OPTICS

PHYS 311

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DIFFRACTION

INTRODUCTION

DIFFRACTION

A general characteristic of wave phenomenon which occurs whenever a portion of a wavefront is obstructed in some way.

***It is any deviation from geometrical optics as a result of obstruction of a wavefront of light*.**

Diffraction is a consequence of the wave character of light.

INTRODUCTION

- **Diffraction is also seen when obstacles causes local variation in the amplitude or phase of the wavefront of the transmitted light. ← Tiny bubbles & imperfection in a glass lens.**

- **Diffraction also cause blurriness on the edge of optical images. ← *Diffraction-limited optics*.**

INTRODUCTION

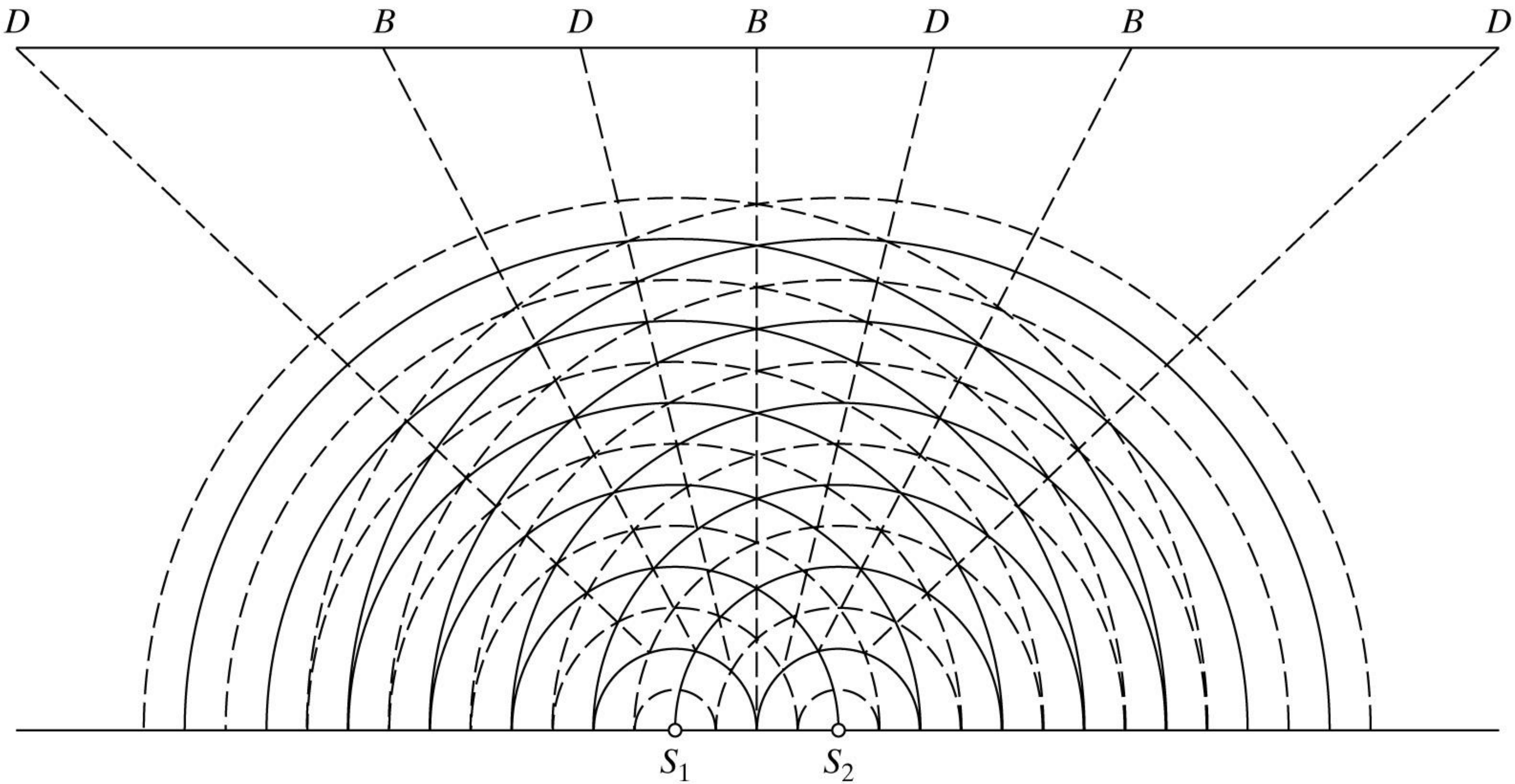
Huygens-Fresnel principle:

Huygens ← every point on a wavefront of light can be considered a source of secondary spherical wavelets.

Fresnel ← The actual field at any point beyond the wavefront is a superposition of all these wavelets, taking into account both their amplitudes and phases.

What is the difference between interference and diffraction??

INTRODUCTION



INTRODUCTION

Classification of Diffraction Effects

Bases on the mathematical <u>approximations</u> possible when calculating the resultant fields.

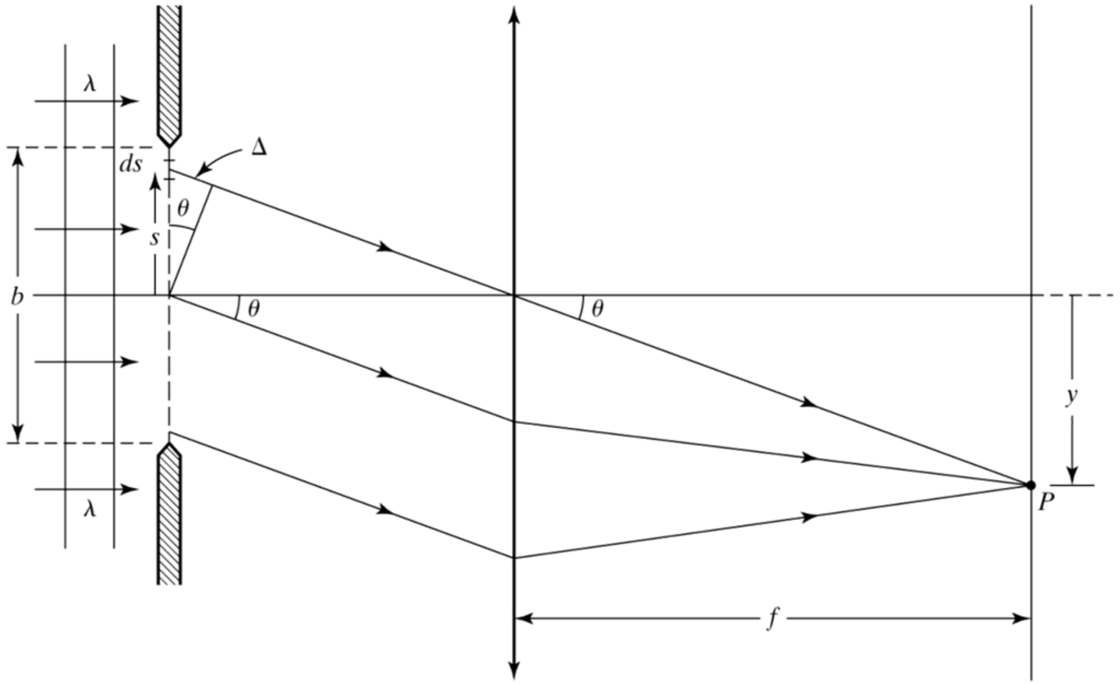
<u>Fraunhofer (far-field) diffraction:</u>

