Chapter 2

Motion in One Dimension

When a particle moves, its displacement is defined by its change in position. As it moves from an initial position x_i to a final position x_f, its displacement is given by

$$\Delta x = x_f - x_i$$

The average velocity of a particle is defined as the particle's displacement Δx divided by the time interval Δt during which that displacement occurred:

$$\bar{v}_x = \frac{\Delta x}{\Delta t}$$

The average velocity of a particle moving in one dimension can be positive or negative, depending on the sign of the displacement.

The velocity and speed are different. Consider a marathon runner who runs 40 km in 2 hours, ends up at his starting point. His average velocity is zero. The average speed is a scalar quantity and defined as the total distance traveled divided by it takes = 40 km /2 h = 20 km/h

The instantaneous velocity v_x equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero.

$$v_x = \lim_{\Delta x \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

The average acceleration of the particle is defined as the *change* in velocity v_x divided by the time interval t during which that change occurred:

$$\bar{a}_x = \frac{\Delta v}{\Delta t}$$

The instantaneous acceleration of the particle is defined as $a = \dfrac{d v_x}{dt}$

Equations of Motion

معادلات الحركة في خط مستقيم وبتسارع ثابت ، كل معادلة يمكن إستخدامها لإيجاد مجهول واحد فقط

$$\upsilon = \upsilon_o + at \qquad (1)$$

$$x - x_o = \upsilon_o t + \frac{1}{2}at^2 \qquad (2)$$

$$\upsilon^2 = \upsilon_o^2 + 2a(x - x_o) \qquad (3)$$

$$x - x_o = \frac{1}{2}(\upsilon_o + \upsilon)t \qquad (4)$$

$$x - x_o = \upsilon t - \frac{1}{2}at^2 \qquad (5)$$

<u>2- Free Fall Equations</u> : معادلات السقوط الحر

هذه المعادلات تعتبر معادلات حركة في الإتجاة الرأسي (y) وإستبدال قيمة التسارع (g) بقيمة تسارع الجاذبية

$$\upsilon = \upsilon_o + at \qquad (1)$$

$$y - y_o = \upsilon_o t + \frac{1}{2}g t^2 \qquad (2)$$

$$\upsilon^2 = \upsilon_o^2 + 2g(y - y_o) \qquad (3)$$

$$y - y_o = \frac{1}{2}(\upsilon_o + \upsilon)t \qquad (4)$$

$$y - y_o = \upsilon t - \frac{1}{2}at^2 \qquad (5)$$

$y_o = 0$, $a = -g$, y is negative في حالة السقوط الحر
$y_o = 0$, $a = +g$, y is positive في حالة الصعود إلى أعلى

Problems

1. A car, initially at rest, travels 20 m in 4 s along a straight line with constant acceleration. The acceleration of the car is:

A. 0.4 m/s^2
B. 1.3 m/s^2
C. 2.5 m/s^2
D. 4.9 m/s^2
E. 9.8 m/s^2

ans: C

2. A racing car traveling with constant acceleration increases its speed from 10 m/s to 50 m/s over a distance of 60 m. How long does this take?

A. 2.0 s
B. 4.0 s
C. 5.0 s
D. 8.0 s
E. The time cannot be calculated since the speed is not constant

ans: B

3. A car starts from rest and goes down a slope with a constant acceleration of 5 m/s^2. After 5 s the car reaches the bottom of the hill. Its speed at the bottom of the hill, in meters per second, is:

A. 1
B. 12.5
C. 25
D. 50
E. 160

ans: C

4. A car moving with an initial velocity of 25 m/s north has a constant acceleration of 3 m/s^2 south. After 6 seconds its velocity will be:

A. 7 m/s north
B. 7 m/s south
C. 43 m/s north
D. 20 m/s north
E. 20 m/s south

ans: A

5. How far does a car travel in 6 s if its initial velocity is 2 m/s and its acceleration is 2 m/s^2 in the forward direction?

A. 12 m
B. 14 m
C. 24 m
D. 36 m
E. 48 m

ans: E

6. A baseball is thrown vertically into the air. The acceleration of the ball at its highest point is:

A. zero
B. g , down
C. g , up
D. 2g , down
E. 2g , up

ans: B

7. An object is thrown straight up from ground level with a speed of 50 m/s. If $g = 10\ m/s^2$ its distance above ground level after 6.0 s is:

A. 0.00 m
B. 270 m
C. 330 m
D. 480 m
E. none of these

ans: E

8. At a location where $g = 9.80\ m/s^2$, an object is thrown vertically down with an initial speed of 1 m/s. After 5 s the object will travele

A. 125 m
B. 127.5 m
C. 245 m
D. 250 m
E. 255 m

ans: B

9. An object is thrown vertically upward at 35 m/s. Taking $g = 10\ m/s^2$, the velocity of the object after 5 s is:

A. 7.0 m/s up
B. 15 m/s down
C. 15 m/s up
D. 85 m/s down
E. 85 m/s up

ans: B

10. A heavy ball falls freely, starting from rest. Between the third and fourth second of time it travels a distance of:

A. 4.9 m
B. 9.8 m
C. 29.4 m
D. 34.3 m

E. 39.8 m

ans: D

11. An object dropped from the window of a tall building hits the ground in 12 s. If its acceleration is 9.80 m/s^2, the height of the window above the ground is:

A. 29.4 m
B. 58.8 m
C. 118 m
D. 353 m
E. 706 m

ans: E

12. A baseball is hit straight up and is caught by the catcher 2.0 s later. The maximum height of the ball during this interval is:

A. 4.9 m
B. 7.4 m
C. 9.8 m
D. 12.6 m
E. 19.6 m

ans: A

13. An object is thrown straight down with an initial speed of 4 m/s from a window which is 8 m above the ground. The time it takes the object to reach the ground is:

A. 0.80 s
B. 0.93 s
C. 1.3 s
D. 1.7 s
E. 2.0 s

ans: B

14. A stone is released from rest from the edge of a building roof 190 m above the ground. The speed of the stone, just before striking the ground, is:

A. 43 m/s
B. 61 m/s
C. 120 m/s
D. 190 m/s
E. 1400 m/s

ans: B

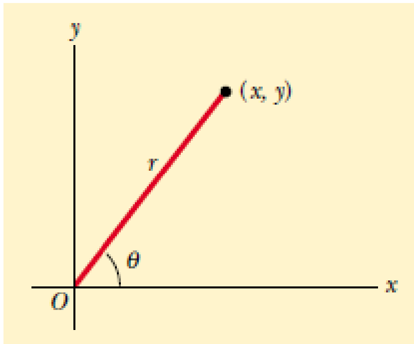
Problem : A particle moves along the x axis according to the equation $x = 2 + 3t - t^2$ where x is in meters and t is in seconds. At t = 3 s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

Polar Cooridnates الإحداثيات القطبية

Starting with the polar coordinates of any point, we can obtain the cartesian coordinates, using the equations

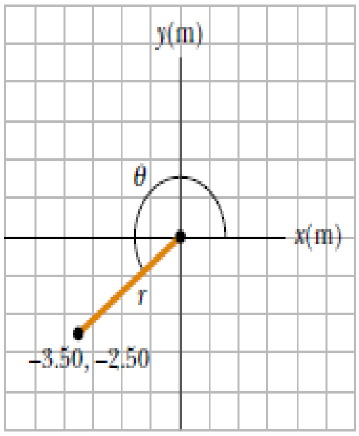
$$x = r\cos\theta$$
$$y = r\sin\theta$$

$$\tan\theta = \frac{y}{x}$$
$$r = \sqrt{x^2 + y^2}$$



Problem

The cartesian coordinates of a point in the x-y plane is (- 3.5, -2.5) m. Find the polar coordinates of this point.



