# CHAPTER 28: MAGNETIC FIELDS

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# Circuits

# What we will learn

- What produces a magnetic field?
- How can we define the magnetic field and the magnetic field lines?
- What happens to a charged particle in a magnetic field?
- Magnetic forces on a current carrying wire.
- Torque on a current loop.
- The magnetic dipole moment.

# Magnetic Fields

Tom Walker/Getty Images, Inc.

*If you are outside on a dark night in the middle to high latitudes, you might be able to see an aurora, a ghostly "curtain" of light that hangs down from the sky. The curtain is not only local; it may be 200 km high and 4000 km long, stretching around Earth in an arc. However, it is only about 100 m thick.*

*What produces this huge display, and what makes it so thin?*

The answers are in this chapter.

# What produces a magnetic field?

| Electric fields | Magnetic fields |
| --- | --- |
| From electric charge<br>(+ , -) | Magnetic charge ??<br>No magnetic monopoles exist |

**Electromagnet**

**(moving electric charge)**

**Permanent magnet**

**(intrinsic magnetic field of elementary particles)**

# The definition of **B**

We can define a **magnetic field, *B*,** by firing a charged particle through the point at which is to be defined, using various directions and speeds for the particle and determining the force that acts on the particle at that point.

The magnetic force on the charged particle, $\boldsymbol{F_B}$, is defined to be:

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

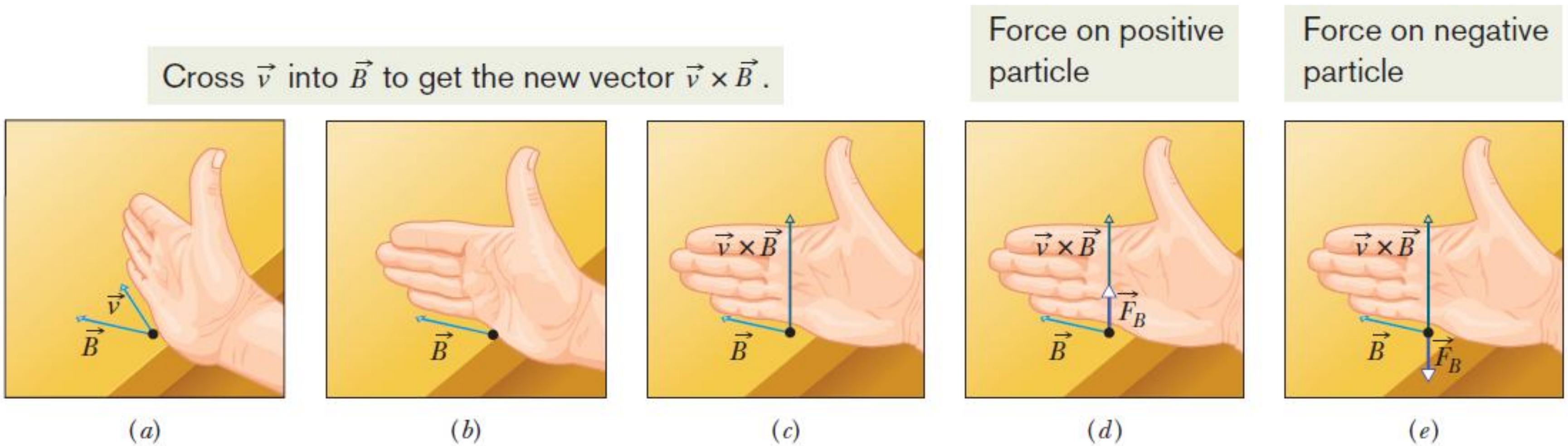
Here $q$ is the charge of the particle, ***v*** is its velocity, and ***B*** the magnetic field in the region. The magnitude of this force is then:

$$F_B = |q|vB\sin\phi,$$

Here $\phi$ is the angle between vectors ***v*** and ***B***.