- **Both source and screen are far enough from the diffraction aperture.**
- **Wavefronts arriving at aperture and screen maybe considered plane.**

•**<u>Fresnel (near-field) diffraction:</u>**

- **Both source and screen are near the diffraction aperture.**
- **The curvature of the wavefronts must be considered.**

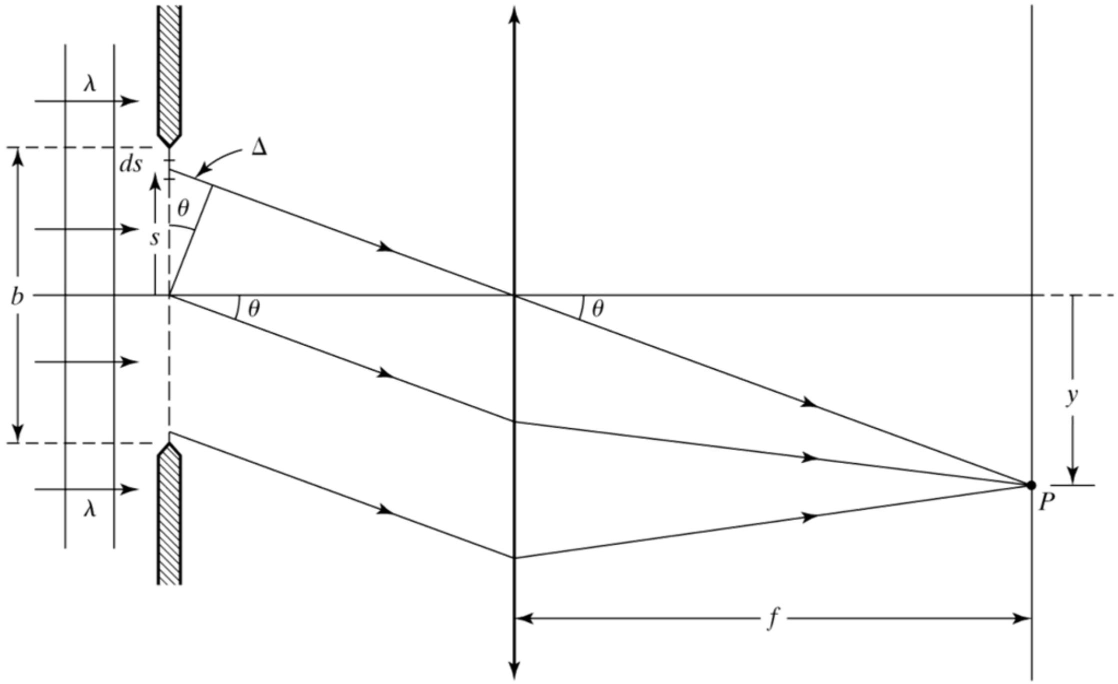
DIFFRACTION FROM A SINGLE SLIT



For Fraunhofer diffraction

- Incident waves are parallel ← How??
- Observation screen at infinity ← How??
- Light reaching P is due to parallel rays from different portions of the wavefront at the slit.
- The waves do NOT arrive at P in phase ← Why??

DIFFRACTION FROM A SINGLE SLIT

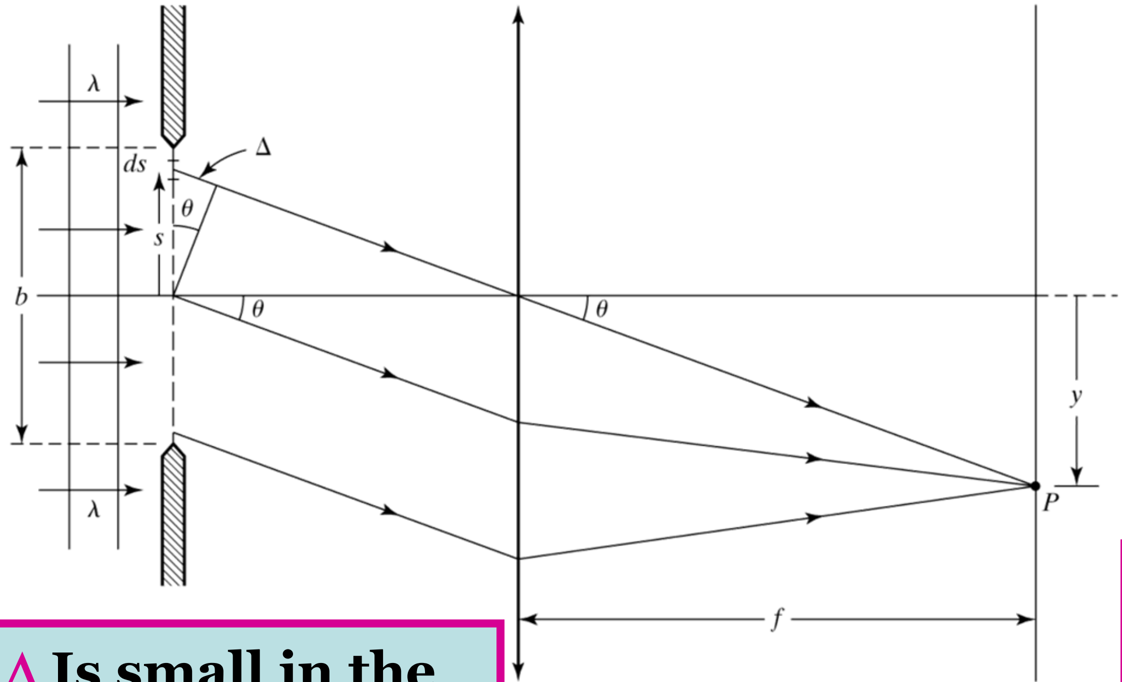


optical path-length of a ray from the center of the slit is shorter than one leaving a distance *s* above the optical axis by an amount Δ

How to find the diffraction effect at P?

