بسم الله الرحمن الرحيم

King Abdulaziz University

Exam One

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- أنقر على Start لبداء الاختبار.
- يحتوي هذا الأختبار على خمسة وأربعين سؤالاً.
- عند الانتهاء من الاختبار أنقر على End للحصول على النتيجة.
- بالتوفيق إن شاء الله.

Calculus I
Math110

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Version 1.0

Enter Name: ____________

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Start Answer each of the following.

1. Solve the inequality $2x - 3 < -7$

- ☐ $(-\infty, -2)$
- ☐ $(-\infty, -2]$
- ☐ $(-2, \infty)$
- ☐ $[-2, \infty)$

2. Solve the inequality $-2 < 2 - 2x < 3$

- ☐ $[-2, 1/2]$
- ☐ $[-1/2, 2]$
- ☐ $(-2, 1/2)$
- ☐ $(-1/2, 2)$

3. Solve the inequality $x^2 + 4x + 3 < 0$

- ☐ $(-\infty, -3) \cup (-1, \infty)$
- ☐ $(-\infty, -3] \cup [-1, \infty)$
- ☐ $(-3, -1)$
- ☐ $[-3, -1]$

4. Solve $|2x + 1| < 1$

- ☐ $[-1, 0]$
- ☐ $(-1, 0)$
- ☐ $(-\infty, -1) \cup (0, \infty)$
- ☐ $(-\infty, -1] \cup [0, \infty)$

5. Solve $|2x+1| > 2$

- ☐ $(-\infty, -3/2) \cup (1/2, \infty)$
- ☐ $(-\infty, -3/2] \cup [1/2, \infty)$
- ☐ $(-3/2, 1/2)$
- ☐ $[-3/2, 1/2]$

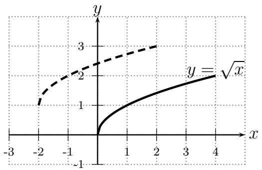
6. Solve $\dfrac{x+2}{x-2} \geq 0.$

- ☐ $(-\infty, -2) \cup (2, \infty)$
- ☐ $(-\infty, -2] \cup (2, \infty)$
- ☐ $(-2, 2)$
- ☐ $(-\infty, -2] \cup [2, \infty)$

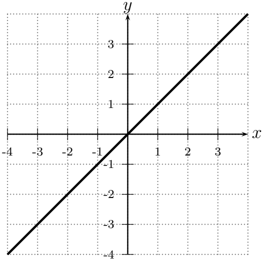
7. The distance between the points $(5,2),(1,-1)$ is

- ☐ 5
- ☐ -5
- ☐ $\sqrt{5}$
- ☐ $-\sqrt{5}$

8. The accompanying figure shows the graph of $y=\sqrt{x}$



- ☐ $y=\sqrt{x+2}-1$
- ☐ $y=\sqrt{x-2}+1$
- ☐ $y=\sqrt{x-2}-1$
- ☐ $y=\sqrt{x+2}+1$



9. The accompanying figure shows the graph of

- [ ] $f(x) = x$
- [ ] $f(x) = |x|$
- [ ] $f(x) = x^3$
- [ ] $f(x) = x^2$

10. Find the slope m of the line through the points $(3, -6)$ and $(1, -1)$.

- [ ] $m = \frac{5}{2}$
- [ ] $m = \frac{-2}{5}$
- [ ] $m = \frac{-5}{2}$
- [ ] $m = \frac{2}{5}$

11. Find a second point on the line with slope $\frac{-1}{2}$ and passes through $(1, 2)$.

- $(3, 1)$
- $(0, 4)$
- $(3, 3)$
- $(4, 0)$

12. Determine if the two lines are parallel, perpendicular, or neither. $y – 3x – 1 = 0$ and $9y + 3x = -6$.

- perpendicular
- parallel
- neither

13. Find an equation of the line through the point $(3, 1)$ and perpendicular to the line $y - 2x = 1$.

- [ ] $y = -2x + 7$
- [ ] $y = 2x - 5$
- [ ] $y = \frac{1}{2}x - \frac{1}{2}$
- [ ] $y = \frac{-1}{2}x + \frac{5}{2}$

14. Find the domain of the function $f(x) = x - 1$.

- [ ] $\mathbb{R}$
- [ ] $(-\infty, 1) \cup (1, \infty)$
- [ ] $\mathbb{R} \setminus \{-1\}$
- [ ] $(-\infty, 1]$