# The definition of **B**

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

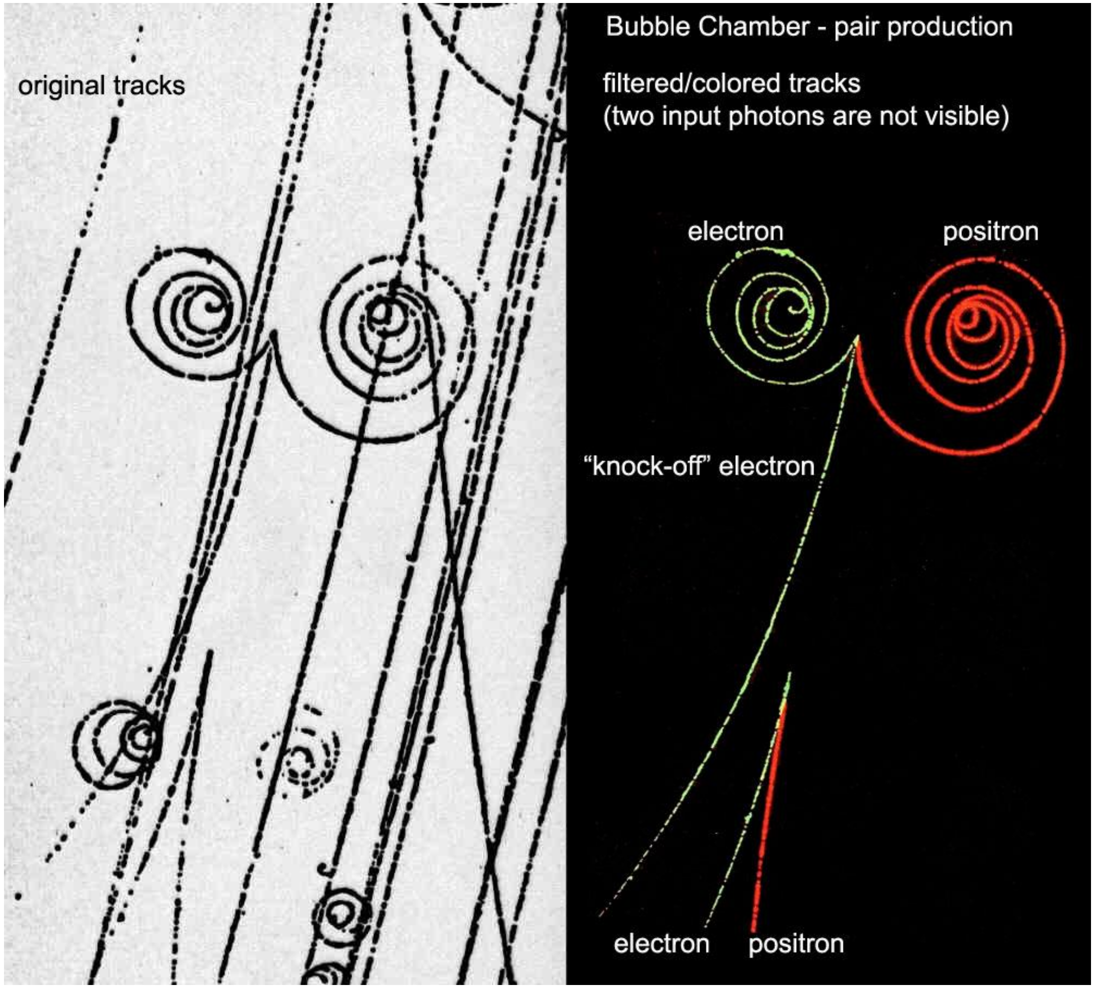



The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

**F** does NOT change the speed of the particle but only change its direction (only this way it can be accelerated)

# The definition of **B**

$$\vec{F}_B = q\vec{v} \times \vec{B}$$



# The definition of **B**

**Table 28-1**

**Some Approximate Magnetic Fields**

| | |
|---|---|
| At surface of neutron star | $10^{8}$ T |
| Near big electromagnet | 1.5 T |
| Near small bar magnet | $10^{-2}$ T |
| At Earth's surface | $10^{-4}$ T |
| In interstellar space | $10^{-10}$ T |
| Smallest value in magnetically shielded room | $10^{-14}$ T |