- **Consider each interval of length *ds* as a source.**
- **Integrate over the entire slit width *b* to find the result of all such sources.**

DIFFRACTION FROM A SINGLE SLIT



Each interval contributes a spherical wavelet at P whose magnitude:

$$dE_p = \left(\frac{E_L ds}{r}\right) e^{i(kr - \omega t)}$$

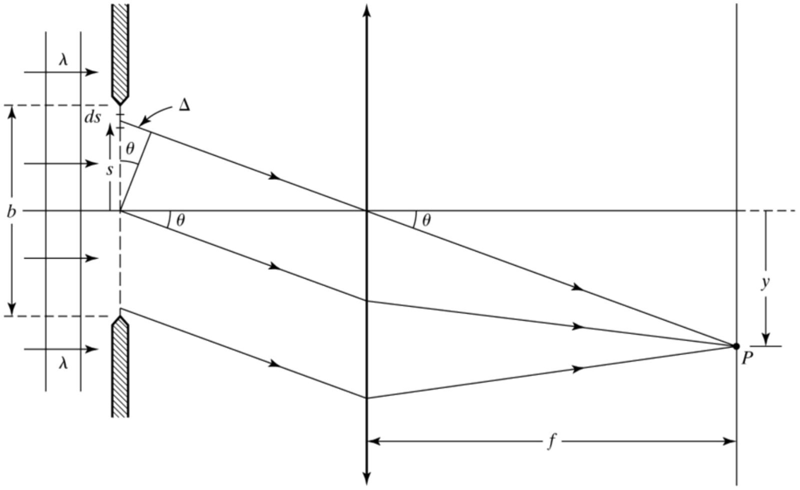
Let us set *r* = *r*$_o$ for the wave from the center of the slit (at *s* = *o*), then:

$$dE_p = \left(\frac{E_L ds}{r_o + \Delta}\right) e^{i[k(r_o + \Delta) - \omega t]} = \left(\frac{E_L ds}{r_o + \Delta}\right) e^{i(kr_o - \omega t)} e^{ik\Delta}$$

Δ Is small in the amplitude factor but can't be ignored in the phase factor!!

Also Δ = s sin θ

DIFFRACTION FROM A SINGLE SLIT



$$dE_p = \left(\frac{E_L ds}{r_o}\right) e^{i(kr_o - \omega t)} e^{iks\sin\theta}$$

Integrate over the width of the slit:

$$E_p = \int_{slit} dE_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \int_{-b/2}^{b/2} e^{iks\sin\theta} ds$$

Inserting the limits of integration:

$$E_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \left(\frac{e^{iks\sin\theta}}{iks\sin\theta}\right)_{-b/2}^{b/2}$$

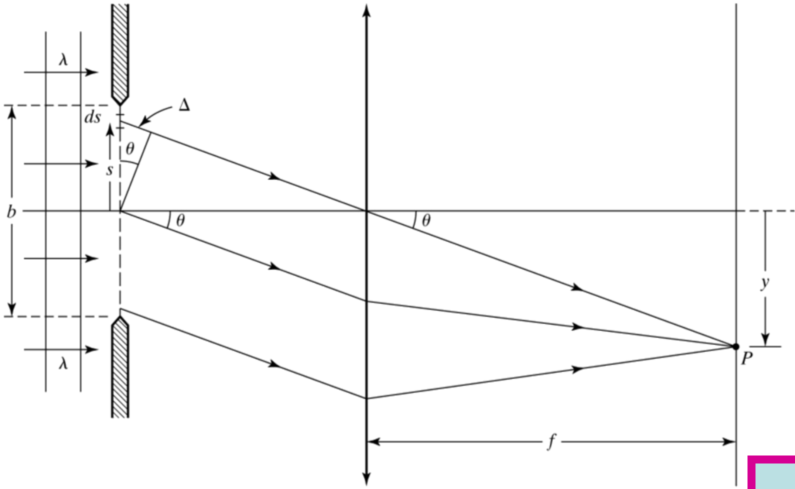
$$\beta \equiv \frac{1}{2} kb\sin\theta$$

$$E_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \frac{e^{(ikb\sin\theta)/2} - e^{-(ikb\sin\theta)/2}}{ik\sin\theta}$$

Using Euler's Equation:

$$E_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \frac{b(e^{i\beta} - e^{-i\beta})}{2i\beta} = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \frac{b(2i\sin\beta)}{2i\beta}$$

DIFFRACTION FROM A SINGLE SLIT



Simplifying..

$$\beta \equiv \frac{1}{2} kb\, sin\,\theta$$

$$E_p = \frac{E_L b}{r_o} \frac{sin\,\beta}{\beta} e^{i(kr_o - \omega t)}$$

β Can be interpreted as a phase difference ← How??

Phase difference = k Δ → Δ=(b/2) sinθ

The amplitude of the electric field at P is:

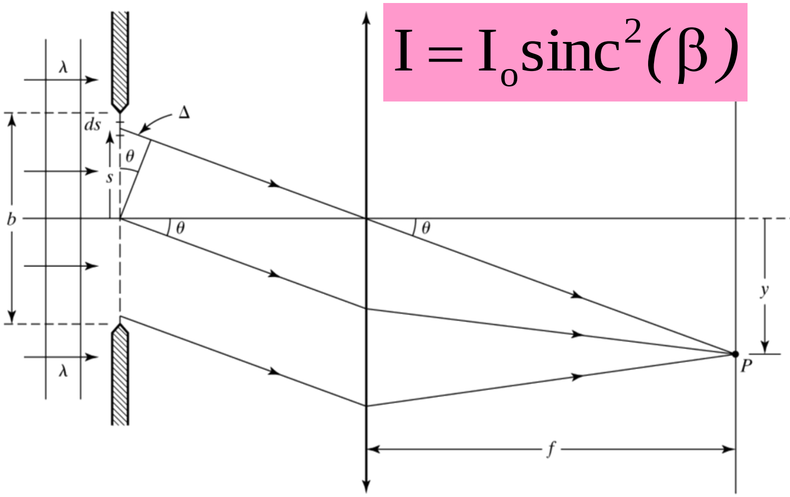
$$E_o = \frac{E_L b}{r_o} \frac{sin\,\beta}{\beta}$$

The irradiance at P is:

$$I = \left(\frac{\varepsilon_o c}{2}\right) E_o^2 = \frac{\varepsilon_o c}{2} \left(\frac{E_L b}{r_o}\right)^2 \frac{sin^2\beta}{\beta^2}$$

$$I = I_o \left(\frac{sin^2\beta}{\beta^2}\right) = I_o \mathbf{sinc}^2(\beta)$$

DIFFRACTION FROM A SINGLE SLIT



$$I = I_o \operatorname{sinc}^2(\beta)$$

We can now plot the variation of irradiance with vertical displacement y from the symmetry axis at the screen..