15. Find the domain of the function $f(x) = \sqrt{2x-3}$.

- ☐ $(3/2, \infty)$
- ☐ $\mathbb{R} \setminus \{3/2\}$
- ☐ $(-\infty, 3/2) \cup (3/2, \infty)$
- ☐ $[3/2, \infty)$

16. Find the domain of the function

$$f(x) = \frac{5x+1}{x^2+4}.$$

- ☐ $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$
- ☐ $\mathbb{R}$
- ☐ $(-\infty, 2) \cup (2, \infty)$
- ☐ $(-\infty, 2) \cup (2, \infty)$

17. Find the domain of the function $f(x) = \sqrt[3]{x^2 - 1}$.

- ☐ $(-\infty, -1) \cup (1, \infty)$
- ☐ $\mathbb{R} \setminus \{-1, 1\}$
- ☐ $[-1, 1]$
- ☐ $\mathbb{R}$

18. Find the domain of the function $f(x) = \dfrac{5x + 1}{x^2 - x - 6}$.

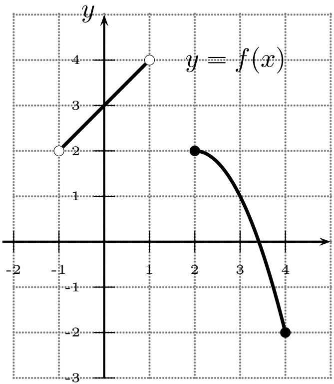
- ☐ $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$
- ☐ $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$
- ☐ $(-\infty, -2) \cup (3, \infty)$
- ☐ $(-\infty, -3] \cup [2, \infty)$

19. The function $f(x) = 4x^3 - 5x^5$, is

- [ ] Even
- [ ] Even and odd
- [ ] Odd
- [ ] Neither even nor odd

20. The accompanying figure shows the graph of $y = f(x)$. Then the domain of f is

- [ ] $[-1, 1) \cup [2, 4]$
- [ ] $[-1, 1] \cup [2, 4]$
- [ ] $(-1, 1] \cup [2, 4]$
- [ ] $(-1, 1) \cup [2, 4]$



21. Find the points of intersection of $y = x^2 - 2x + 4$ and $y = 3x - 2$.

- ☐ $(-2, -8), (-3, -11)$
- ☐ $(-3, -11)$
- ☐ $(-2, -8)$
- ☐ $(2, 4), (3, 7)$

22. Covert the $120°$ to radian

- ☐ $2\pi/3$
- ☐ $\pi/3$
- ☐ $4\pi/3$
- ☐ $\pi/6$

23. Covert the $\frac{4\pi}{3}$ to degree

- ☐ 140°
- ☐ 160°
- ☐ 240°
- ☐ 210°

24. $\cos\left(\frac{\pi}{4}\right) =$

- ☐ $\frac{1}{2}$
- ☐ $\frac{1}{\sqrt{2}}$
- ☐ $-\frac{1}{\sqrt{2}}$
- ☐ $\frac{\sqrt{3}}{2}$

25. $\sin\left(\dfrac{5\pi}{3}\right) =$

- ☐ $\frac{1}{2}$
- ☐ $\frac{\sqrt{3}}{2}$
- ☐ $-\frac{\sqrt{3}}{2}$
- ☐ $-\frac{1}{2}$

26. $\cos(-x) =$

- ☐ $-\cos x$
- ☐ $\cos x$
- ☐ $\sin x$
- ☐ $-\sin x$

27. If $\tan x = \frac{5}{12}, \quad 0 < x < \frac{\pi}{2}$ then $\cos x =$

- ☐ $\frac{13}{12}$
- ☐ $\frac{12}{13}$
- ☐ $\frac{5}{13}$
- ☐ $\frac{13}{5}$

28. If $f(x) = x^2 + 1$ and $g(x) = \sqrt{x-1}$, then $(g - f)(x) =$.

- ☐ $\sqrt{x-1} - x^2 - 1$
- ☐ $x^2 + 1 - \sqrt{x-1}$
- ☐ $\sqrt{x-1} - x^2 + 11$
- ☐ $\sqrt{x-1} + x^2 + 1$

29. If $f(x) = \sqrt{x-3}$ and $g(x) = x - 5$, then domain $\left(\frac{f}{g}\right)$ is

- ☐ $\mathbb{R}$
- ☐ $(-\infty, 3)$
- ☐ $[3, 5) \cup (5, \infty)$
- ☐ $[3, \infty)$