The SI unit for ***B*** that follows is newton per coulomb-meter per second. For convenience, this is called the **tesla (T):**

$$1 \text{ tesla} = 1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}$$

$$= 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}$$

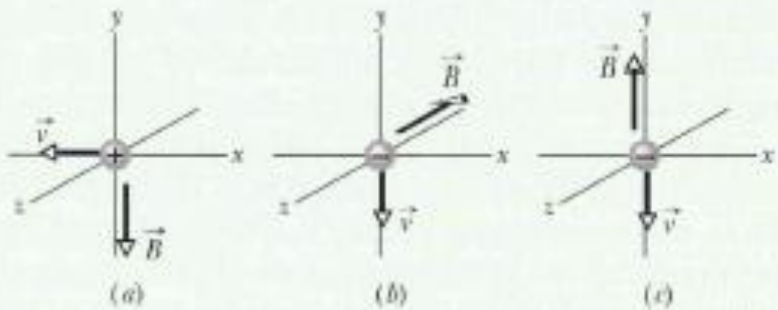
An earlier (non-SI) unit for ***B*** is the *gauss (G),* and

$$1 \text{ tesla} = 10^{4} \text{ gauss.}$$

# The definition of **B**

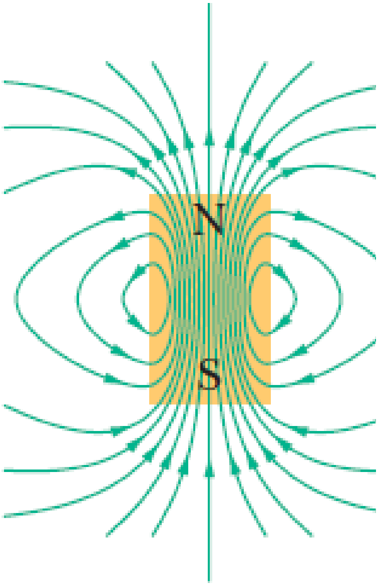
**CHECKPOINT 1**

The figure shows three situations in which a charged particle with velocity $\vec{v}$ travels through a uniform magnetic field $\vec{B}$. In each situation, what is the direction of the magnetic force $\vec{F}_B$ on the particle?



# The magnetic field lines

- ❑The direction of the tangent to a magnetic field line at any point gives the direction of ***B*** at that point.
- ❑The spacing of the lines represents the magnitude of ***B*** —the magnetic field is stronger where the lines are closer together, and conversely.




(*a*)

(*b*)

Opposite magnetic poles attract each other, and like magnetic poles repel each other.

# The magnetic field lines

A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is $1.67 \times 10^{-27}$ kg. (Neglect Earth's magnetic field.)

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**KEY IDEAS**

Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}_B$ can act on it. Because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}_B$ is not simply zero.

***Magnitude:*** To find the magnitude of $\vec{F}_B$, we can use Eq. 28-3 ($F_B = |q|vB \sin \phi$) provided we first find the proton's speed $v$. We can find $v$ from the given kinetic energy because $K = \frac{1}{2}mv^2$. Solving for $v$, we obtain

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{1.67 \times 10^{-27} \text{ kg}}}$$
$$= 3.2 \times 10^7 \text{ m/s}.$$

Equation 28-3 then yields

$$\begin{aligned} F_B &= |q|vB \sin \phi \\ &= (1.60 \times 10^{-19} \text{ C})(3.2 \times 10^7 \text{ m/s}) \\ &\quad \times (1.2 \times 10^{-3} \text{ T})(\sin 90°) \\ &= 6.1 \times 10^{-15} \text{ N}. \qquad \text{(Answer)} \end{aligned}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration; namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 3.7 \times 10^{12} \text{ m/s}^2.$$

***Direction:*** To find the direction of $\vec{F}_B$, we use the fact that $\vec{F}_B$ has the direction of the cross product $q\vec{v} \times \vec{B}$. Because the charge $q$ is positive, $\vec{F}_B$ must have the same direction as $\vec{v} \times \vec{B}$, which can be determined with the right-hand rule for cross products (as in Fig. 28-2$d$). We know that $\vec{v}$ is directed horizontally from south to north and $\vec{B}$ is directed vertically up. The right-hand rule shows us that the deflecting force $\vec{F}_B$ must be directed horizontally from west to east, as Fig. 28-6 shows. (The array of dots in the figure represents a magnetic field directed out of the plane of the figure. An array of Xs would have represented a magnetic field directed into that plane.)

If the charge of the particle were negative, the magnetic deflecting force would be directed in the opposite direction—that is, horizontally from east to west. This is predicted automatically by Eq. 28-2 if we substitute a negative value for $q$.