We know that:

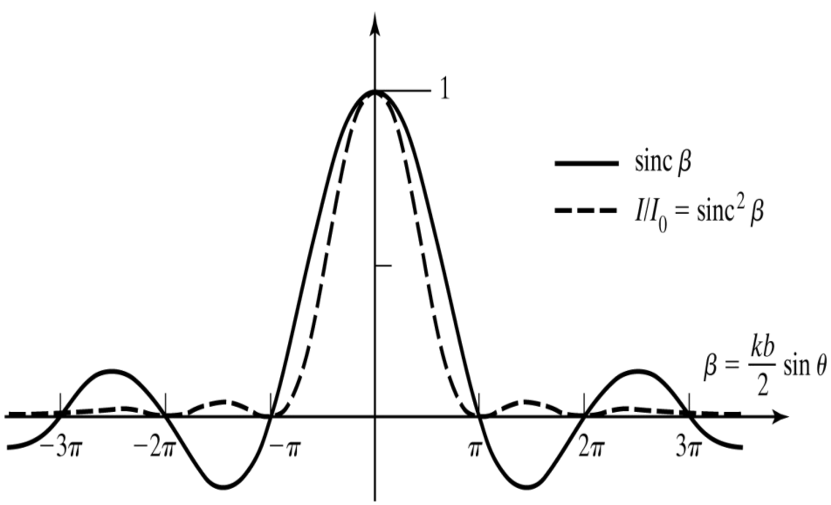
$$\lim_{\beta\to 0} \operatorname{sinc}(\beta) = \lim_{\beta\to 0}\left(\frac{\sin\beta}{\beta}\right) = 1$$

And..

$$\operatorname{sinc}(\beta) = 0 \quad \text{when} \quad \sin\beta = 0$$

That is..

$$\beta = \frac{1}{2}(kb\sin\theta) = m\pi \quad , m = \pm 1, \pm 2, \ldots$$



The zeros of the Irradiance is:

$$m\lambda = b\sin\theta \quad , m = \pm 1, \pm 2, \ldots$$

DIFFRACTION FROM A SINGLE SLIT

The zeros of the Irradiance is:

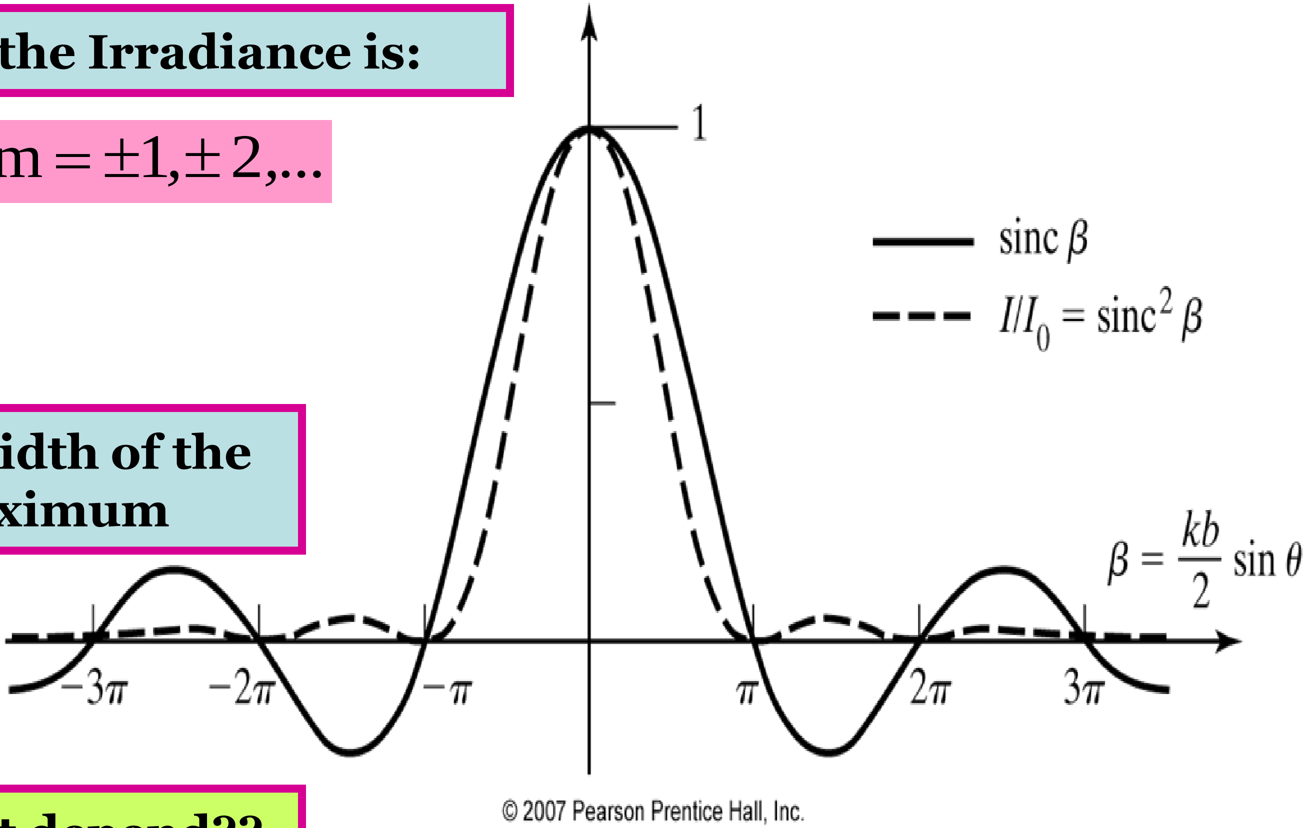
$$m\lambda = b\,sin\,\theta \quad , m = \pm 1, \pm 2, \ldots$$

$$y_m \cong \frac{m\lambda f}{b}$$

The angular width of the central maximum

$$\Delta\theta = \frac{2\lambda}{b}$$

On what does it depend??



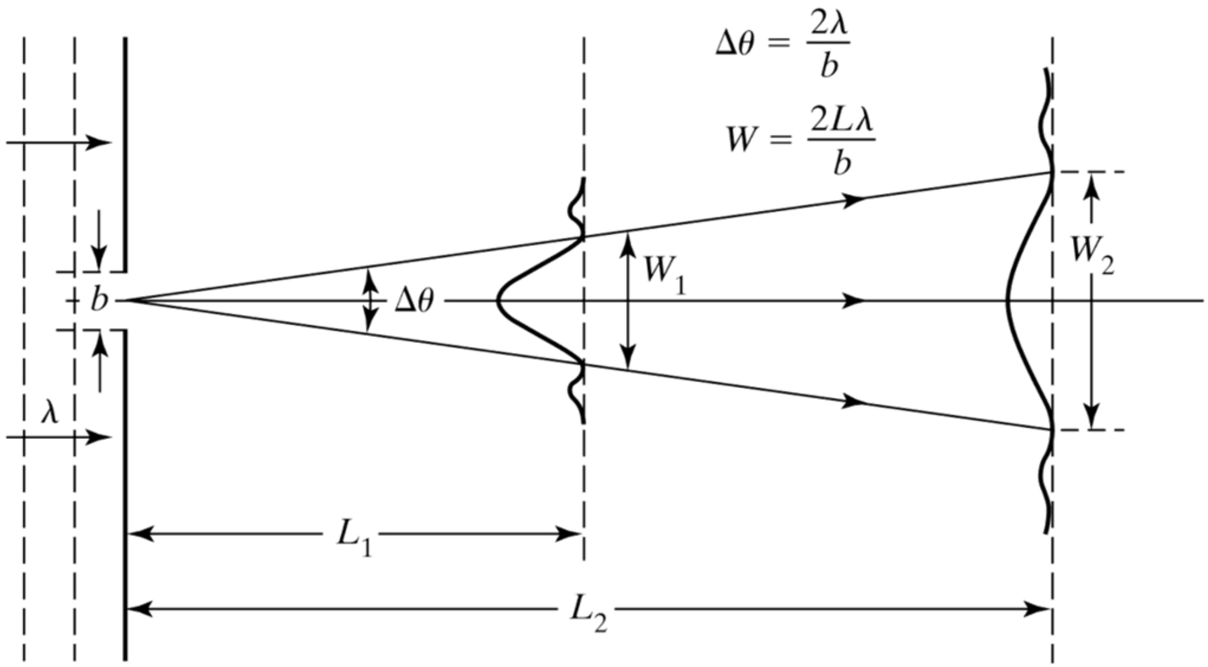
- **The central maximum represent the image of the slit..**
- **Secondary maxima cause the blurring of the image due to diffraction..**

