Effect of Integral action on the closed-loop Response

For *Closed-Loop Servo Response y(s) changes but d(s)=0*

$$y(s) = \frac{g_c(s)g_p(s)g_f(s)}{1+g_c(s)g_p(s)g_f(s)g_m(s)}\, y_{sp}(s)$$

$g_c(s) = \dfrac{Kc}{\tau_I s}$, $g_f(s) = 1$, $g_m(s) = 1$, $g_P(s) = \dfrac{K_P}{\tau_P s + 1}$

$$y(s) = \frac{\dfrac{Kc}{\tau_I s}\cdot\dfrac{K_P}{\tau_P s+1}.1}{1+\dfrac{Kc}{\tau_I s}\cdot\dfrac{K_P}{\tau_P s+1}.1.1}\, y_{sp}(s)$$

$$y(s) = \frac{K`}{\tau`^2 s^2 + 2\zeta`\tau`s + 1}\, y_{sp}(s)$$

Where

$$\tau` = \sqrt{\frac{\tau_I \tau_P}{K_P K_c}} \qquad \zeta` = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau_P K_P K_c}} \qquad \text{and} \quad K` = 1$$

Consider a setpoint change of magnitude = M

$y_{sp}(t)$ =M   L. T. $y_{sp}(s) = \frac{M}{s}$

$$y(s) = \frac{1}{\tau`^2 s^2 + 2\zeta`\tau`s + 1} \cdot \frac{M}{s}$$

Using final value theorem

$$\lim_{t \to \infty} [y(t)] = \lim_{s \to 0} [sy(s)] = \lim_{s \to 0} \frac{s.1}{\tau`^2 s^2 + 2\zeta`\tau`s + 1} \cdot \frac{M}{s} = M$$

Offset = y_{sp} - Ultimate value of the response = M - M = 0

Summary:

- integral action increases order of the system in closed-loop
- integral action eliminates offset

$$\tau` = \sqrt{\frac{\tau_I \tau_P}{K_P K_c}} \qquad \zeta` = \frac{1}{2}\sqrt{\frac{\tau_I}{\tau_P K_P K_c}}$$

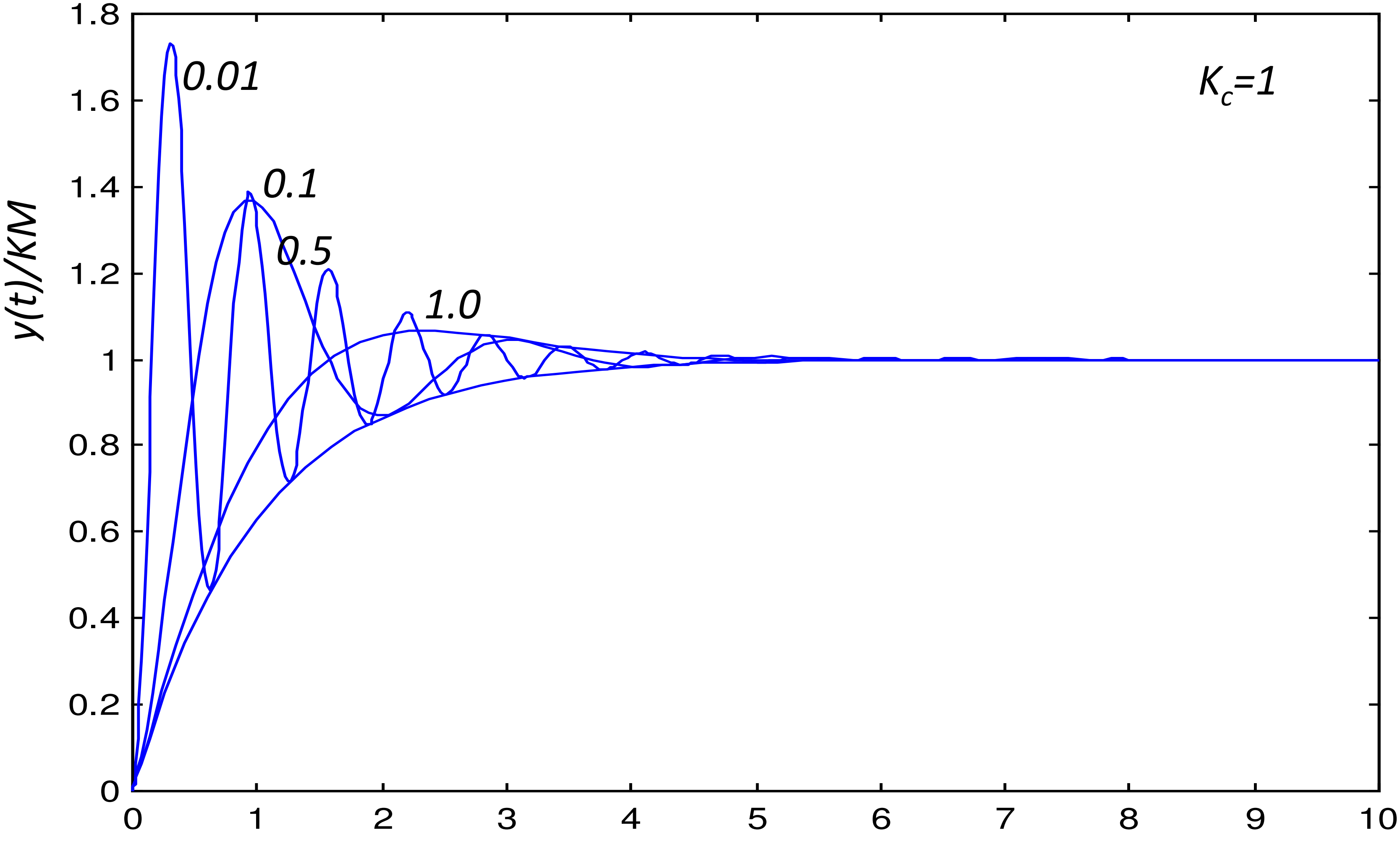
τ_I = constant , as $K_c \uparrow$ $\tau` \downarrow$ $\zeta` \downarrow$ response is faster but more oscillatory