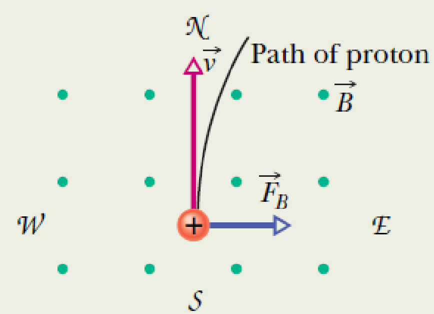
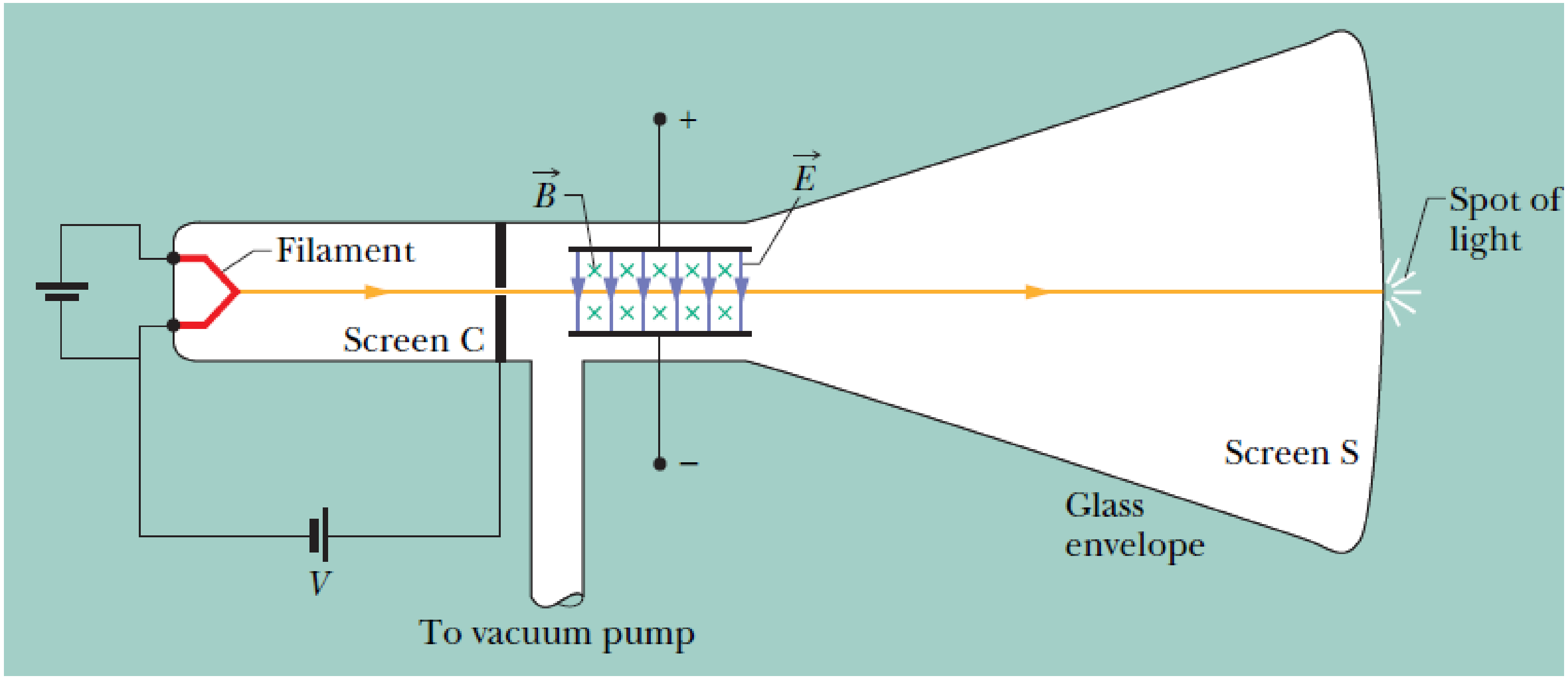



**Fig. 28-6** An overhead view of a proton moving from south to north with velocity $\vec{v}$ in a chamber. A magnetic field is directed vertically upward in the chamber, as represented by the array of dots (which resemble the tips of arrows). The proton is deflected toward the east.

# Crossed fields: Discovery of the electron



# A circulating charged particle

A particle of charge magnitude $|q|$ and mass $m$ moving perpendicular to a uniform magnetic field $\boldsymbol{B}$, at speed $v$.