BEAM SPREADING

$$\Delta\theta = \frac{2\lambda}{b}$$

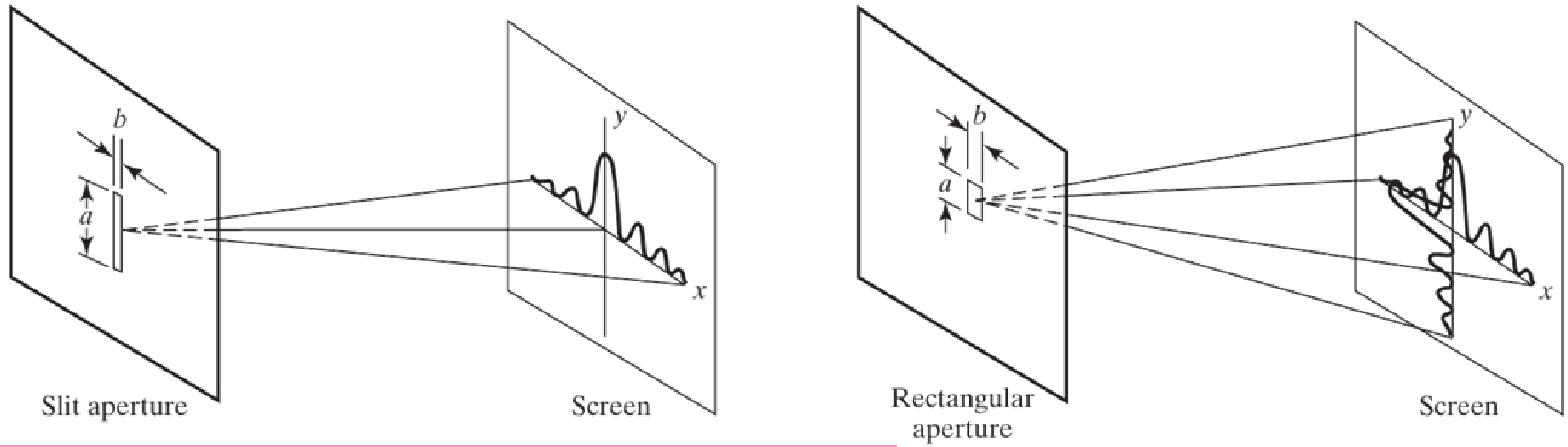
The angular spread $\Delta\theta$ of the central maximum in the far field is independent of the distance between the aperture and screen.

$$W = L\Delta\theta = \frac{2L\lambda}{b}$$



RECTANGULAR AND CIRCULAR APERTURES

When both dimension of the slit are comparable and small, each produces appreciable spreading.



$$I = I_o\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad \text{where} \quad \alpha \equiv \left(\frac{k}{2}\right) a \sin\theta$$

For aperture dimension a, we write for irradiance

$$y_m = \frac{m\lambda f}{b} \qquad \text{or} \qquad x_n = \frac{n\lambda f}{a}$$

The two dimensional pattern gives zero irradiance for points x and y.

$$I = I_o(\text{sinc}^2\beta)(\text{sinc}^2\alpha)$$

Irradiance over the screen is the product of the irradiance in each dimension

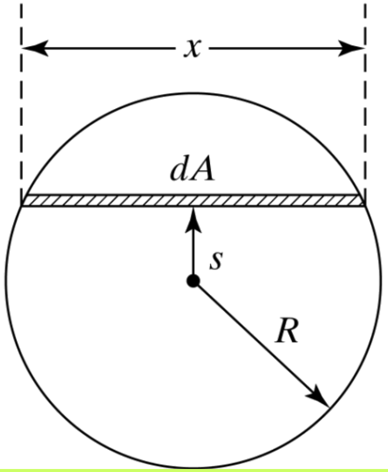
RECTANGULAR AND CIRCULAR APERTURES

In the case of a circular apertures:

1. **We replace of the incremental electric field amplitude $E_L\, ds/r_o$ by $E_A\, dA/r_o$.**
2. **We integrate over the aperture area.**

$$E_p = \frac{E_A}{r_o} e^{i(kr_o - \omega t)} \iint_{Area} e^{iks\sin\theta} dA$$

$$x = 2\sqrt{R^2 - s^2}$$



We take the rectangular strip of area *dA=x ds*

$$E_p = \frac{2E_A}{r_o} e^{i(kr_o - \omega t)} \int_{-R}^{R} e^{iks\sin\theta} \sqrt{R^2 - s^2}\, ds$$

Substituting $\upsilon = s/R$ and $\gamma = kR\sin\theta$

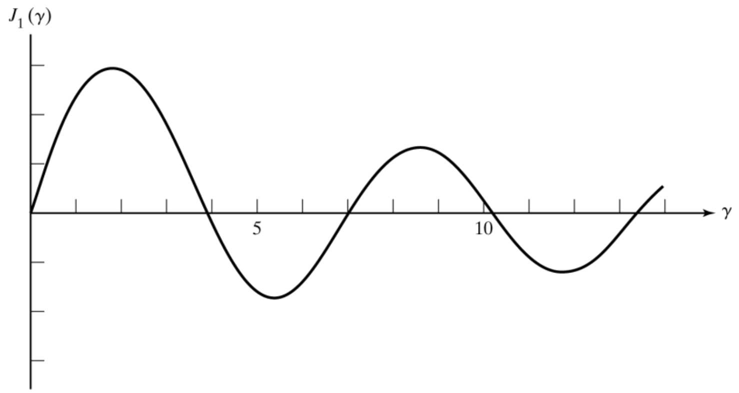
$$E_p = \frac{2E_A R^2}{r_o} e^{i(kr_o - \omega t)} \int_{-1}^{+1} e^{i\gamma\upsilon} \sqrt{1 - \upsilon^2}\, d\upsilon$$

$$\int_{-1}^{+1} e^{i\gamma\upsilon} \sqrt{1 - \upsilon^2}\, d\upsilon = \frac{\pi J_1(\gamma)}{\gamma}$$

RECTANGULAR AND CIRCULAR APERTURES

$$J_1(\gamma) = \frac{\gamma}{2} - \frac{(\gamma/2)^3}{1^2.2} + \frac{(\gamma/2)^5}{1^2.2^2.3} - \dots$$

$$\int_{-1}^{+1} e^{i\gamma\upsilon}\sqrt{1-\upsilon^2}\,d\upsilon = \frac{\pi J_1(\gamma)}{\gamma}$$



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J1(γ)/γ has the limit of ½ as γ → 0