Solution :

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3.5)^2 + (-2.5)^2} = 4.3\ m$$
$$\tan\theta = \frac{y}{x} = \frac{-2.5}{-3.5} = 0.714$$
$$\therefore \quad \theta = 216^o$$

Vector and Scaler Quantities

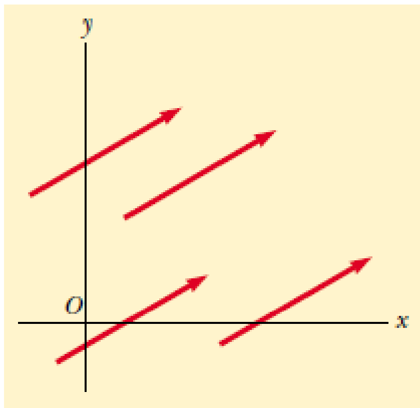
A scalar quantity is specified by a single value with an appropriate unit and has no direction. Examples of scalar quantities are volume, mass, and time intervals.

A vector quantity has both magnitude and direction, such as displacement, velocity and force.

SOME PROPERTIES OF VECTORS

1- Equality

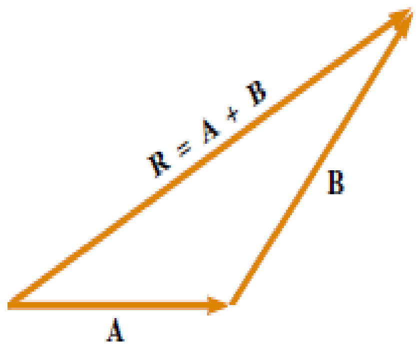
These four vectors are equal because they have equal lengths and point in the same



2- Adding Vectors

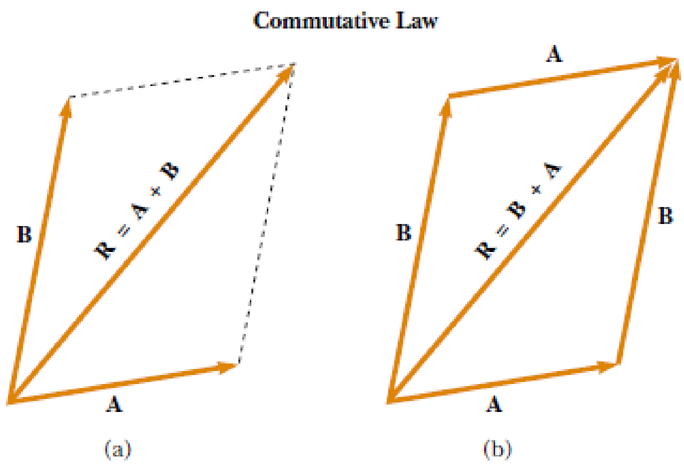
When vector B is added to vector A , the resultant R is the vector that runs from the tail of A to the tip of B.

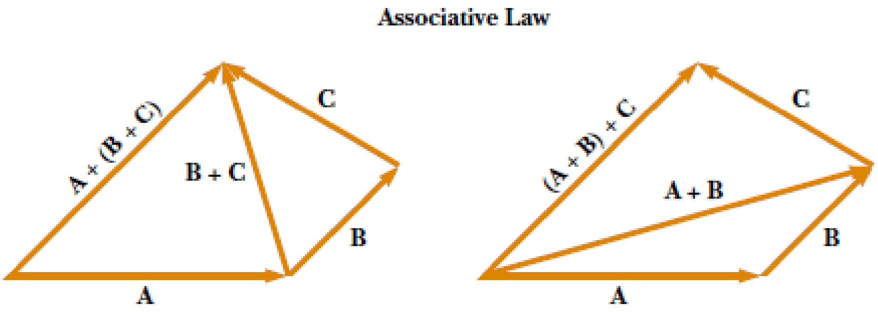
المحصلة هي المتجة الذي يبدأ من ذيل
المتجة A وينتهي عند رأس المتجة B



3- Commutative law

$$A + B = B + A$$

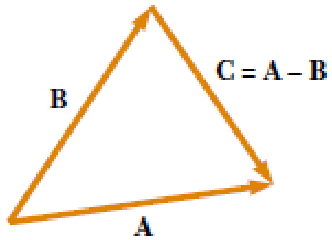




$$A + (B + C) = (A + B) + C$$

4- <u>Subtracting Vectors</u> طرح المتجهات

The operation of vector subtraction makes use of the negative vector. The operation $A - B = A + (-B)$ as vector – B added to A.



<u>Problems</u>

P1- The vector $-\vec{A}$ is:

A. greater than $\vec{A}$ in magnitude
B. less than $\vec{A}$ in magnitude
C. in the same direction as $\vec{A}$
D. in the direction opposite to $\vec{A}$
E. perpendicular to $\vec{A}$

P2- A vector has a magnitude of 12. When its tail is at the origin it lies between the positive x axis and the negative y axis and makes an angle of 30^o with the x axis. Its y component is:

A. $6/\sqrt{3}$
B. $-6\sqrt{3}$
C. 6
D. -6
E. 12

3- Vector $\overrightarrow{V_3}$ in the diagram is equal to

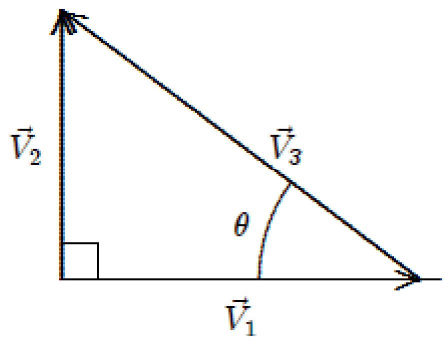
$A.\ \overline{V_1} - \overline{V_2}$

$B.\ \overline{V_1} + \overline{V_2}$

$C.\ \overline{V_2} - \overline{V_1}$

$D.\ \overline{V_1} \cos\theta$

$E.\ \overline{V_1} / \cos\theta$



5- Multiplying a Vector by a Scalar

If vector $\vec{A}$ is multiplied by a positive scalar quantity m, then the product $m\vec{A}$ is a vector that has the same direction as $\vec{A}$ and magnitude mA .

6- <u>Components of a Vector and Unit Vectors</u> :

If a vector A has

x-component $A_x = A \cos \theta$

y-component $A_y = A \sin \theta$

the vector can be expressed in unit–vector form as

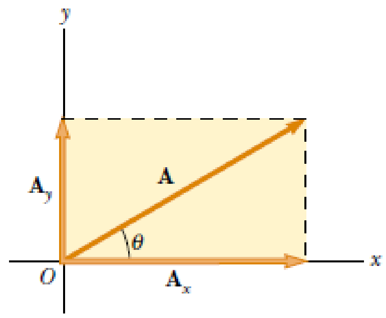
$$A = A_x\, i + A_y\, j$$

In this notation,

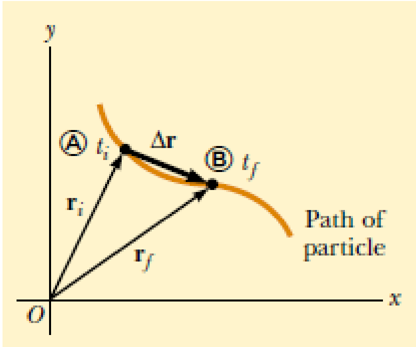
i unit vector pointing positive x- direction,

j unit vector pointing positive y-direction.