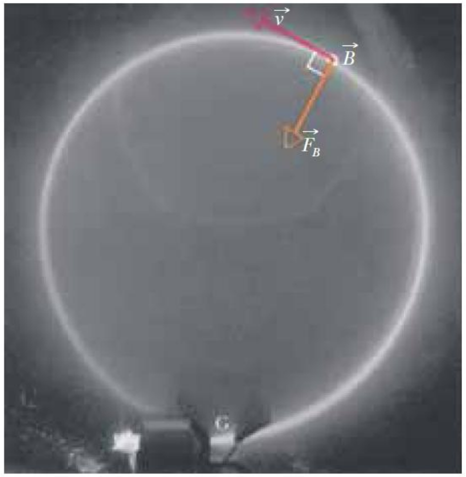
$\mathbf{F_B}$ continuously deflects the particle,
$\boldsymbol{B} \perp \boldsymbol{v}$ always → particle follows a circular path.

The magnetic force acting on the particle has a magnitude of $|q|vB$.

For uniform circular motion

$$F = m\frac{v^2}{r},$$

$$|q|vB = \frac{mv^2}{r}.$$




$$r = \frac{mv}{|q|B} \qquad \text{(radius)}.$$

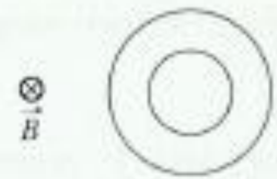
$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \qquad \text{(period)}.$$

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \qquad \text{(frequency)}.$$

$$\omega = 2\pi f = \frac{|q|B}{m} \qquad \text{(angular frequency)}.$$

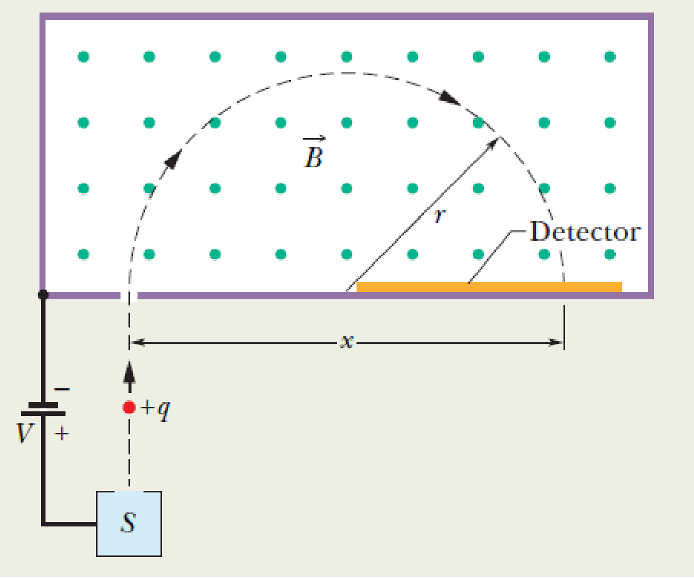
# A circulating charged particle

**CHECKPOINT 3** The figure here shows the circular paths of two particles that travel at the same speed in a uniform magnetic field $\vec{B}$, which is directed into the page. One particle is a proton; the other is an electron (which is less massive). (a) Which particle follows the smaller circle, and (b) does that particle travel clockwise or counterclockwise?



# A circulating charged particle

Figure 28-12 shows the essentials of a *mass spectrometer,* which can be used to measure the mass of an ion; an ion of mass $m$ (to be measured) and charge $q$ is produced in source $S$. The initially stationary ion is accelerated by the electric field due to a potential difference $V$. The ion leaves $S$ and enters a separator chamber in which a uniform magnetic field $\vec{B}$ is perpendicular to the path of the ion. A wide detector lines the bottom wall of the chamber, and the $\vec{B}$ causes the ion to move in a semicircle and thus strike the detector. Suppose that $B = 80.000$ mT, $V = 1000.0$ V, and ions of charge $q = +1.6022 \times 10^{-19}$ C strike the detector at a point that lies at $x = 1.6254$ m. What is the mass $m$ of the individual ions, in atomic mass units (Eq. 1-7: 1 u $= 1.6605 \times 10^{-27}$ kg)?