For a circular aperture, instead of a sine function we use a Bessel function.

$$E_p = \frac{2E_A R^2}{r_o} e^{i(kr_o - \omega t)} \frac{\pi J_1(\gamma)}{\gamma}$$

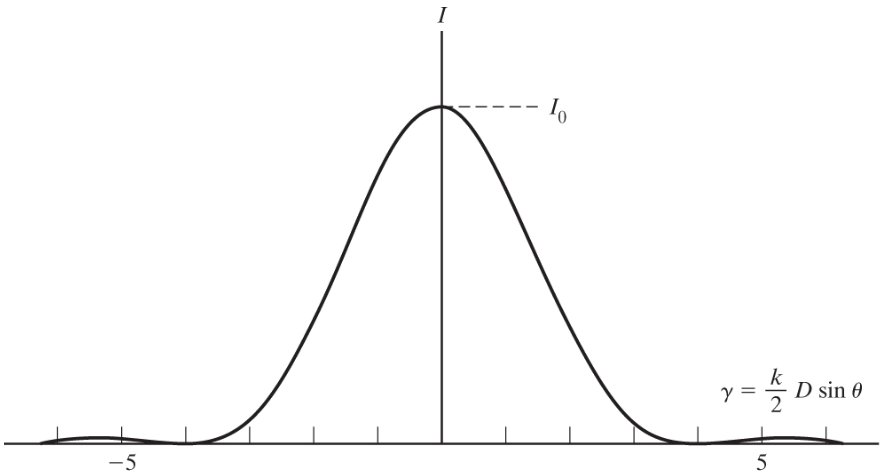
$$\gamma = kR\sin\theta$$

$$I = I_o\left(\frac{2J_1(\gamma)}{\gamma}\right)^2 \quad , \quad \text{where } \gamma \equiv \frac{1}{2}kD\sin\theta$$

RECTANGULAR AND CIRCULAR APERTURES

The pattern is symmetrical about the optical axis through the center of the circular aperture and has its first zero when γ= 3.832

$$I = I_o\left(\frac{2J_1(\gamma)}{\gamma}\right)^2 \quad , \quad \text{where } \gamma \equiv \frac{1}{2} kD\,sin\,\theta$$



(a)

	γ	$I/I_0 = (2J_1(\gamma)/\gamma)^2$
1st Maximum	0	1
1st Zero	3.832	0
2nd Maximum	5.136	0.0175
2nd Zero	7.016	0
3rd Maximum	8.417	0.00416
3rd Zero	10.173	0
4th Maximum	11.620	0.00160
4th Zero	13.324	0

(b)

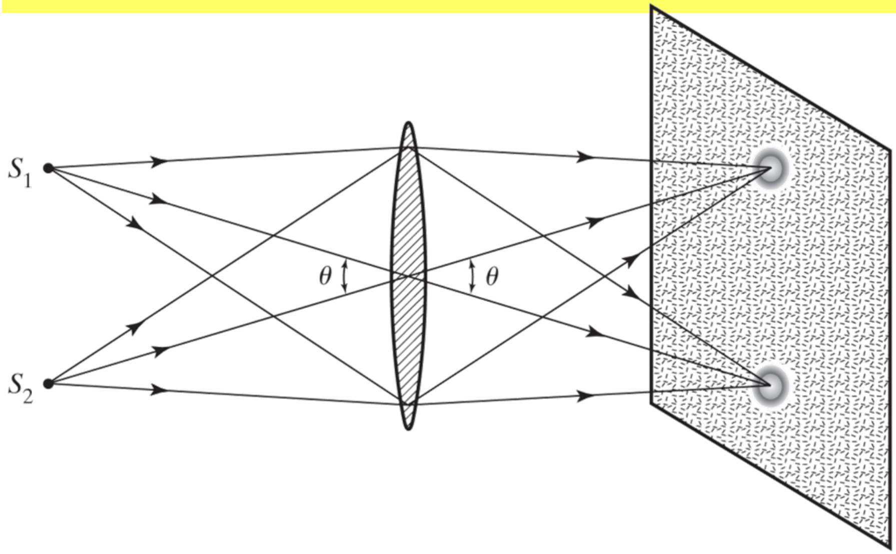
$$\gamma = \left(\frac{k}{2}\right) D\,sin\,\theta = 3.832 \quad \text{or} \quad D\,sin\,\theta = 1.22\lambda$$

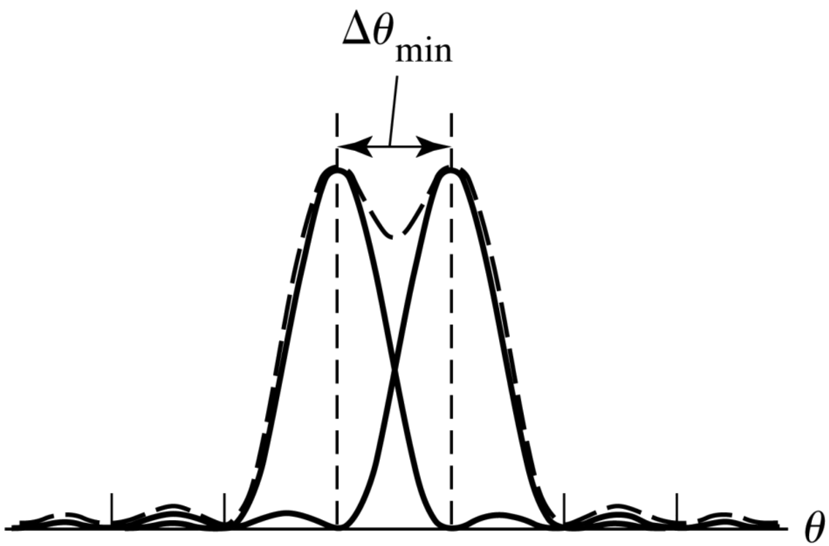
$$m\lambda = b\,sin\,\theta \quad , m = \pm 1, \pm 2, ...$$

The diffracted image of the circular aperture is called the Airy disc. Its angular half-width is:

$$\Delta\theta_{1/2} = \frac{1.22\lambda}{D}$$

RESOLUTION





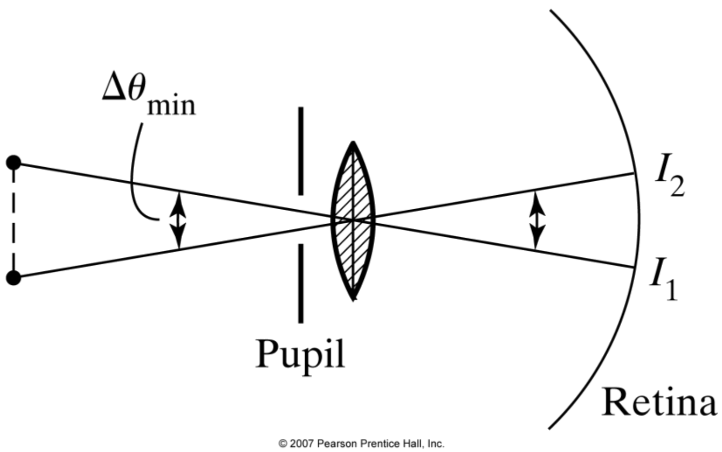
The point objects and the centers of their Airy discs are both separated by the angle θ.
If θ is large enough, two distinct images will be clearly seen.