$|i| = |j| = 1$



Motion in Two Dimension

The displacement of a particle as it moves from A to B in the time interval $\Delta t = t_f - t_i$ is equal to the vector $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$	

The average velocity of a particle during time interval Δt as the displacement of the particle Δr divided by that time interval

$$\vec{v} = \frac{\Delta\vec{r}}{\Delta t}$$ average velocity is independent of path

The instantaneous velocity v is defined as السرعة اللحظية

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \qquad (1)$$

The instantaneous acceleration a is defined as التسارع اللحظي

$$a = \frac{dv}{dt} = \frac{d(dr/dt)}{dt} = \frac{d^2 r}{dt^2} \qquad (2)$$

Two-Dimensional Motion with Constant Acceleration

Let us consider two-dimensional motion during which the acceleration remains constant in both magnitude and direction. The position vector of a particle moving in the *xy* plane is

$$\vec{r} = x.\hat{i} + y.\hat{j} \qquad (3)$$

the velocity of the particle is given by

$$\vec{v} = v_x \hat{i} + v_y \hat{j} \qquad (4)$$

If a particle moves with *constant* acceleration a and has velocity v_i and position r_i at t = 0, its velocity and position vectors at some later time t are

$$\vec{v}_f = \vec{v}_i + \vec{a}.t \qquad (5)$$

$$\vec{r}_f = \vec{r}_i + \vec{v}_i.t + \frac{1}{2}\vec{a}.t^2 \qquad (6)$$

Prolebms:

1. Which of the following is a scalar quantity?

A. Speed
B. Velocity
C. Displacement
D. Acceleration
E. None of these

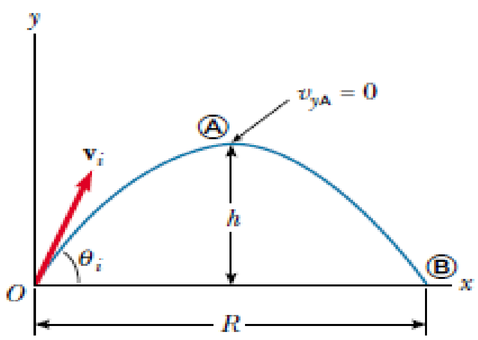
2. Which of the following is a vector quantity?

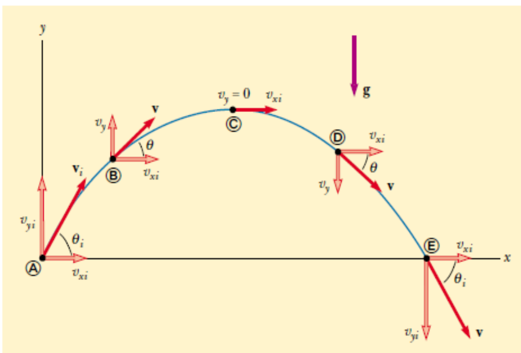
A. Mass
B. Density
C. Speed
D. Temperature
E. None of these

3. A particle goes from (x = −2m, y = 3m) to (x = 3m, y = −1m) , Its displacement is:

A. (1 m)ˆi + (2 m)ˆj
B. (5 m)ˆi − (4 m)ˆj
C. −(5 m)ˆi + (4 m)ˆj
D. −(1 m)ˆi − (2 m)ˆj
E. −(5 m)ˆi − (2 m)ˆj

Projectiles المقذوفات





The projectile leaves the origin with a velocity v_i
The velocity v changes with time in both magnitude and direction.
The x component of velocity remains constant with time
The y component of velocity is zero at the peak of the path

معادلة أقصى إرتفاع

$$h = \frac{v_i^2 \sin^2\theta}{2g}$$

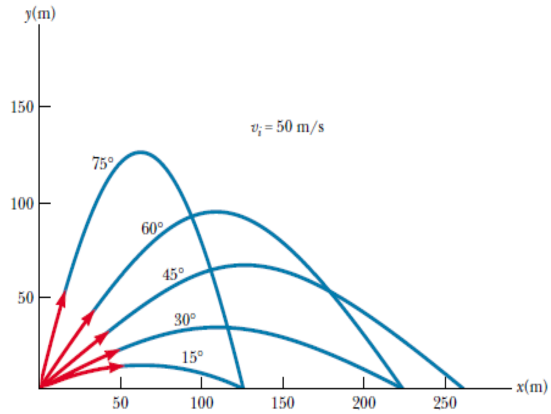
معادلة المدى أفقي

$$R = \frac{v_i^2 \sin 2\theta}{g}$$

A projectile fired from the origin with initial speed of 50 m/s with different θ

The maximum range occurs when

θ = 45°



Problems

1- A large cannon is fired from ground level at an angle of 30 ° above the horizontal. The muzzle speed is 980 m/s. The horizontal distance the projectile will travel before striking the ground, assume g = 9.8 m/s^2

A. 4.3 km
B. 8.5 km
C. 43 km
D. 85 km
E. 170 km

ans: D

2- A boy on a rough of building 20 m high kicks a ball horizontally outward with a speed of 20 m/s. It strikes the ground at horizontal distance from the wall, assume g = 10 m/s^2

A. 10 m
B. 40 m
C. 50 m
D. 50√5 m
E. none of these

ans: B

3- A projectile is fired from ground level with an initial velocity that has a vertical component of 20 m/s and a horizontal component of 30 m/s. Using $g = 10\ m/s^2$, the distance from launching to landing points is:

A. 40 m
B. 60 m
C. 80 m
D. 120 m
E. 180 m

Uniform Circular Motion

A particle is in uniform circular motion if it travels around a circle at constant speed. Although the speed does not change, the particle is accelerated because the velocity changes direction. The acceleration is directed toward center of the circle (مركز الدائرة) and given by

$$a_r = \frac{v^2}{r}$$ centripetal acceleration (تسارع مركزي)

Where r is the radius of the circle and v is the speed of the particle.
The time required to complete one circle is called

The periodic time (الزمن الدوري) $T = \dfrac{2\pi r}{v}$

The frequency (عدد الدورات في الثانية الواحدة) , $f = \dfrac{1}{T}$

Problems

1- A car rounds a 20 m radius curve at 10 m/s. The magnitude of its acceleration is:

A. 0
B. 0.2 m/s^2
C. 5 m/s^2
D. 40 m/s^2
E. 400 m/s^2

ans: C

2- A stone is tied to 50 cm string (خيط) and whirled (يدور) at constant speed of 4 m/s in a vertical circle. Its acceleration at the bottom of the circle is

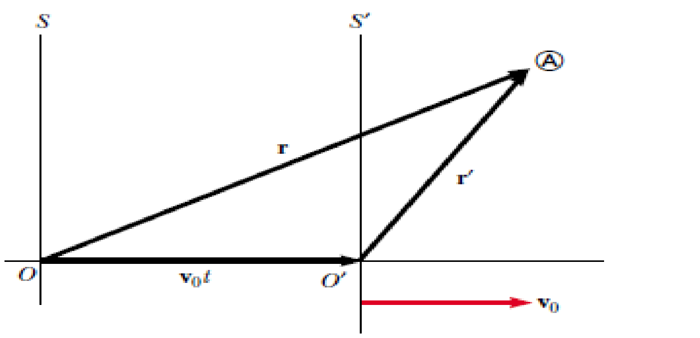
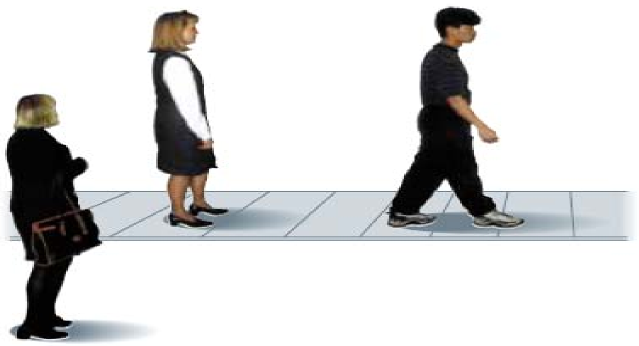
A. 9.8 m/s^2 , up

