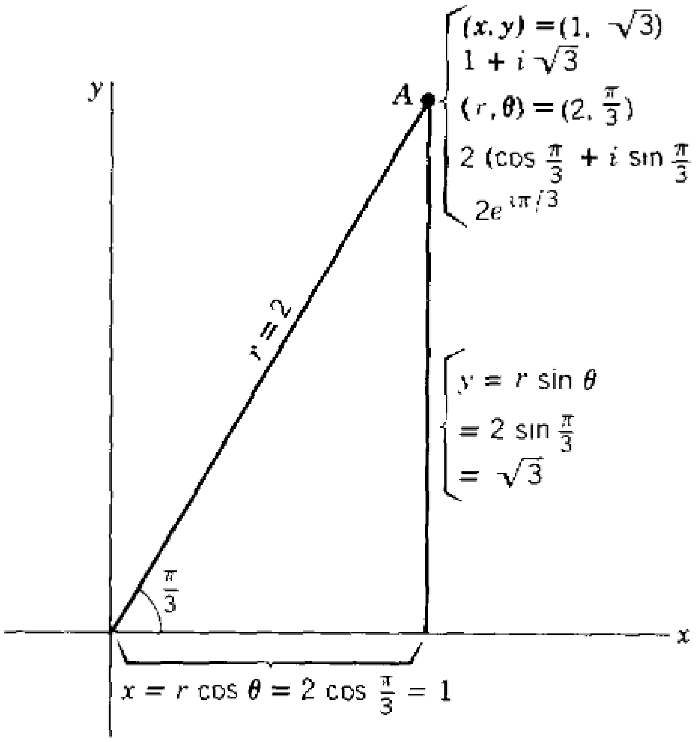

FIGURE 3.3

Section (5) P. (49/50,51): 1-6,10, 15,16,19,23, 26, 30, 31, 32

PROBLEMS, SECTION 5

Follow the instructions for the problems in Section 4. *Hint:* First simplify each number to the form $x + iy$ or to the form $re^{i\theta}$. Use a calculator as needed to find r, θ, from x, y, or to find x, y, from r, θ; be careful to set it in radian mode or degree mode as appropriate.

1. $\dfrac{1}{1+i}$

2. $\dfrac{1}{i-1}$

3. i^4

4. $i^2 + 2i + 1$

5. $(i + \sqrt{3})^2$

6. $\left(\dfrac{1+i}{1-i}\right)^2$

10. $\dfrac{3i-7}{i+4}$ *Careful! Not* $3 - 7i$

15. $\dfrac{5-2i}{5+2i}$

16. $\dfrac{1}{0.5(\cos 40^\circ + i \sin 40^\circ)}$

Find each of the following in rectangular $(a + bi)$ form if $z = 2 - 3i$; if $z = x + iy$.

19. z^{-1}

23. $\dfrac{1+z}{1-z}$

Use equation (5.1) to find the absolute value of

26. $\dfrac{2i-1}{i-2}$

30. $\dfrac{3i}{i-\sqrt{3}}$

31. $\dfrac{5-2i}{5+2i}$

32. $(2-3i)^4$

Supporting Materials:

(5.1) $$|z| = r = \sqrt{x^2 + y^2} = \sqrt{z\bar{z}}.$$

Section (6) P. (62): 1-6, 29, 31, 32, 35, 37

PROBLEMS, SECTION 9

Express the following complex numbers in the $x + iy$ form. This is most easily done by plotting the points as in the examples above.

1. $e^{-i\pi/4}$
2. $e^{i\pi/2}$
3. $9e^{3\pi i/2}$
4. $e^{(1/3)(3+4\pi i)}$
5. $e^{5\pi i}$
6. $e^{-2\pi i} - e^{-4\pi i} + e^{-6\pi i}$

Use Problems 27 and 28 to evaluate:

29. $|e^{i\pi/2}|$

31. $|5e^{2\pi i/3}|$

32. $|3e^{2+4i}|$

35. $|3e^{5i} \cdot 7e^{-2i}|$

37. $\left|\frac{1+i}{1-i}\right|$

Supporting Materials:

27. Show that for any real y, $|e^{iy}| = 1$. Hence show that $|e^z| = e^x$ for every complex z.

28. Show that the absolute value of a product of two complex numbers is equal to the product of the absolute values. Also show that the absolute value of the quotient of two complex numbers is the quotient of the absolute values. *Hint::* Write the numbers in the $re^{i\theta}$ form.

Section (10) P. (66): 3, 4, 7

PROBLEMS, SECTION 10

Find all the values of the indicated roots and plot them.

3. $\sqrt[4]{1}$ 4. $\sqrt[4]{16}$

7. $\sqrt[8]{16}$ (8 answers)

Section (11) P. (68): 4, 7, 9

PROBLEMS, SECTION 11

Find each of the following in rectangular form $x + iy$.

4. $e^{3 \ln 2 - i\pi}$

9. $\sin(\pi - i \ln 3)$

7. $\cos(i\pi)$

Disclaimer:

All the problems and excerpts above have been borrowed from the book of "Mathematical Methods in the Physical Sciences" ۲nd Edition by Mary L. Boas, and it was done only for the purpose of outlining the students' assignments.