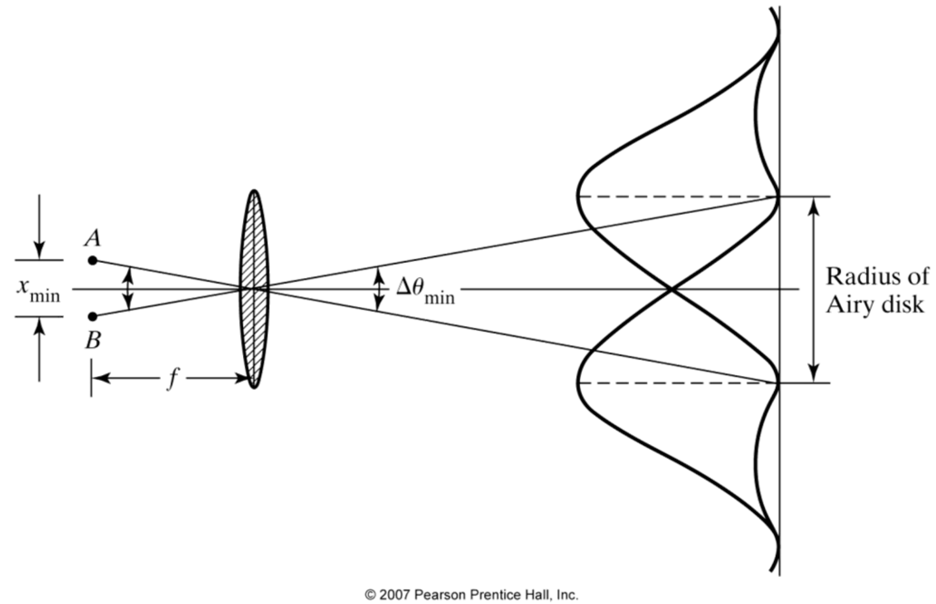
The Rayleigh's criterion:
The angular separation of the centers of the image patterns be no less than the angular radius of the Airy disc

$$(\Delta\theta)_{min} = \frac{1.22\lambda}{D}$$

Limit of resolution
D: diameter of the lens

RESOLUTION





Diffraction by the eye with pupil as aperture limits the resolution of objects subtending angle $\Delta\theta_{min}$

$$x_{min} = f(\Delta\theta)_{min} = f\left(\frac{1.22\lambda}{D}\right)$$

D/f : The numerical aperture (~1.2)

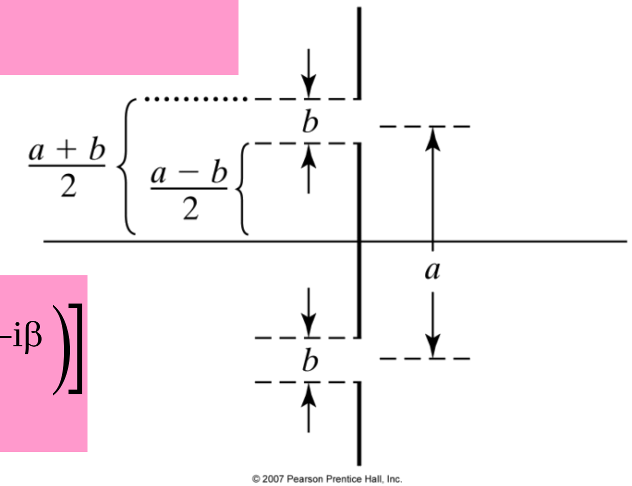
$$x_{min} = \lambda$$

DOUBLE-SLIT DIFFRACTION

$$E_p = \frac{E_L}{r_o} e^{i(kr_o-\omega t)} \int_{-(1/2)(a+b)}^{-(1/2)(a-b)} e^{isk\,sin\theta} ds + \frac{E_L}{r_o} e^{i(kr_o-\omega t)} \int_{(1/2)(a-b)}^{(1/2)(a+b)} e^{isk\,sin\theta} ds$$

$$E_p = \frac{E_L}{r_o} e^{i(kr_o-\omega t)} \frac{1}{ik\,sin\,\theta} [e^{(1/2)ik(-a+b)sin\theta} - e^{(1/2)ik(-a-b)sin\theta} + e^{(1/2)ik(a+b)sin\theta} - e^{(1/2)ik(a-b)sin\theta}]$$

$$\beta \equiv \frac{1}{2} kb\,sin\,\theta \qquad \alpha \equiv \frac{1}{2} ka\,sin\,\theta$$



$$E_p = \frac{E_L}{r_o} e^{i(kr_o-\omega t)} \frac{b}{2i\beta} \left[e^{i\alpha}\left(e^{i\beta} - e^{-i\beta}\right) + e^{-i\alpha}\left(e^{i\beta} - e^{-i\beta}\right)\right]$$

$$E_p = \frac{E_L}{r_o} e^{i(kr_o-\omega t)} \frac{b}{2i\beta} (2i\,sin\,\beta)(2\,cos\,\alpha)$$

DOUBLE-SLIT DIFFRACTION

$$E_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \frac{2b \sin\beta}{\beta} \cos\alpha \qquad E_o = \frac{E_L}{r_o} \frac{2b \sin\beta}{\beta} \cos\alpha$$

$$I = \left(\frac{\varepsilon_o c}{2}\right) E_o^2 = \left(\frac{\varepsilon_o c}{2}\right)\left(\frac{2E_L b}{r_o}\right)^2 \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha$$

$$I = 4I_o \left(\frac{\sin\beta}{\beta}\right)^2 \cos^2\alpha \qquad I_o = \left(\frac{\varepsilon_o c}{2}\right)\left(\frac{E_L b}{r_o}\right)^2$$

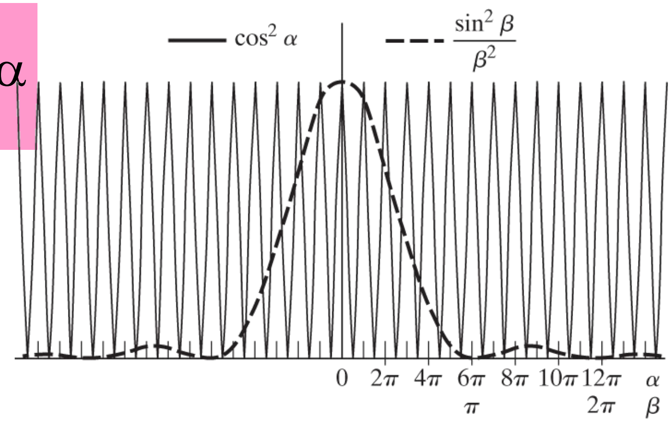
$$\cos^2\alpha = \cos^2\left[\frac{ka(\sin\theta)}{2}\right] = \cos^2\left[\frac{\pi a(\sin\theta)}{\lambda}\right]$$