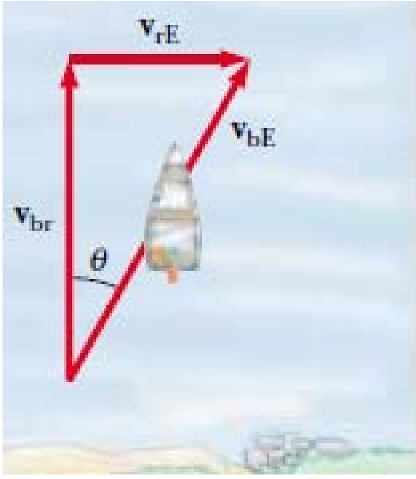
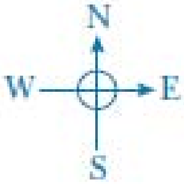
B. 9.8 m/s^2 , down
C. 8.0 m/s^2 , up
D. 32 m/s^2 , up
E. 32 m/s^2 , down
ans: D

3- A stone is tied to the end of a string and is swung with constant speed around a horizontal circle with a radius of 1.5 m. If it makes two complete revolutions each second, its acceleration is
A. 0.24 m/s^2
B. 2.4 m/s^2
C. 24 m/s^2
D. 240 m/s^2
E. 2400 m/s^2
ans: D

Relative Velocity and Relative Acceleration

The position of a particle A relative to the *S* frame with the position vector r and that relative to the S' frame with the position vector r' **.** The vectors r and r' are related to each other at time t

$r' = r - v_o t$	
$\frac{dr'}{dt} = \frac{dr}{dt} - v_o$ $v' = v - v_o$	

Example: A boat heading north crosses a river with speed of 10 km/h relative to the water. The river has speed of 5.00 km/h to east relative to the Earth. Determine velocity of the boat relative to an observer standing on bank	 

The relationship between the three velocities is

$$v_{bE} = v_{br} + v_{rE}$$

$$v_{bE}^{2} = v_{br}^{2} + v_{rE}^{2} = (10)^2 + (5)^2 = 125$$

$$\tan\theta = \frac{v_{rE}}{v_{br}} = \frac{5}{10} \quad , \quad \theta = \tan^{-1}(\frac{1}{2})$$

Laws of Motion

1- <u>Newton's first law</u> :

In the absence of external forces, an object at rest remains at rest and an object in motion continues in motion with the same velocity.

2- <u>Newton's second law</u> :

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

$$\sum F = m\,a$$

The SI unit of force is the newton, which is defined as the force acting on a 1-kg mass, produces an acceleration of 1 m/s**²**.

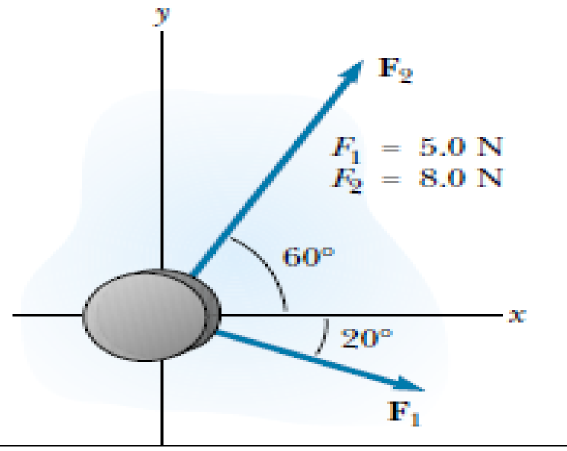
$$1\,N = 1\,kg\,.\,m\,/\,s^2$$

Example :

A hockey puck having a mass of 0.30 kg slides on a horizontal, frictionless surface of an ice rink. Two forces act on the puck, as shown in Figure. Determine both the magnitude and direction of the puck's acceleration.

$$\Sigma F_x = F_1 \cos 20^o + F_2 \cos 60^o =$$
$$= 5\cos 20^o + 8\cos 60^o = 8.7\ N$$

$$\Sigma F_y = F_1 \sin 20^o + F_2 \sin 60^o =$$
$$= 5\sin 20^o + 8\sin 60^o = 5.2\ N$$



$$a_x = \frac{\Sigma F_x}{m} = \frac{8.7\ N}{0.3 kg} = 29\ m\,/\,s^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{5.2\ N}{0.3 kg} = 17\ m\,/\,s^2$$

$$a = \sqrt{(29)^2 + (17)^2} = 34\ m\,/\,s^2$$

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{17}{29}\right) = 30^o$$

<u>Problems</u>

1. A force of 1N is:
   A. 1 kg/s
   B. 1 kg · m/s
   C. 1 kg · m/s^2
   D. 1 kg · m^2/s
   E. 1 kg · m^2/s^2

ans: C

2. When a certain force is applied to the standard kilogram its acceleration is 5 m/s^2. When the same force is applied to another object its acceleration is one-fifth as much. The mass of the object is:
   A. 0.2 kg
   B. 0.5 kg
   C. 1 kg
   D. 5 kg
   E. 10 kg

ans: D

3. A constant force of 8 N is exerted (تؤثر) for 4 s on a 16 kg object initially at rest. The change in speed of this object will be:
   A. 0.5 m/s
   B. 2 m/s
   C. 4 m/s
   D. 8 m/s
   E. 32 m/s

ans: B

4. Two forces, given by F_1 = (- 6i - 4j) N and F_2 = (- 3i + 7j) N, act on a particle of mass 2 kg that is initially at rest at the origin. (a) What are the components of the particle's velocity at t = 10 s. (b) In what direction is the particle moving at t = 10 s , (c) What displacement does the particle make.

Force of Gravity and Weight

The weight of an object is defined as $F_g = m\,g$

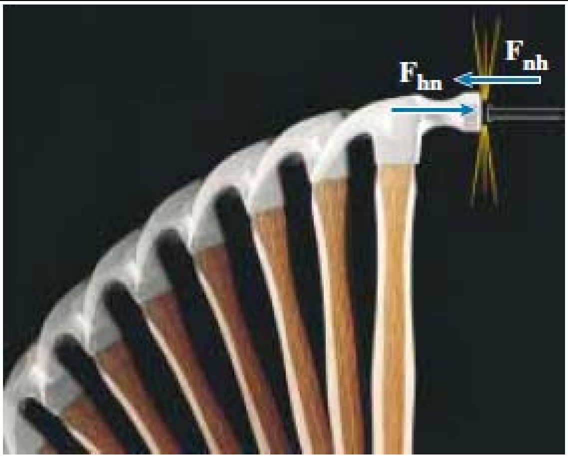
where g is the gravity acceleration equal = 9.8 m/s^2 or ≈ 10 m/s^2

Because weight $F_g = m\,g$, we can compare the masses of two objects by measuring their weights on a spring scale. The ratio of the weights of two objects equals the ratio of their masses.

Newton's Third Law :

If two objects interact, the force F_{12} exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force F_{21} exerted by object 2 on object 1:

$$F_{12} = -F_{21}$$

The force F_{hn} exerted by the hammer on the nail is equal and opposite the force F_{nh} exerted by the nail on the hammer.	

Weighings in Elevator

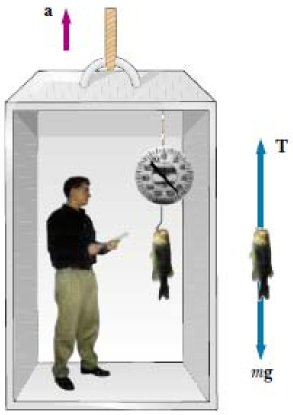
A person weighs a fish of mass *m* on a spring scale attached to the elevator. If the elevator accelerates either upward or downward, the scale gives the weight of the fish.

Solution

1- If the elevator is either at rest or moving at constant velocity, Newton's second law gives

$$F_g = m\,g = T$$

2- If the elevator moves upward with an acceleration, Newton's second law applied to the fish gives the net force



$$F_y = T - mg = m\,a_y$$
$$T = mg + m\,a_y = m\,(g + a_y) \qquad (2)$$

The scale reading T is greater than the weight mg if a_y is upward (+) , and the reading is less than mg if a_y is downward (–).

