

Lecture#11

Discrete Probability Distribution:

Example: Binomial Distribution with $n=10$ and $\theta = 0.4$

$$n := 10 \quad \theta := 0.4$$

$$f(x) := \text{combin}(n, x) \theta^x (1 - \theta)^{n-x}$$

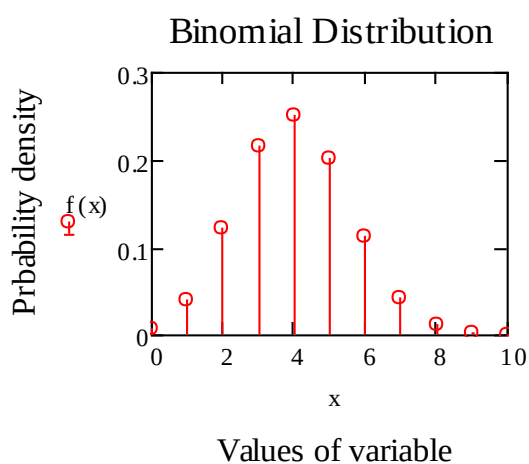
$$x := 0..n$$

x =	f(x) =
0	$6.047 \cdot 10^{-3}$
1	0.04
2	0.121
3	0.215
4	0.251
5	0.201
6	0.111
7	0.042
8	0.011
9	$1.573 \cdot 10^{-3}$
10	$1.049 \cdot 10^{-4}$

From a table we can find for example:

1. $f(2) = 0.121$
2. $P(x=5)=f(5)=0.201$
3. $P(x<2)=P(x=0)+P(x=1)=0.006047+0.04=0.046047$
4. $P(x=18) = 0$
5. $P(2 \leq x \leq 3) = P(x=2)+P(x=3) = 0.121+0.215=0.336$

Graph the probability density function:



$$\text{mean} := n \cdot \theta = 4$$

$$\text{variance} := n \cdot \theta \cdot (1 - \theta) = 2.4$$

$$SD := \sqrt{\text{variance}} = 1.549$$

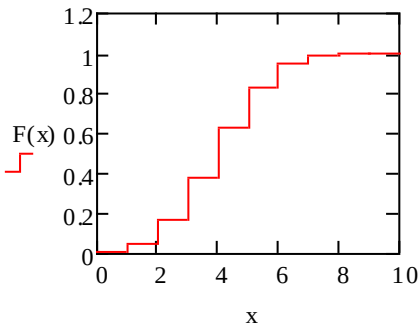
Write and draw the function of cumulative distribution:

$$F(x) := \sum_{x=0}^x f(x)$$

F(x) =

6.047·10 ⁻³
0.046
0.167
0.382
0.633
0.834
0.945
0.988
0.998
1
1

$P(x \leq 4) = 0.633, \quad P(x < 7) = P(x \leq 6) = 0.945, \quad P(x > 6) = 1 - P(x \leq 6) = 1 - 0.945 = 0.055$



x =

0
1
2
3
4
5
6
7
8
9
10

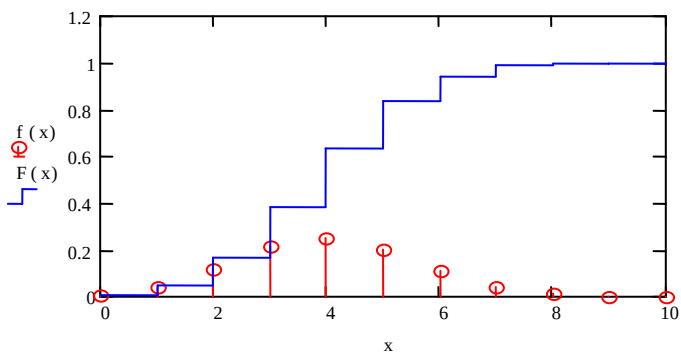
f(x) =

6.047·10 ⁻³
0.04
0.121
0.215
0.251
0.201
0.111
0.042
0.011
1.573·10 ⁻³
1.049·10 ⁻⁴