30. Find the domain of the function

$$f(x) = \sqrt{25 - x^2} - \sqrt{x^2 - 4}.$$

- ☐ $\mathbb{R}$
- ☐ $[-5, -2] \cup [2, 5]$
- ☐ $\mathbb{R} \setminus \{\pm 2, \pm 5\}$
- ☐ $(-\infty, -5] \cup [5, \infty)$

31. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+3}$, then $(f \circ g)(x) =$

- ☐ $x + 2$
- ☐ $\sqrt{x^2 + 2}$
- ☐ $\sqrt{x+3} - 1$
- ☐ $\sqrt{\sqrt{x^2 - 1} + 2}$

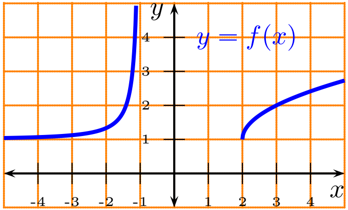
32. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+3}$, then $D(f \circ g) =$

- ☐ $(-\infty, -3) \cup (-3, \infty)$
- ☐ $\mathbb{R}$
- ☐ $(-3, \infty)$
- ☐ $[-3, \infty)$

33. Let $f(x) = \sqrt{x}$ and $g(x) = \sqrt{x-1}$, then $(g \circ f)(x) =$

- ☐ 1
- ☐ $x - 1$
- ☐ $\sqrt{\sqrt{x}}$
- ☐ $\sqrt{\sqrt{x} - 1}$

34. Use the graph of $y = f(x)$ to evaluate $f(3)$



- ☐ 1
- ☐ 2
- ☐ -1
- ☐ 0

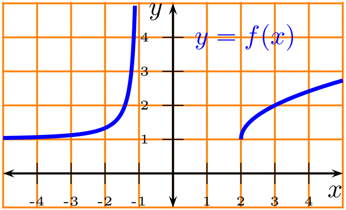
35. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+3}$, then $(g \circ f) =$

- ☐ 1
- ☐ $x + 2$
- ☐ $\sqrt{x+2}$
- ☐ $\sqrt{x^2+2}$
- ☐ $x^2 + 2$

36. Let $f(x) = x^2 - 1$ and $g(x) = \sqrt{x+3}$, then $D(g \circ f)(x) =$

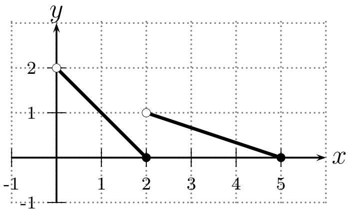
- ☐ $(-\infty, -3) \cup (-3, \infty)$
- ☐ $\mathbb{R}$
- ☐ $(-3, \infty)$
- ☐ $[-3, \infty)$

37. Using the graph to find the domain of f.



- $[1, \infty)$
- $(-\infty, -1] \cup [2, \infty)$
- $(-\infty, -1) \cup [2, \infty)$
- $(1, \infty)$

38. A formula for the function graphed to the right is



☐ $f(x) = \begin{cases} -x+2, & \text{if } 0 \le x \le 2; \\ \frac{-1}{3}x + \frac{5}{3}, & \text{if } 2 \le x \le 5. \end{cases}$

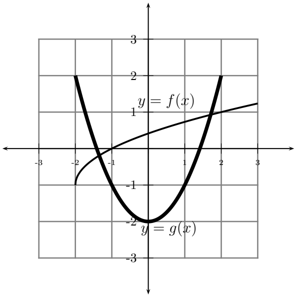
☐ $f(x) = \begin{cases} x-2, & \text{if } 0 \le x \le 2; \\ \frac{-1}{3}x - \frac{5}{3}, & \text{if } 2 \le x \le 5. \end{cases}$

☐ $f(x) = \begin{cases} -x+2, & \text{if } 0 < x \le 2; \\ \frac{-1}{3}x + \frac{5}{3}, & \text{if } 2 < x \le 5. \end{cases}$

☐ $f(x) = \begin{cases} x-2, & \text{if } 0 < x \le 2; \\ \frac{-1}{3}x - \frac{5}{3}, & \text{if } 2 < x \le 5. \end{cases}$