Problems :

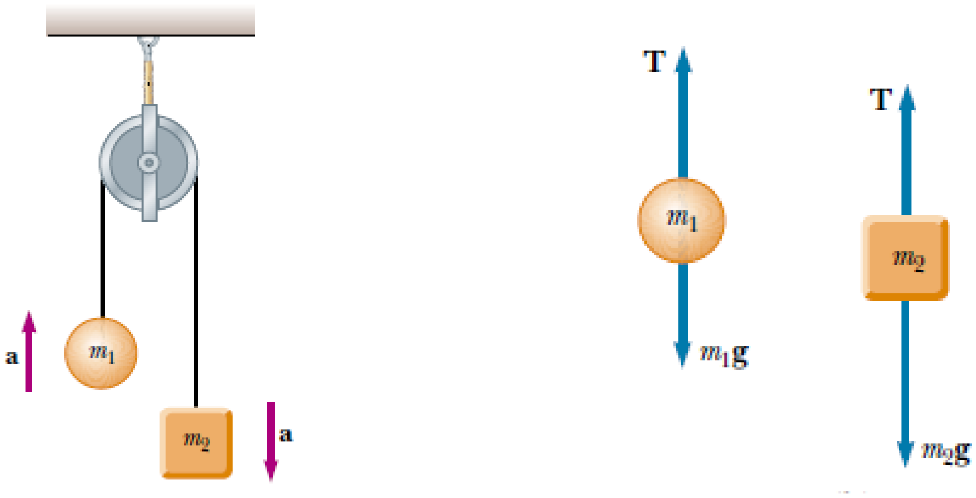
1. 90 kg man stands in an elevator that has a downward acceleration of 1.4 m/s^2. The force exerted by him on the floor is about:

   A. zero
   B. 90N
   C. 760N
   D. 880N
   E. 1010N

ans: C

Atwood's Machine

When two objects of unequal mass are hung (تعلق) vertically over a frictionless pulley, the arrangement is called an *Atwood machine*.



When Newton's second law is applied to object 1, we obtain

$$T - m_1 g = m_1 a_y \qquad (1)$$

Similarly, for object 2 we find

$$m_2 g - T = m_2 a_y \qquad (2)$$

When (2) added to (1), T is dropped and get

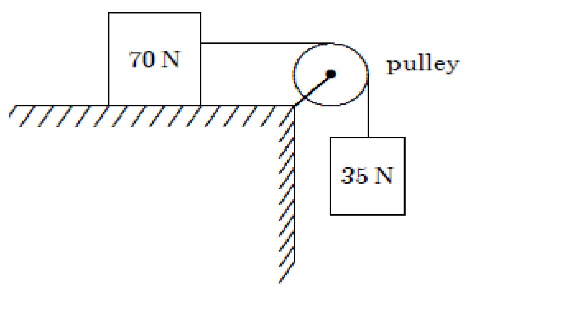
$$a_y = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g \qquad (3)$$

When (3) is substituted into (1), we obtain

$$T = \left(\frac{2 m_1 m_2}{m_1 + m_2}\right) g \qquad (4)$$

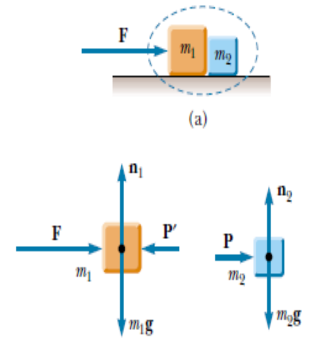
2. 70 N block and 35 N block are connected by a string. If the pulley is massless and the surface is frictionless, the magnitude of the acceleration of the 35 N block is:

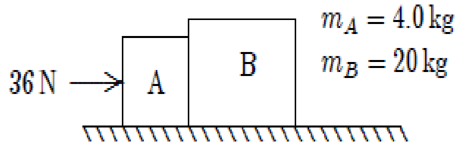
A. 1.6 m/s^2
B. 3.3 m/s^2
C. 4.9 m/s^2
D. 6.7 m/s^2
E. 9.8 m/s^2
ans: B



3. Two blocks (A and B) are in contact on a horizontal frictionless surface. A 36 N constant force is applied to A. The force of A on B is

A. 1.5 N
B. 6.0 N
C. 29 N
D. 30 N
E. 36 N
Ans: D

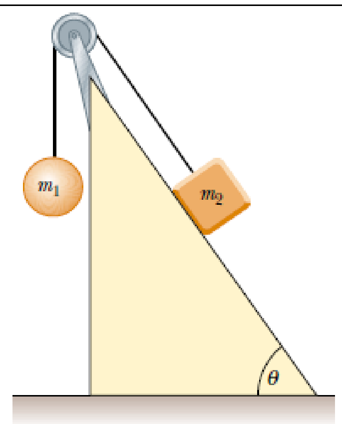




$$a_x = \frac{F}{m_1 + m_2}$$

4.
$m_1 = 2$ kg , $m_2 = 6$ kg
$\theta = 55°$
frictionless pulley

<u>Find</u> :
(a) the accelerations of the masses,
(b) the tension in the string,
(c) the speed of each mass 2.00 s after being released from rest.



For the first body

$$T - m_1 g = m_1 a \qquad (1)$$

For the second body

$$m_2 g \sin\theta - T = m_2 a \qquad (2)$$

Add equations (1) and (2), gives the acceleration. Substitute the value of a in the first equation gives the tension in the string. To find the speed after 2 s apply the law, $v = v_o + at = 0 + 2a = \quad m/s$

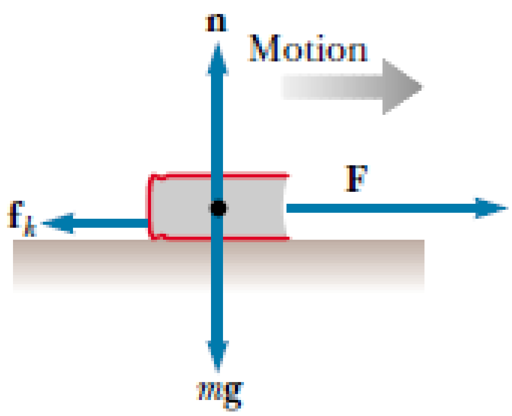
Force of Friction

To push a heavy desk on rough floor, it takes a greater force to start moving than it takes keep moving. Increasing F, the magnitude of F_s increases along with it, keeping the book in place. When it is on the verge of moving, F_s has a maximum value. When F exceeds F_{sm} , the book accelerates to the right. Once the book is in motion, the retarding frictional force is called kinetic friction F_k and smaller than F_{sm}.

If $F = F_k$, the book moves to the right with constant speed.

If $F > F_k$, the unbalanced force ($F - F_k$) accelerates the desk to the right.

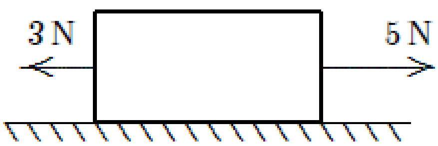
If the applied force F is removed, then the frictional force F_k decelerating (تقلل السرعة) the desk and eventually brings it to rest.



$$F_s = \mu_s F_N \quad , \textit{where } \mu_s \textit{ is the coefficient of static friction}$$
$$F_k = \mu_k F_N \quad , \textit{where } \mu_k \textit{ is the coefficient of kinetic friction}$$
$$F_N \quad \textit{is the normal force}$$

Problems

1. A block moves with constant velocity on a horizontal rough surface. The frictional force necessary to keep a constant velocity is:

A. 0 B. 2 N, leftward C. 2 N, rightward D. slightly more than 2N, leftward E. slightly less than 2N, leftward ans: B	

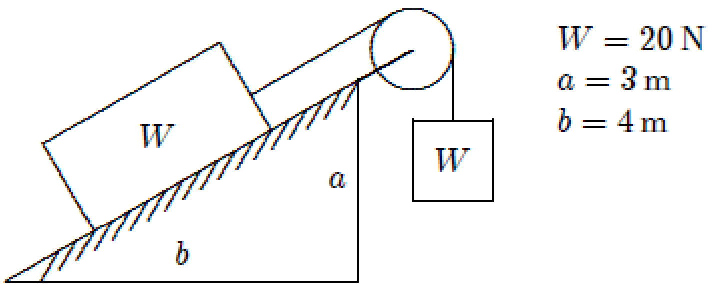
2. A person pushes horizontally with a force of 220 N on a 55 kg crate to move it across a level floor. The coefficient of kinetic friction is 0.35 what is the magnitude of the frictional force and the acceleration.