K_c = constant as $\tau_I \downarrow$ $\tau` \downarrow$ $\zeta` \downarrow$ response is faster but more oscillatory

- Integral action Has two control parameters K_c, τ_I (more complicated than P)

- Integral action can be de-stabilizing (because of oscillations)

Therefore,
Integral control action eliminates offset



Effect of Derivative action on the closed-loop Response

For *Closed-Loop Servo Response y(s) changes but d(s)=0*

$$y(s)=\frac{g_c(s)g_p(s)g_f(s)}{1+g_c(s)g_p(s)g_f(s)g_m(s)}y_{sp}(s)$$

$g_c(s) = K_c\tau_D s$ $g_f(s) = 1$, $g_m(s) = 1$,

$$g_P(s)=\frac{K_P}{\tau_P s+1}$$

$$y(s)=\frac{K_c\tau_D s.\frac{K_P}{\tau_P s+1}.1}{1+K_c\tau_D s.\frac{K_P}{\tau_P s+1}.1.1}y_{sp}(s)$$

$$y(s)=\frac{K_P K_c\tau_D s}{(\tau_P+K_P K_c\tau_D)s+1}y_{sp}(s)$$

$$y(s)=\frac{K_P K_c\tau_D s}{\tau`s+1}y_{sp}(s)$$

$\tau` = \tau_P + K_P K_c\tau_D$

Order is the same

$\tau` > \tau$ response of closed loop is slower than the original process

The effect of derivative action will be clear for second order process

For *Closed-Loop Servo Response y(s) changes but d(s)=0*

$$y(s) = \frac{g_c(s)g_p(s)g_f(s)}{1 + g_c(s)g_p(s)g_f(s)g_m(s)} y_{sp}(s)$$

$g_c(s) = K_c \tau_D s$, $g_f(s) = 1$, $g_m(s) = 1$,

$$g_P(s) = \frac{K_P}{\tau^2 s + 2\zeta\tau s + 1}$$

$$y(s) = \frac{K_c \tau_D s . \dfrac{K_P}{\tau^2 s + 2\zeta\tau s + 1} .1}{1 + K_c \tau_D s . \dfrac{K_P}{\tau^2 s + 2\zeta\tau s + 1} .1.1} y_{sp}(s)$$

$$y(s) = \frac{K_c \tau_D s . K_P .1}{\tau^2 s^2 + 2\zeta\tau s + 1 + K_c \tau_D s . K_P .1.1} y_{sp}(s)$$

$$\boxed{y(s) = \frac{K_c K_P \tau_D s}{\tau^2 s^2 + (2\zeta\tau + K_c K_P \tau_D)s + 1} y_{sp}(s)}$$