39. Use the graph to find $(f \circ g)(0)$.

- -1
- -2
- 1
- 0



40. The domain of the function $f(x) = \frac{x^2-1}{\sqrt{4-x^2}}$

- $(-\infty, -2) \cup (2, \infty)$
- $(-2, 2)$
- $[-2, 2]$
- $(-\infty, 2)$

41. If the graph of $y = x^2 - 1$ is compress horizontally by a factor of 2. An equation for the new graph is

- ☐ $y = \frac{1}{2}x^2 - \frac{1}{2}$
- ☐ $y = \frac{1}{4}x^2 - 1$
- ☐ $y = 4x^2 - 1$
- ☐ $y = 2x^2 - 2$

42. If the graph of $y = \sqrt{4 - x}$, is reflected across the $y-$axis, then an equation of the new graph is

- ☐ $y = \sqrt{4 + x}$
- ☐ $y = -\sqrt{4 - x}$
- ☐ $y = \sqrt{-4 + x}$
- ☐ $y = -\sqrt{4 + x}$

43. If the domain of $y = f(x)$ is $[-2, 6]$ and $g(x) = f(x-2)$, then domain g is

- ☐ $[-4, 4]$
- ☐ $[0, 8]$
- ☐ $[-4, 12]$
- ☐ $[-1, 3]$

44. Let $f(x) = \dfrac{\sqrt{x-2}}{\sqrt{x-7}}$ Then domain f is

- ☐ $[2, \infty)$
- ☐ $(7, \infty)$
- ☐ $\mathbb{R}$
- ☐ $[7, \infty)$

45. Let $f(x) = \sqrt{\dfrac{x-2}{x+7}}$ Then domain f is

- ☐ $(-\infty, -7) \cup (2, \infty)$
- ☐ $(-\infty, -7) \cup [2, \infty)$
- ☐ $(2, \infty)$
- ☐ $(-7, 2) \cup (2, \infty)$

End

Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to 1.

$$\begin{aligned} 2x - 3 &< -7 \qquad \text{add } 3 \\ 2x &< -7 + 3 \\ 2x &< -4 \qquad \text{divide by } 2 \\ x &< -2 \end{aligned}$$

Hence the solution is $(-\infty, -2)$. ■

Solution to 2.

$$
\begin{aligned}
-2 < 2-2x &< 3 && \text{subtract 2}\\
-4 < \quad -2x &< 1 && \text{divide by } -2 \text{ and switch the inequality}\\
& && \text{because we are dividing by negative number}\\
2 > \qquad x &> \frac{-1}{2} && \text{rewrite the inequality}\\
\frac{-1}{2} < \qquad x &< 2
\end{aligned}
$$

Hence the solution is $(-1/2, 2)$. ■

Solution to 3.

First, we write	$x^2 + 4x + 3 < 0$	we are looking for $(-)$ sign.
Second, set	$x^2 + 4x + 3 = 0$	to find the zeroes.
We have	$(x+3)(x+1) = 0$	by factor.
Hence	$x = -3, -1.$	

Third, use the real line to find the sign of $x+3$ and $x+1$.

$-\infty$	$-\infty < x < -3$	$-3 < x < -1$	$-1 < x < \infty$
Test Value	-4	-2	1
sign of $x+3$	$-4+3=-1$ $(-)$	$-2+3=1$ $(+)$	$1+3=4$ $(+)$
sign of $x+1$	$-4+1=-3$ $(-)$	$-2+1=-1$ $(-)$	$1+1=2$ $(+)$
sign of $(x+3)(x+1)$	$(-)\cdot(-)=+$	$(+)\cdot(-)=-$	$(+)\cdot(+)=+$

Hence the solution is $(-3,-1)$. ■

Solution to 4.

$$
\begin{aligned}
|2x+1| < 1 &\Leftrightarrow -1 < 2x+1 < 1 && \text{Use } |x| < k \Leftrightarrow -k < x < k. \\
&\Leftrightarrow -2 < 2x < 0 && \text{subtract 1 from all sides} \\
&\Leftrightarrow -1 < x < 0 && \text{divide all sides by 2}
\end{aligned}
$$

Hence the interval is $(-1, 0)$. ■

Solution to 5.

$$\begin{aligned} |2x+1| > 2 &\Leftrightarrow 2x+1 > 2 \quad \text{or} \quad 2x+1 < -2. \\ &\Leftrightarrow 2x > 1 \quad \text{or} \quad 2x < -3 \\ &\Leftrightarrow x > \frac{1}{2} \quad \text{or} \quad x < \frac{-3}{2} \end{aligned}$$

Hence the interval is $(-\infty, -3/2) \cup (1/2, \infty)$. ■

Solution to 6. Since $\dfrac{x+2}{x-2} \geq 0$, then we are looking for $(+)$ sign. Next, we find the zeros of the numerator and the denominator. The real zeros of the numerator and the denominator are $x+2=0 \Leftrightarrow x=-2$ and $x-2=0 \Leftrightarrow x=2$. So the expression's test intervals are $(-\infty,-2], [-2,2)$, and $(2,\infty)$. We excluded -2 because we have bigger than sign and excluded 2 because it makes the denominator equal zero and dividing by zero is not allowed. Now, we use the real line to find the sign of $x+2$ and $x-2$.