3. 5 kg crate is on an incline that makes an angle of 30° with the horizontal. If the coefficient of static friction is 0.5, the maximum force that can be applied parallel to the plane without moving the crate is:

A. 0
B. 3.3 N
C. 30 N
D. 46 N
E. 55 N

ans: D

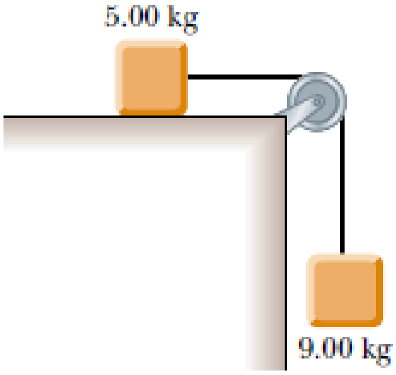
4. The system shown remains at rest. Each block weighs 20 N. The force of friction on the upper block is:



A. 4N
B. 8N
C. 12N
D. 16N
E. 20N

ans: B

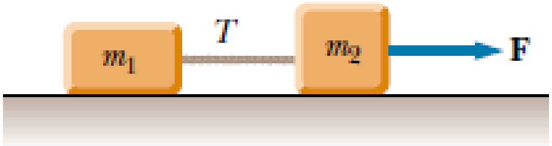
5. 9 kg hanging weight is connected by a string over a pulley to a 5 kg block that is sliding on a flat table. If the coefficient of kinetic friction is 0.2 , find the tension in the string.



6. Two blocks connected by a rope of negligible mass are being dragged by a horizontal force F = 68 N, m_1 = 12 kg, m_2 = 18 kg , and the coefficient of kinetic friction between each block and the surface is 0.1

Determine :

(a) the tension T , (b) the acceleration of the system.



6. The magnitude of the force required to cause a 0.04 kg object to move at 0.6 m/s in a circle of radius 1.0 m is:

A. 2.4 × 10−2 N
B. 1.4 × 10−2 N
C. 1.4π × 10−2 N
D. 2.4π2 × 10−2 N
E. 3.13N

ans: B

7. 0.2-kg stone is attached to a string and swung in a circle of radius 0.6m on a horizontal and frictionless surface. If the stone makes 150 revolutions per minute, the tension force of the string on the stone is:

A. 0.03 N
B. 0.2 N

C. 0.9 N
D. 1.96 N
E. 30 N

ans: E

Work and Kinetic Energy

Kinetic Energy :

For an object of mass m and speed v , its kinetic energy K is given by

$$K = \frac{1}{2} m v^2 \qquad J \ \text{(Joule = 1 kg . m}^2\text{/s}^2\text{)}$$

Work done :

Work done by a constant force is the product of the component of the force in the direction of displacement and the magnitude of the displacement:

$$W = F . d \cos\theta \qquad J$$

If an applied force F acts along the direction of the displacement, then θ = 0 , and cos θ = 1. The above equation gives

$$W = F . d$$

Work is scalar quantity, and its units are force multiplied by length. (N.m).

The change in kinetic energy = net work done on the particle

$$\Delta K = W$$

$$K_f - K_i = W$$

Example : A 0.3 kg ball has a speed of 15.0 m/s. (a) What is its kinetic energy? (b) If the speed is doubled, what would be its kinetic energy?

Solution : (a) - $K = \frac{1}{2} m v_1^2 = \frac{1}{2}(0.3)(15)^2 = \qquad J$

(b)- $K = \frac{1}{2} m v_2^2 = \frac{1}{2}(0.3)(30)^2 = \qquad J$

Example : A mechanic pushes (ميكانيكي يدفع) 2500 kg car, moving it on frictionless floor from rest with constant acceleration. He does 5000 J of work in the process. During this time, the car moves 25 m. (a) What horizontal force did he exert on the car ? , (b) what is the final speed of the car ?

Solution :

$$W = F.d\cos\theta = F.d\cos 0$$
$$5000 = F\ .\ 25$$
$$\therefore \quad F = 200\ N$$

$$\because \quad F = m\,a$$
$$200\ N = 2500.\,a$$
$$\therefore\ a = 0.04\ m/s^2$$
$$v^2 = v_o^2 + 2ax = 0 + 2(0.04)(25) = \qquad m/s$$

Example : A force $F = (6i - 2j)$ N acts on a particle that moves a displacement $d = (3i + j)$ m. Find (a) the work done by the force on the particle and (b) the angle between F and d.

<u>**Work done by Gravitional Force**</u> الشغل المبذول بالجاذبيةالأرضية

The work done by gravity can be given by $W = F.d\cos\phi$

The value of F is the gravitational force, $F_g = mg$. In case of rising an object, F_g is directed opposite the displacement d,

$\phi = 180°$

$$W = F.d\cos\phi = mg.d\cos 180^\circ$$
$$W = -mg.d \qquad (1)$$

When the object is falling back down, the angle between F_g and d is zero , $\phi = 0°$

$$W = F.d\cos\phi = mg.d\cos 0^\circ$$
$$W = +mg.d \qquad (2)$$

– sign indicates that the gravitational force transfer amount of $mg.d$ from the kinetic energy of the object during rising, while the + sign tells us that gravity add amount of $mg.d$ to kinetic energy of the object.

If an external force F is used to lift an object, the applied force tends to transfer energy to the object while the gravitational force F_g tends to transfer energy from it. The change in the kinetic energy of the object

$$\Delta K = K_f - K_i = W_a - W_g$$

<u>Work done by Spring</u> الشغل المبذول بالزمبرك

1-The Spring Force

A block on a horizontal, frictionless surface is connected to a spring. If the spring is either stretched or compressed a small distance from its unstretched (equilibrium) configuration, it exerts on the block a force of magnitude

$$F_x = -k\,x \quad \ldots\ldots\ldots\ldots\ldots \quad \textbf{Hooke's law}$$

where x is the displacement of the block from its unstretched (x = 0) position and k is a positive constant called " constant of the spring". The negative sign means that the force exerted by the spring is always directed *opposite* the displacement.

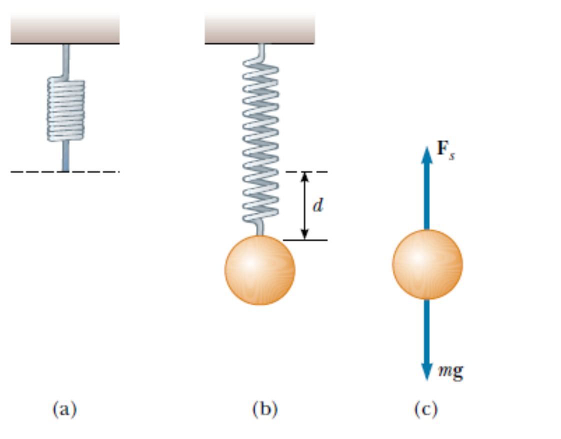
<u>Measuring spring constant</u> :

The spring is hung vertically. An object of mass m is attached to its end. Under the action of “load” mg, the spring stretches distance d.