diffraction minima : $m\lambda = b \sin\theta$

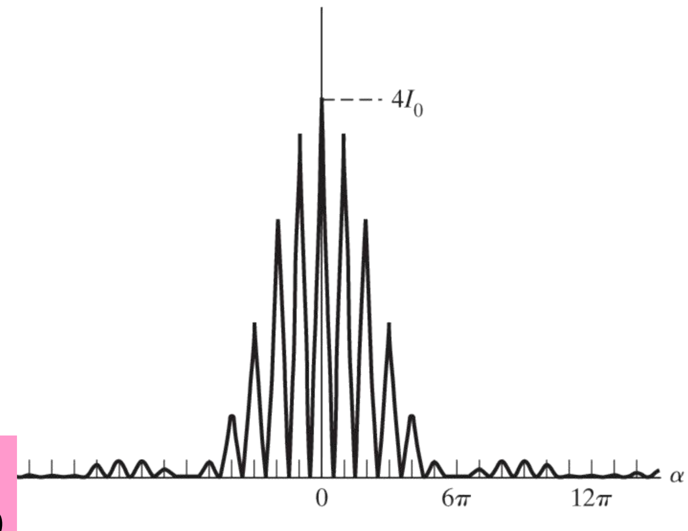
interference maxima : $p\lambda = a \sin\theta$

Condition of missing orders

$$a = \left(\frac{p}{m}\right) b$$



(a)



(b)

DIFFRACTION FROM MANY SLITS

For an aperture of multiple slits (a grating) we extend the integration over N slits.

$$E_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)} \sum_{j=1}^{N/2} \left\{ \int_{[-(2j-1)a-b]/2}^{[-(2j-1)a+b]/2} e^{isk\,sin\theta} ds + \int_{[(2j-1)a-b]/2}^{[(2j-1)a+b]/2} e^{isk\,sin\theta} ds \right\}$$

$$K = \frac{1}{ik\,sin\theta}\left[e^{-ik\,sin\theta[(2j-1)a-b)]/2} - e^{-ik\,sin\theta[(2j-1)a+b)]/2}\right] + \frac{1}{ik\,sin\theta}\left[e^{ik\,sin\theta[(2j-1)a+b)]/2} - e^{-ik\,sin\theta[(2j-1)a-b)]/2}\right]$$

$$K = \frac{b}{2i\beta}\left[e^{-i(2j-1)\alpha}\left(e^{i\beta} - e^{-i\beta}\right) + e^{i(2j-1)\alpha}\left(e^{i\beta} - e^{-i\beta}\right)\right]$$

$$K = 2b\frac{sin\beta}{\beta} Re\left[e^{i(2j-1)\alpha}\right]$$

$$S = 2b\frac{sin\beta}{\beta} Re\sum_{j=1}^{N/2} e^{i(2j-1)\alpha}$$

DIFFRACTION FROM MANY SLITS

For an aperture of multiple slits (a grating) we extend the integration over N slits.

$$S = 2b\frac{\sin\beta}{\beta} Re\sum_{j=1}^{N/2} e^{i(2j-1)\alpha}$$

$$S = 2b\frac{\sin\beta}{\beta} Re\left[e^{i\alpha} + e^{i3\alpha} + e^{i5\alpha} + \ldots + e^{i(N-1)\alpha}\right]$$

$$a\left(\frac{r^n - 1}{r - 1}\right) = e^{i\alpha}\left[\frac{\left(e^{2i\alpha}\right)^{N/2} - 1}{e^{2i\alpha} - 1}\right] = \frac{e^{iN\alpha} - 1}{e^{i\alpha} - e^{-i\alpha}}$$

$$\frac{(\cos N\alpha - 1) + i\sin N\alpha}{2i\sin\alpha} = \frac{i(\cos N\alpha - 1) - \sin N\alpha}{-2\sin\alpha}$$

$$S = b\frac{\sin\beta}{\beta}\frac{\sin N\alpha}{\sin\alpha}$$

$$E_p = \frac{E_L}{r_o} e^{i(kr_o - \omega t)}\left\{\frac{b\sin\beta}{\beta}\frac{\sin N\alpha}{\sin\alpha}\right\}$$

$$I = I_o\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

DIFFRACTION FROM MANY SLITS

$$I = I_o\left(\frac{\sin\beta}{\beta}\right)^2\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$

diffraction ***interference***

$$\lim_{\alpha\to m\pi}\frac{\sin N\alpha}{\sin\alpha} = \lim_{\alpha\to m\pi}\frac{N\cos N\alpha}{\cos\alpha} = \pm N$$

Irradiance at the principal maxima ∝ N²

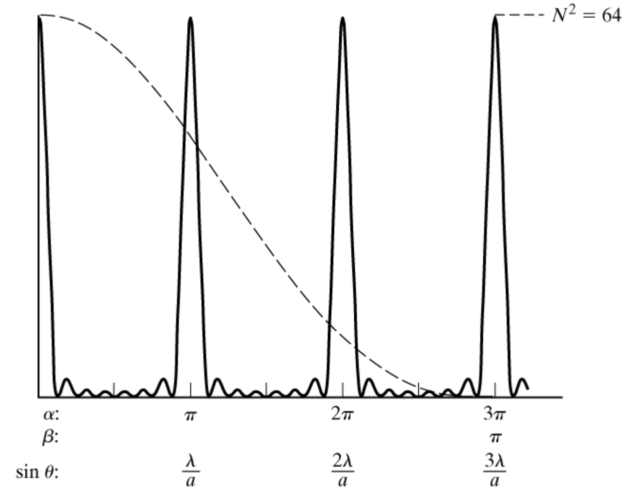
Location: α = 0, ± π, ±2π, ±3π, ….

$$\alpha = \frac{p\pi}{N} \quad , \quad p = 0, \pm 1, \pm 2, \ldots, \pm N \ldots \pm 2N$$

Principal maxima $p = 0, \pm N, \pm 2N, \ldots$

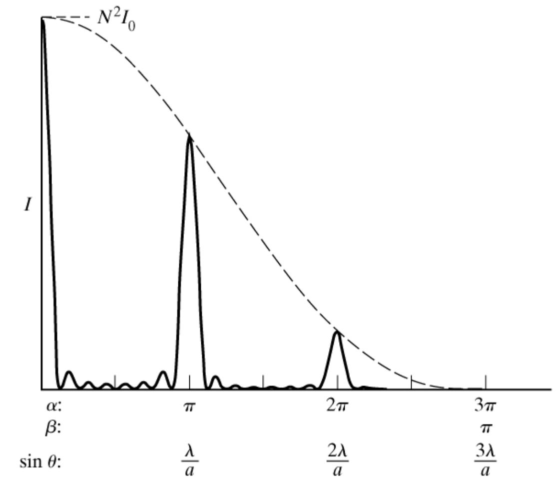
$$\alpha = m\pi \quad , \quad m = 0, \pm 1, \pm 2, \ldots$$

$$m\lambda = a\sin\theta$$



(a)

There are N-2 secondary peak



(b)