F(x) =

6.047·10 ⁻³
0.046
0.167
0.382
0.633
0.834
0.945
0.988
0.998
1
1

Density and Cumulative distribution



Values of variables

Using Built-in functions:

Generating random numbers from any distribution:

```
a := rbinom(10,50,0.5)
```

a =

	0
0	26
1	26
2	26
3	17
4	26
5	24
6	27
7	26
8	20
9	25

```
rbinom(10,20,0.5) =
```

	0
0	14
1	10
2	8
3	9
4	11
5	11
6	13
7	9
8	10
9	11

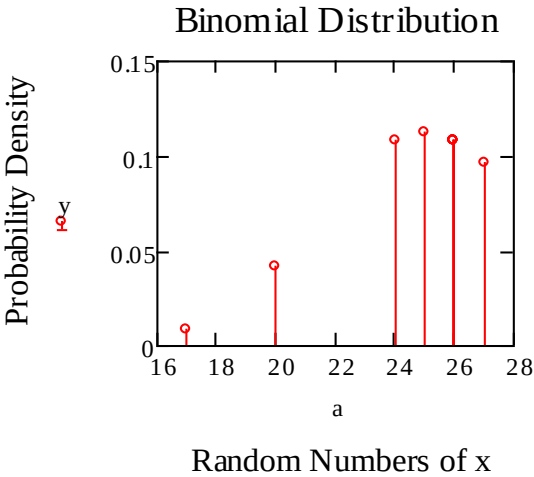
Calculate the probability density function of any value of the variable:

```
dbinom(26, 50, 0.5) = 0.108
```

```
y := dbinom(a, 50, 0.5)
```

y =

	0
0	0.108
1	0.108
2	0.108
3	8.746·10 ⁻³
4	0.108
5	0.108
6	0.096
7	0.108
8	0.042
9	0.112



Calculate the cumulative function for any value of variable:

$$\text{pbinom}(5, 50, 0.5) = 2.105 \times 10^{-9}$$

$$\text{pbinom}(a, 50, 0.5) =$$

	0
0	0.664
1	0.664
2	0.664
3	0.016
4	0.664
5	0.444
6	0.76
7	0.664
8	0.101
9	0.556

By using Built-in function:

If $x \sim \text{Binomial}(15, 0.8)$, then:

1. Drew a random sample of size 17:

$\text{rbinom}(17, 15, 0.8)$

2. $P(x=3)$

$\text{dbinom}(3, 15, 0.8) = 9.542 \times 10^{-7}$

3. $P(x \leq 2)$

$\text{pbinom}(5, 15, 0.8) = 0.0001123$

or

$\text{dbinom}(0, 15, 0.8) + \text{dbinom}(1, 15, 0.8) + \text{dbinom}(2, 15, 0.8)$

4. $P(x < 5)$

$P(x < 5) = P(x \leq 4) = \text{pbinom}(4, 15, 0.8) = 0.00001246$

5. $P(2 \leq x \leq 4)$

$P(2 \leq x \leq 4) = P(x \leq 4) - P(x \leq 2) = \text{pbinom}(4, 15, 0.8) - \text{pbinom}(2, 15, 0.8)$

Rules for discrete probability distribution:

$$P(a < x < b) = P(x < b) - P(x < a)$$

$$P(a \leq x < b) = P(x < b) - P(x \leq a)$$

$$P(a < x \leq b) = P(x \leq b) - P(x < a)$$

$$P(a \leq x \leq b) = P(x \leq b) - P(x \leq a)$$

$$P(x > a) = 1 - P(x \leq a) = 1 - F(a)$$

$$P(x \geq a) = 1 - P(x < a)$$

$$P(x < b) = 1 - P(x \geq b)$$

$$P(x \leq b) = 1 - P(x > b) = F(b)$$

$$P(x < a) = P(x \leq a - 1)$$