Force spring = weight of load

$$-k\,.x = m\,g$$

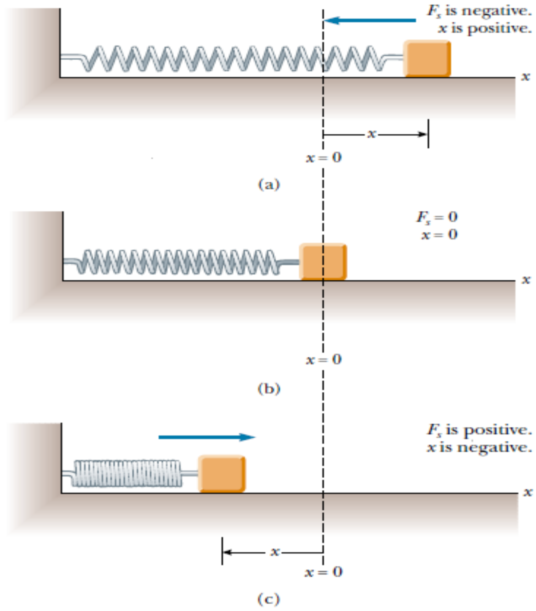
$$\therefore k = \frac{m\,g}{x}$$



The work done by the spring is given by

$$W = \frac{1}{2} k \ x^2 \quad ,$$

where $\boldsymbol{x}$ is the change in the spring



Power (القدرة)

Power is the time rate of doing work المعدل الزمني لعمل الشغل

$$P = \frac{dW}{dt} \qquad (\ Joule/s \ = watt)$$

$$P = \frac{F.dx.\cos\phi}{dt} = F.\cos\phi.\frac{dx}{dt}$$

$$P = F.v.\cos\phi$$

Example: A 0.6 *kg* particle has a speed of 2 m/s at point *A* and kinetic energy of 7.5 J at point *B*. What is

(a) its kinetic energy at *A*?

(b) its speed at *B*?

(c) the total work done on the particle as it moves from *A* to *B*?

Solution :

(a) - $K_A = \frac{1}{2} m v^2 = \frac{1}{2}(0.6)(2)^2 = 1.2 \ J$

(b) -

$$K_B = \frac{1}{2} m v_B^2 = \frac{1}{2}(0.6) v_B^2 =$$

$$\therefore \quad 7.5 = 0.3\, v_B^2$$

(c) -

$$\Delta W = W_B - W_A =$$

$$= 7.5 - 1.2 = 6.3 \ J$$

<u>Problem</u> :

1- Express the unit of the force constant of a spring in terms of the basic units meter, kilogram, and second.

2- when 4 *kg* mass is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.5 cm. If the 4 *kg* mass is removed, (a) how far will the spring stretch if 1.5 *kg* mass is hung on it and (b) how much work to stretch the same spring 4 cm from its unstretched position?

3- A 15 kg block is dragged over a rough, horizontal surface by 70 N force acting at 20° above the horizontal. The block is displaced 5 m, and the coefficient of kinetic friction is 0.3 . Find the work done by

(a) the 70 N force, (b) the normal force, (c) the force of gravity.

(d) What is the energy loss due to friction?

(e) Find the total change in the block's kinetic energy.

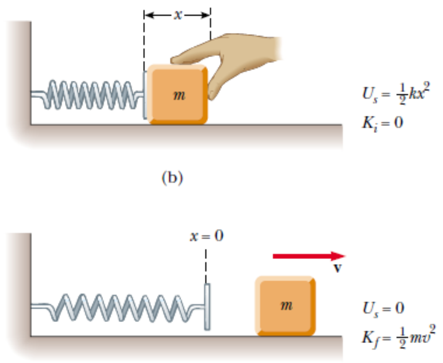
Potential Energy

As an object falls toward the Earth, the gravitational force does work on the object and increases the object’s kinetic energy. The conversion (التحول) from potential energy to kinetic energy occurs continuously over the entire fall. The product of the gravitational force mg and height y of the object gives the potential energy ($U_g = mg.y$). For either rise or fall, the change in the potential energy ΔU is equal to the negative of the work done

$$\Delta U = -W$$
$$\Delta U = mg.y_2 - mg.y_1 =$$
$$\Delta U = mg\,(y_2 - y_1)$$

In a block-spring system, if we push the block to left, the spring force acts rightward and transfers kinetic energy of the block to elastic potential energy of the spring-block system. The block slows and stops and begins to move rightward. The transfer of energy is reversed, from elastic to kinetic energy.

$$U(x) = \frac{1}{2}k\,x^2$$
$$\Delta U = \frac{1}{2}k\,x_f^2 - \frac{1}{2}k\,x_i^2 =$$
$$\Delta U = \frac{1}{2}k\,(x_f^2 - x_i^2)$$



Conservation of Mechanical Energy

The mechanical energy of a system is the sum of its potential energy U and the kinetic energy K of the object

$$E_{\mathbf{mec}} = K + U$$

For isolated system which means that no external force, from outside the system, causes energy changes inside the system.

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

$$\Delta K = -\Delta U$$

$$K_2 - K_1 = -(U_2 - U_1)$$

$$K_2 + U_2 = K_1 + U_1 = cons\,\text{tan}\,t$$

This is called the (قانون حفظ الطاقة الميكانيكية)

"**Principle of Conservation of Mechanical Energy**"

Example : A ball of mass m is dropped from height h above the ground. Determine the speed of the ball when it is at a height y above the ground.

Solution : At the instant the ball is released, its kinetic energy is $K_i = 0$ and the potential energy of the system is $U_i = mgh$. When the ball is at a distance y above the ground,

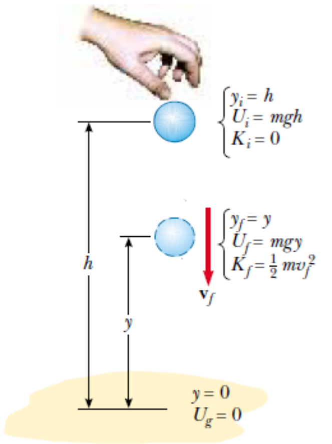
(kinetic energy + potential energy)$_f$ = (initial kinetic + potential energy)$_i$

Solution :

$$K_i + U_i = K_f + U_f$$

$$0 + mgh = \frac{1}{2} m v_f^2 + mgy$$

$$\therefore\ v_f^2 = 2g(h - y)$$



Work Done by an External Force

When you lift a book through some distance, the force you apply does work W on the book, while the gravitational force does work W_g on the book. The net work done on the book is related to the change in its kinetic energy as

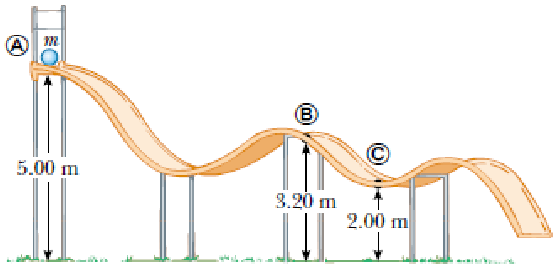
$$W + W_g = \Delta K$$

Because the gravitational force is conservative, the work done by the gravitational force equals $-\Delta U$. Substituting this into the above equation gives

$$W = \Delta U + \Delta K$$

The right side of the equation represents the change in the mechanical energy of the book–Earth system. This result indicates that your applied force transfers energy to the system in the form of kinetic energy of the book and gravitational potential energy of the book–Earth system.

Example : A particle of mass $m = 5\ kg$ is released from point A and slides on frictionless track. Determine (a) the particle's speed at points B and C and (b) the net work done by the force of gravity in moving the particle from A to C .



Solution : Because the particle moves under conservative force

$$K_A + U_A = K_B + U_B = K_C + U_C$$

$$0 \quad + mgh_A = \frac{1}{2} m v_B^2 + mgh_B = \frac{1}{2} m v_C^2 + mgh_C$$

$$2gh_A = v_B^2 + 2gh_B = v_C^2 + 2gh_C$$

(a) Substituting for the height values at A , B, C by 5 , 3.2 , 2 m , we can find v_A and v_B .