$$y(s) = \frac{K_c K_P \tau_D s}{\tau`^2 s^2 + 2\zeta`\tau`s + 1} y_{sp}(s)$$

Order is the same

$$\tau` = \tau$$

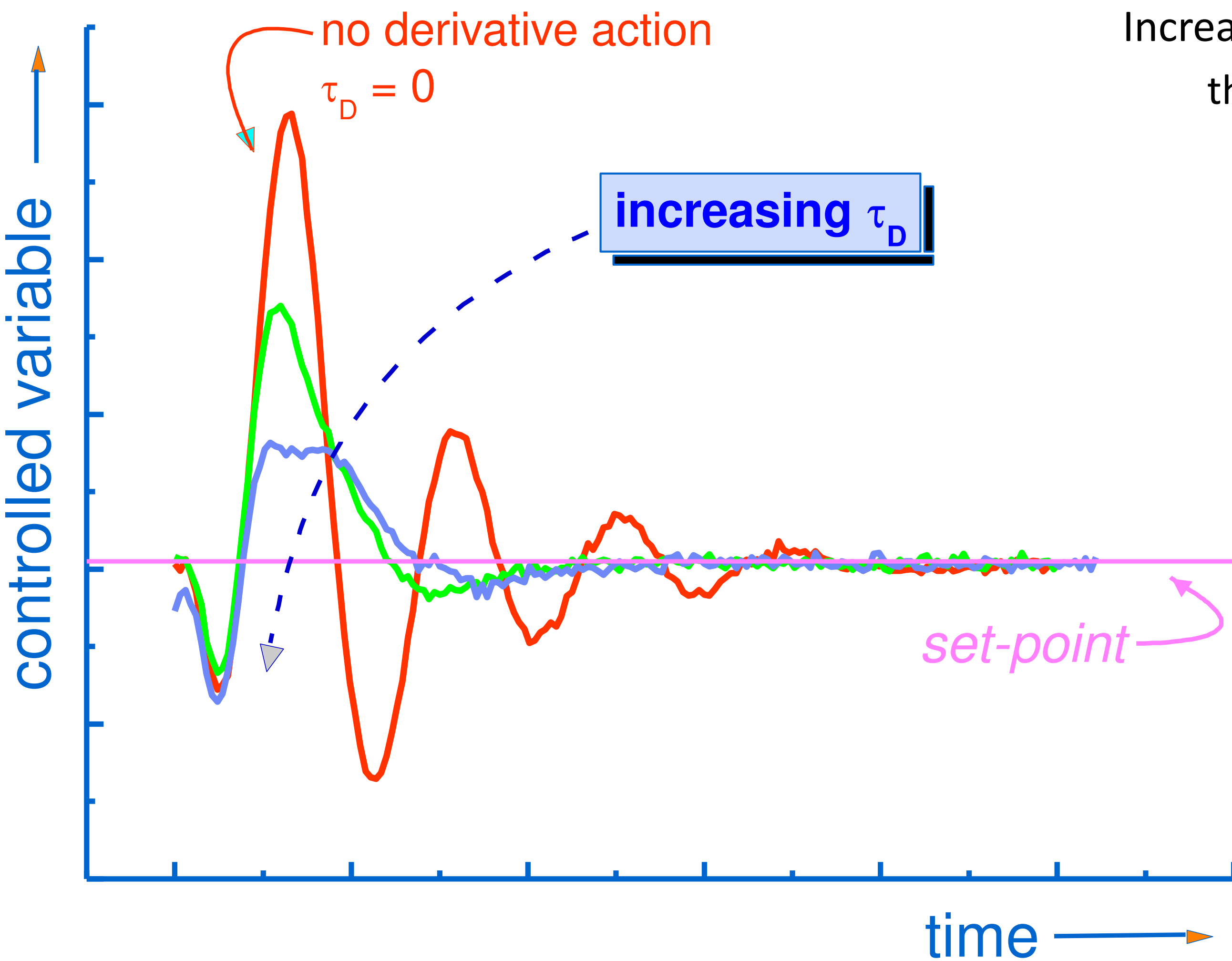
$$2\zeta`\tau` = 2\zeta\tau + K_c K_P \tau_D \qquad \zeta` = \zeta + \frac{1}{2\tau`} K_c K_P \tau_D \qquad \zeta` > \zeta$$

Therefore, the closed loop is more damped (i.e. Less oscillatory)

$\zeta`$ ↑ as Kc ↑ and τ_D↑

Performance of PID controllers

Response to a disturbance step change



Increasing τ_D :

the oscillations caused by the integral action are dampened

<u>CAUTION</u>

Noisy measurements may disrupt the controller performance !

Summary for PID control

☺ **Advantages**

oscillations can be dampened with respect to PI control

☹ **disadvantages**

tuning is harder than PI (three parameters must be specified, K_C , τ_I and τ_D)

the derivative action may amplify measurement noise ⇒ potential wear on the final control element

😐 **Use of derivative action**

avoid using the D action when the controlled variable has a noisy measure or when the process is not sluggish $\theta/\tau_P < 0.5$

Controller selection recommendations

- When steady state offsets can be tolerated, use a P-only controller (many liquid level loops are on P control)

- When offset *cannot* be tolerated, use a PI controller (a large proportion of feedback loops in a typical plant are under PI control)

- When it is important to compensate for some natural sluggishness in the system, and the process signal are relatively noise-free, use a PID controller