$-\infty$	-2 (included)	2 (excluded)	∞
Test Value	-3	0	3
sign of $x+2$	$-3+2=-1$ $(-)$	$0+2=2$ $(+)$	$3+2=5$ $(+)$
sign of $x-2$	$-3-2=-5$ $(-)$	$0-2=-2$ $(-)$	$3-2=1$ $(+)$
sign of $(x+2)/(x-2)$	$(-)/(-)=+$	$(+)/(-)=-$	$(+)/(+)=+$

We find the $(+)$signs in the interval $(-\infty,-2]$ or $(2,\infty)$. Hence the solution is $(-\infty,-2]\cup(2,\infty)$. ■

Solution to 7.

$$\begin{aligned} d((5,2),(1,-1)) &= \sqrt{(5-1)^2 + (2-(-1))^2} \\ &= \sqrt{(4)^2 + (3)^2} \\ &= \sqrt{16+9} \\ &= \sqrt{25} \\ &= 5. \end{aligned}$$

■

Solution to 8. The graph of $y = \sqrt{x}$ is shifted to the left 2 units and up one unit. Now, left 2 units means replace x by $x + 2$ and up 1 unit means add 1. Hw=ence the equation is $y = \sqrt{x+2} + 1$. ■

Solution to 9. Clearly it is the graph of $y = x$. ■

Solution to 10. The slope of the line through the points $(3, -6)$ and $(1, -1)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{-1 - (-6)}{1 - 3} = \frac{5}{-2} = -\frac{5}{2}.$$

■

Solution to 11. Since $m = \frac{\Delta y}{\Delta x} = \frac{-1}{2}$, this mean that to get another point from the point $(1, 2)$ just add (or subtract) $\Delta x = 2$ to the x-coordinate of the point $(1, 2)$ and add (or subtract) $\Delta y = -1$ to the y-coordinate of the point $(1, 2)$. Hence $(1 + 2, 2 + (-1)) = (3, 1)$ is a point on the line. ■

Solution to 12.

$$\begin{aligned} y - 3x - 1 &= 0 & 9y + 3x &= -6 \\ y &= 3x + 1 & 9y &= -3x - 6 \\ & & y &= \frac{-1}{3}x - 3 \\ m_1 &= 3 & m_2 &= \frac{-1}{3} \end{aligned}$$

Hence the lines are perpendicular. ■

Solution to 13. we find the slope of the line $y - 2x = 1$

$$\begin{aligned} y - 2x &= 1 && \text{Isolate } y \text{ term.} \\ y &= 2x + 1 && \text{Point-Slope form} \end{aligned}$$

Now, since the line is perpendicular to $y - 2x = 1$ then $m = \frac{-1}{2}$ and passes through the point $(3, 1)$.

$$\begin{aligned} y - y_1 &= m(x - x_1) && \text{Point-Slope form} \\ y - 1 &= -\frac{1}{2}(x - 3) && \text{Substitute} \\ y - 1 &= -\frac{1}{2}x + \frac{3}{2} && \text{Simplify} \\ y &= -\frac{1}{2}x + \frac{3}{2} + 1 && \text{Simplify} \\ Y &= -\frac{1}{2}x + \frac{5}{2} \end{aligned}$$

■

Solution to 14. The function $f(x) = f(x) = x - 1$ is a polynomial, hence

$$D(f) = \mathbb{R}.$$

■

Solution to 15. $f(x) = \sqrt{2x{-}3}$ is even root function. Then it is defined if

$$\begin{aligned} 2x{-}3 &\geq 0 \\ 2x &\geq 3 \\ x &\geq 3/2 \end{aligned}$$

Hence $D(f) = [3/2, \infty)$. ■

Solution to 16. The function is a rational function. The domain is $\mathbb{R} \setminus \{$ zeros of $x^2+4\}$. Now, if $x^2+4=0 \Leftrightarrow x^2=-4$, but this is impossible because the square of any real number is bigger than or equal to zero and never negative. Hence $x^2+4 \neq 0$. Hence $D(f)=\mathbb{R}$. ■