(b) – The net work done by gravity = 0

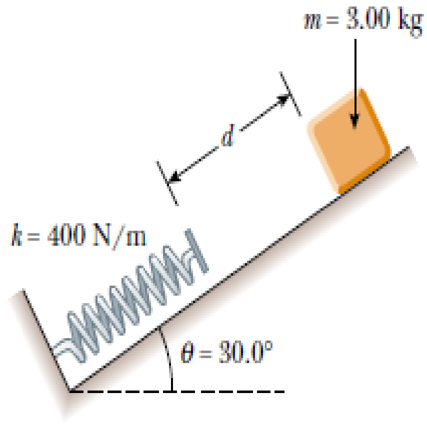
Example : 3 *kg* mass starts from rest and slides a distance *d* down a frictionless 30° incline. While sliding, it comes into contact with unstressed spring. The mass slides an additional 0.2 *m* before rest by compression of the spring (k = 400 N/m). Find the initial separation d between the mass and the spring.

Solution : The force acting on the block

$F = mg\sin\theta = 5 \times 9.8 \times \sin 30 = 24.5$ N

$F = mg\sin\theta = m\,a$

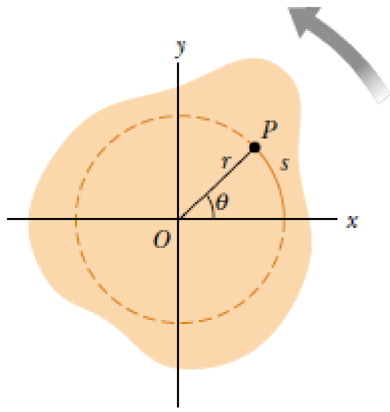
$a = g\sin\theta = 9.8 \times \sin 30^\circ = 4.9$ m/s^2

$\because v^2 = v_o^2 + 2ax^2$ $v^2 = 0 + 2(4.9)(d)^2 = 9.8d^2$ $\because \frac{1}{2}mv^2 = \frac{1}{2}k\,x^2$ $\frac{1}{2}5(9.8d^2) = \frac{1}{2}(400)(0.2)^2$ $\therefore d = \frac{16}{49}$ m	

Rotation

Consider a flat and rigid object of arbitrary shape rotating about a fixed axis through (o). A particle at P is at a fixed distance r from the origin and rotates about it in a circle of radius r. In polar coordinates, the point P is given by (r, θ), where r is the distance from the origin to P and θ is measured *counterclockwise* from the positive x axis. During rotation, θ changes with time and r remains constant. (In cartesian coordinates, both x and y vary in time). As the particle moves along the circle from the positive x axis ($\theta = 0$) to P, it moves through an arc of length S, which is related to the angular position θ through the relationship

$$\theta = \frac{s}{r}$$



Because θ is the ratio of an arc length and radius of the circle, it is a pure number. Commonly, we give θ an artificial unit **radian** (rad), where one radian is the angle subtended by an arc length equal to the radius of the arc.

$$\theta(rad) = \frac{\pi}{180^o}\,\theta\,(\deg)$$

$$For\ example:\ 60^o = \pi/3\ rad \quad , \quad 45^o = \pi/4\ rad$$

In analogy to linear speed, the instantaneous angular speed ω is defined as $\omega = \frac{d\,\theta}{d\,t}$

Angular speed has units of radians per second (rad/s), or rather second s^{-1} , because radians are not dimensional. The angular acceleration is defined as

$$\alpha = \frac{d\,\omega}{d\,t} = \frac{d}{dt}\left(\frac{d\,\theta}{dt}\right) =$$

$$\alpha = \frac{d^2\theta}{d\,t^2}$$

Angular position (θ), angular speed (ω), and angular acceleration (α) are analogous to linear position (x), linear speed (v), and linear acceleration (a). The variables θ, ω, and α differ dimensionally from the variables x, v, and a only by a factor having the unit of length.

$$x = \theta . r$$

$$v = \omega . r$$

$$a = \alpha . r$$

Rotation with Constant Angular Acceleration

As we consider equations 2.1 – 2.5 are the basic equations for constant linear acceleration, equations 10-1 – 10.5 are the basic equations for constant angular acceleration. To solve any problem, we choose an equation for which the only unknown variable will be the variable requested in the problem.

$\upsilon = \upsilon_o + at$	(2.1)	$\omega = \omega_o + \alpha t$	(10.1)
$x - x_o = \upsilon_o t + \frac{1}{2} a t^2$	(2.2)	$\theta - \theta_o = \omega_o t + \frac{1}{2} \alpha t^2$	(10.2)
$\upsilon^2 = \upsilon_o^2 + 2a(x - x_o)$	(2.3)	$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o)$	(10.3)
$x - x_o = \frac{1}{2}(\upsilon_o + \upsilon)t$	(2.4)	$(\theta - \theta_o) = \frac{1}{2}(\omega_o + \omega)t$	(10.4)
$x - x_o = \upsilon t - \frac{1}{2} a t^2$	(2.5)	$\theta - \theta_o = \omega t - \frac{1}{2} \alpha t^2$	(10.5)

Example : A wheel rotates with a constant angular acceleration of 3.5 rad/s^2. If the angular speed of the wheel is 2 rad/s at $t_i = 0$, (a) through what angle does the wheel rotate in 2 s? (b) What is the angular speed at $t = 2$ s?

Solution :

$$\theta_f - \theta_i = \omega_o t + \frac{1}{2} \alpha t^2 =$$

$$= (2\,rad/s)(2s) + \frac{1}{2}(3.5\,rad/s^2)(2s)^2 =$$

$$= 11\,rad = 630^o$$

$$\omega = \omega_o + \alpha t$$

$$\omega = (2\,rad/s) + (3.5\,rad/s^2)(2s)$$

$$\omega = 9\,rad/s$$

Rotational Kinetic Energy

Let us now look at the kinetic energy of a rotating rigid object, considering the object as a collection of particles and assuming it rotates about a fixed *z* axis with an angular speed ω. Each particle has kinetic energy determined by its mass and linear speed. If the mass of the i^{th} particle is m_i and its linear speed is v_i, its kinetic energy is

$$K_i = \frac{1}{2} m_i v_i^2$$

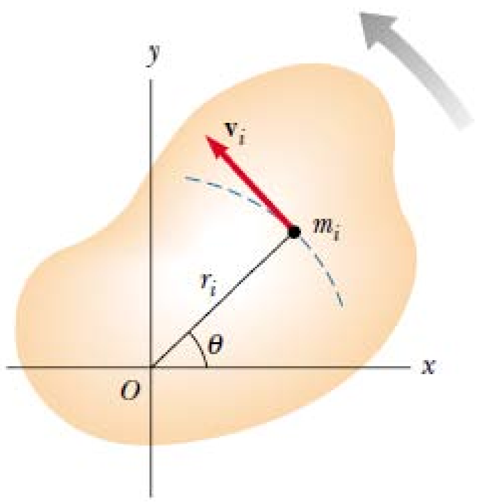
To proceed further, we must recall that although every particle in the rigid object has the same angular speed ω, the individual linear speeds depend on the distance r_i from the axis of rotation according to the expression $v_i = r_i\,\omega$. The *total* kinetic energy of the rotating rigid object is the sum of the kinetic energies of the individual particles:

$$K_R = \sum K_i = \sum \frac{1}{2} m_i v_i^2 =$$

$$K_R = \sum \left(\frac{1}{2} m_i\, r_i\, \omega^2\right) =$$

$$K_R = \frac{1}{2} \omega^2 \sum m_i\, r_i = \frac{1}{2} I\ \omega^2$$

where we have factored _2 from the sum because it is common to every particle.


y
$\mathbf{v}_i$
m_i
r_i
θ
x
O