Solution to 17. The function $f(x) = \sqrt[3]{x^2 - 1}$ is an odd function and hence $D(f) = \mathbb{R}$. ■

Solution to 18. The function is a rational function. The domain is $\mathbb{R} \setminus \{$ zeros of $x^2 - x - 6\}$.

$$\begin{aligned} x^2 - x - 6 &= 0 \\ (x-3)(x+2) &= 0 \\ x - 3 &= 0 \text{ or } x + 2 = 0. \\ x &= 3 \text{ or } \qquad x = -2. \end{aligned}$$

Hence $D(f) = \mathbb{R} \setminus \{-2, 3\} = (-\infty, -2) \cup (-2, 3) \cup (3, \infty)$. ∎

Solution to 19.

$$\begin{aligned} f(-x) &= 4(-x)^3 - 5(-x)^5 \\ &= -4x^3 + 5x^5 \neq f(x) \text{ not even} \\ -f(-x) &= -(-4x^3 + 5x^5) = 4x^3 - 5x^5 = f(x) \end{aligned}$$

Hence f is odd. ■

Solution to 20. Looking at the graph we can see that any vertical line between -1 and 1, $-1, 1$, are no included is going to intersects the graph of f and any vertical line between 2 and 4 $2, 4$ are included is going to intersects the graph of f Hence the domain of f is $(-1, 1) \cup [2, 4]$ ■

Solution to 21. To determine the points of intersection of the two graphs, we set the two functions equal and solve for x :

$$\begin{aligned} x^2 - 2x + 4 &= 3x - 2 \\ x^2 - 5x + 6 &= 0 \\ (x-3)(x-2) &= 0 \\ x - 3 = 0 \text{ or } &\quad x - 2 = 0 \\ x = 3 \text{ or } &\quad x = 2. \end{aligned}$$

Now, if $x = 2 \Rightarrow y = 3(2) - 2 = 6 - 2 = 4$, and if $x = 3 \Rightarrow y = 3(3) - 2 = 9 - 2 = 7$. Hence the points of intersection are $(2, 4), (3, 7)$. ■

Solution to 22.

$$
\begin{aligned}
120^\circ &= \cancel{120}^{2} \cdot \frac{\pi}{\cancel{180}^{3}} \\
&= \frac{2\pi}{3}
\end{aligned}
$$

■

Solution to 23.

$$\frac{4\pi}{3} = \frac{4\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{180}^{60}}{\cancel{\pi}}$$
$$= 4 \cdot 60 = 240^\circ$$

■

Solution to 24. We see from the table below that

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$$

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin x$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos x$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0

■

Solution to 25. Since $\frac{5\pi}{3} = 2\pi - \frac{\pi}{3}$, hence $\frac{5\pi}{3}$ lies in the fourth quadrant. Therefore $\sin\left(\frac{5\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.

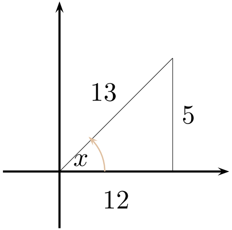
Another Solution:

$$\begin{aligned}\sin\left(\frac{5\pi}{3}\right) &= \sin\left(2\pi - \frac{\pi}{3}\right)\\ &= \sin(2\pi)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{3}\right)\cos(2\pi)\\ &= (0)(\frac{1}{2}) - (\frac{\sqrt{3}}{2})(1)\\ &= -\frac{\sqrt{3}}{2}\end{aligned}$$

■

Solution to 26. Since $\cos x$ is an even function, then $\cos(-x) = \cos x$ ■

Solution to 27. Since $\tan x = \dfrac{5}{12} = \dfrac{\text{opposite}}{\text{adjacent}}$ we draw a triangle is shown to the right: Now, $H^2 = 5^2 + 12^2 = 25 + 144 = 169 \Rightarrow H = 13$. Since $x \in \left(0, \frac{\pi}{2}\right)$, then $\sin x \geq 0$ and $\cos x \geq 0$. Hence $\cos x = \frac{12}{13}$.



■

Solution to 28.

$$\begin{aligned}(g-f)(x) &= g(x)-f(x) && \text{by definition.}\\ &= \sqrt{x-1}-(x^2+1)\\ &= \sqrt{x-1}-x^2-1.\end{aligned}$$

■

Solution to 29. The function $f(x) = \sqrt{x-3}$ is an even root function, hence f is defined if

$$x - 3 \geq 0 \Leftrightarrow x \geq 3.$$

Hence $D(f) = [3, \infty)$. The function $g(x) = x - 5$, is a polynomial, hence $D(g) = \mathbb{R}$. Now,

$$\begin{aligned} D(f/g) &= (D(f) \cap D(g)) \setminus \{x : g(x) = 0\} \\ &= (\mathbb{R} \cap [3, \infty)) \setminus \{x : x - 5 = 0\} \\ &= [3, \infty) \setminus \{x : x - 5 = 0\} = [3, \infty) \setminus \{5\} \\ &= [3, 5) \cup (5, \infty). \end{aligned}$$

■

Solution to 30. The function $f(x) = \sqrt{25 - x^2} - \sqrt{x^2 - 4}$ is the difference of $\sqrt{25 - x^2}$ and $\sqrt{x^2 - 4}$.

Hence $D(f) = D(\sqrt{25 - x^2}) \cap D(\sqrt{x^2 - 4})$.

The function $\sqrt{25 - x^2}$ is an even root function, then

$$
\begin{aligned}
25 - x^2 \geq 0 &\Leftrightarrow x^2 \leq 25 && \text{move } x^2 \text{ to the other side} \\
&\Leftrightarrow \sqrt{x^2} \leq 5 && \text{take the square root} \\
&\Leftrightarrow |x| \leq 5 && \sqrt{x^2} = |x| \text{ use properties of} \\
&\Leftrightarrow -5 \leq x \leq 5 && \text{absolute value inequality}
\end{aligned}
$$

Hence $D(\sqrt{25 - x^2}) = [-5, 5]$.

The function $\sqrt{x^2 - 4}$ is an even root function, then

$$
\begin{aligned}
x^2 - 4 \geq 0 &\Leftrightarrow \sqrt{x^2} \geq 2 && \text{take the square root} \\
&\Leftrightarrow |x| \geq 2 && \sqrt{x^2} = |x| \text{ use properties of} \\
&\Leftrightarrow x \geq 2 \text{ or } x \leq -2 && \text{absolute value inequality}
\end{aligned}
$$

Hence $D(\sqrt{x^2 - 4}) = (-\infty, -2] \cup [2, \infty)$.

$$\begin{aligned} D(f) &= D(\sqrt{25 - x^2}) \cap D(\sqrt{x^2 - 4}) \\ &= [-5, 5] \cap (-\infty, -2] \cup [2, \infty) \\ &= [-5, -2] \cup [2, 5]. \end{aligned}$$

■

Solution to 31.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(\sqrt{x+3}) \\ &= (\sqrt{x+3})^2 - 1 \\ &= x + 3 - 1 = x + 2.\end{aligned}$$

■

Solution to 32. $D(f \circ g) = D(g) \cap D(f(g(x)))$. Since $(f \circ g)(x) = f(g(x)) = x + 2$, then $D(g(f(x))) = \mathbb{R}$. Since $g(x) = \sqrt{x+3}$, is an even root function,
then $x + 3 \geq 0 \Leftrightarrow x \geq -3$. Hence $D(f) = [-3, \infty)$.

$$\begin{aligned} D(f \circ g) &= D(g) \cap D(f(g(x))) \\ &= [-3, \infty) \cap \mathbb{R} \\ &= [-3, \infty). \end{aligned}$$

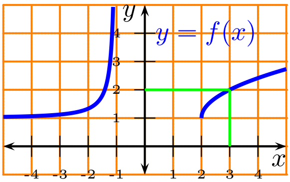
■

Solution to 33.

$$\begin{aligned}(g \circ f)(x) &= g(f(x)) \\ &= g(\sqrt{x}) \\ &= \sqrt{\sqrt{x} - 1}.\end{aligned}$$

■

Solution to 34. From the graph we see that $f(3) = 2$.



■

Solution to 35.

$$
\begin{aligned}
(g \circ f)(x) &= g(f(x)) \\
&= g(x^2 - 1) \\
&= \sqrt{x^2 - 1 + 3} = \sqrt{x^2 + 2}.
\end{aligned}
$$

■

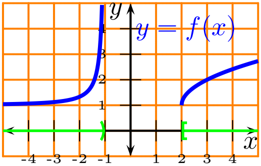
Solution to 36. $D(g \circ f) = D(f) \cap D(g(f(x)))$. Since $(g \circ f)(x) = g(f(x)) = \sqrt{x^2+2}$, then $D(g(f(x))) = \mathbb{R}$. Since $f(x) = x^2 - 1$, is a polynomial, then $D(f) = \mathbb{R}$.

$$\begin{aligned} D(g \circ f) &= D(f) \cap D(g(f(x))) \\ &= \mathbb{R} \cap \mathbb{R} \\ &= \mathbb{R}. \end{aligned}$$

■

Solution to 37. Looking at the graph we see that

$$D(f) = (-\infty, -1) \cup [2, \infty).$$



Solution to 38. You should do it yourself....................good luck ■

Solution to 39. You should do it yourself....................good luck ■

Solution to 40. The function $f(x) = \frac{x^2-1}{\sqrt{4-x^2}}$ is the quotient of $x^2 - 1$ and $\sqrt{4-x^2}$. Since $x^2 - 1$ is a polynomial, hence $D(x^2 - 1) = \mathbb{R}$. The function $\sqrt{4-x^2}$ is defined if

$$\begin{aligned} 4 - x^2 \geq 0 &\Leftrightarrow -x^2 \geq -4 \\ &\Leftrightarrow x^2 \leq 4 \\ &\Leftrightarrow |x| \leq 2 \\ &\Leftrightarrow -2 \leq x \leq 2. \end{aligned}$$

Hence $D(\sqrt{4-x^2}) = [-2, 2]$. Thus

$$\begin{aligned} D(f) &= \{D(x^2 - 1) \cap D(\sqrt{4-x^2})\} \setminus \{x : \sqrt{4-x^2} = 0\} \\ &= \{\mathbb{R} \cap [-2, 2]\} \setminus \{x : 4 - x^2 = 0\} \\ &= [-2, 2] \setminus \{-2, 2\} = (-2, 2). \end{aligned}$$

■

Solution to 41. Since the graph compressed horizontally by factor of 2. We replace x by $2x$ in the original equation. So we get $y = (2x)^2 - 1 = 4x^2 - 1$. ■

Solution to 42. Reflection about the $y-$axis mean replace x by $-x$ in the original equation.
Thus $y = \sqrt{4-(-x)} = \sqrt{4+x}$. ■

Solution to 43. g is the shift of f to the right 2 units. Hence $D(g) = 2 + D(f) = 2 + [-2, 6] = [0, 8]$ ■

Solution to 44. The $f(x) = \dfrac{\sqrt{x-2}}{\sqrt{x-7}}$ is the quotient of $\sqrt{x-2}$ and $\sqrt{x-7}$. Hence $D(f) = D(\sqrt{x-2}) \cap D(\sqrt{x-7})$ Now, $x - 2 \geq 0 \Leftrightarrow x \geq 2$, hence $D(\sqrt{x-2}) = [2, \infty)$ and $x - 7 \geq 0 \Leftrightarrow x \geq 7$, hence $D(\sqrt{x-7}) = [7, \infty)$.

Thus $D(f) = D(\sqrt{x-2}) \cap D(\sqrt{x-7}) \setminus \{x : x - 7 = 0\} = [2, \infty) \cap [7, \infty) \setminus \{7\} = (7, \infty)$ ■

Solution to 45. The function $f(x)=\sqrt{\dfrac{x-2}{x+7}}$ is an even root function, hence $\dfrac{x-2}{x+7}\geq 0$ (we are looking for $(+)$ sign). Now, to solve the inequality $\dfrac{x-2}{x+7}\geq 0$, we find the zeros of the numerator and the denominator. $x-2=0 \Rightarrow x=2$ is the zero of numerator and $x+7=0 \Leftrightarrow x=-7$ is the zero of the denominator. Now, we use the real line to find the sign of each expression $x-2$ and $x+7$.

$-\infty$	-7	2	∞
Test Value	-8	0	3
sign of $x-2$	$-8-2=-10$ $(-)$	$0-2=-2$ $(-)$	$3-2=1$ $(+)$
sign of $x+7$	$-8+7=-1$ $(-)$	$0+7=7$ $(+)$	$3+7=10$ $(+)$
sign of $\frac{x-2}{x+7}$	$(-)/(-)=+$	$(-)/(+)=-$	$(+)/(+)=+$

Hence $D(f)=(-\infty,-7)\cup[2,\infty)$. ■