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***Finding speed:*** When the ion emerges from the source, its kinetic energy is approximately zero. At the end of the acceleration, its kinetic energy is $\frac{1}{2}mv^2$. Also, during the acceleration, the positive ion moves through a change in potential of $-V$. Thus, because the ion has positive charge $q$, its potential energy changes by $-qV$. If we now write the conservation of mechanical energy as

$$\Delta K + \Delta U = 0,$$

we get

$$\tfrac{1}{2}mv^2 - qV = 0$$

or

$$v = \sqrt{\frac{2qV}{m}}. \tag{28-22}$$

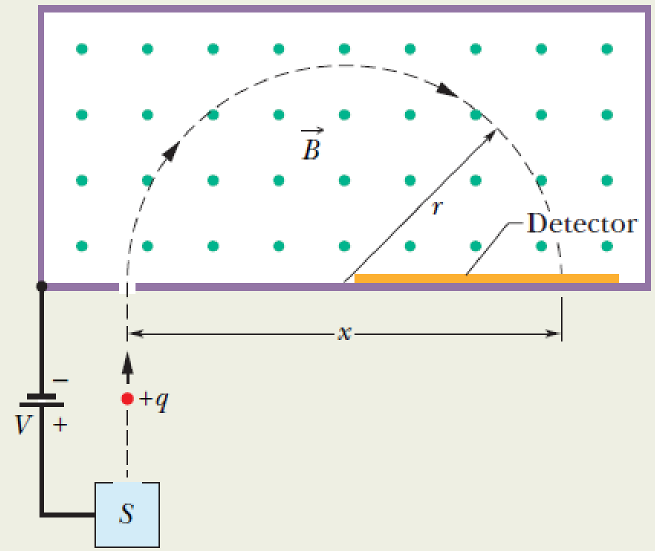



**Fig. 28-12** Essentials of a mass spectrometer. A positive ion, after being accelerated from its source $S$ by a potential difference $V$, enters a chamber of uniform magnetic field $\vec{B}$. There it travels through a semicircle of radius $r$ and strikes a detector at a distance $x$ from where it entered the chamber.

***Finding mass:*** Substituting this value for $v$ into Eq. 28-16 gives us

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}.$$

Thus,

$$x = 2r = \frac{2}{B}\sqrt{\frac{2mV}{q}}.$$

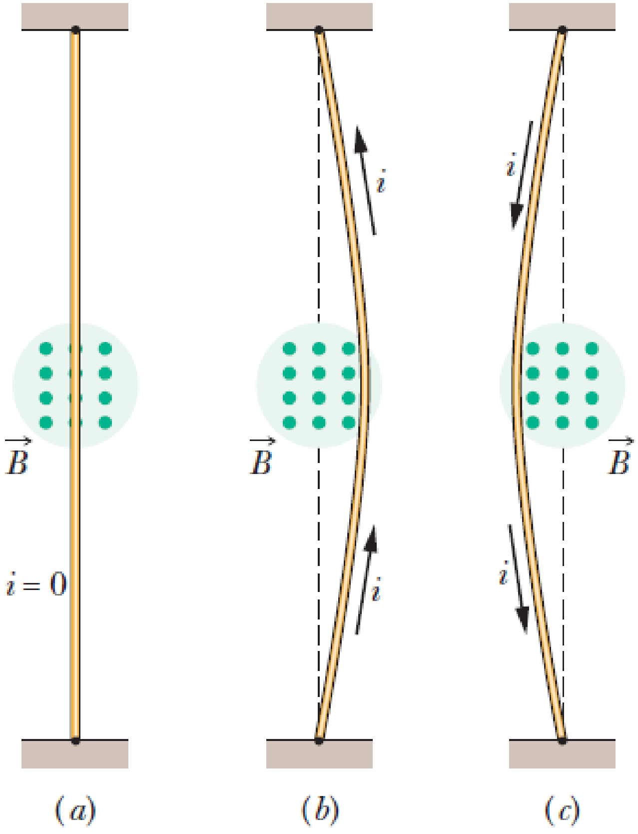
Solving this for $m$ and substituting the given data yield

$$\begin{aligned} m &= \frac{B^2qx^2}{8V} \\ &= \frac{(0.080000\ \text{T})^2(1.6022 \times 10^{-19}\ \text{C})(1.6254\ \text{m})^2}{8(1000.0\ \text{V})} \\ &= 3.3863 \times 10^{-25}\ \text{kg} = 203.93\ \text{u}. \end{aligned} \qquad \text{(Answer)}$$

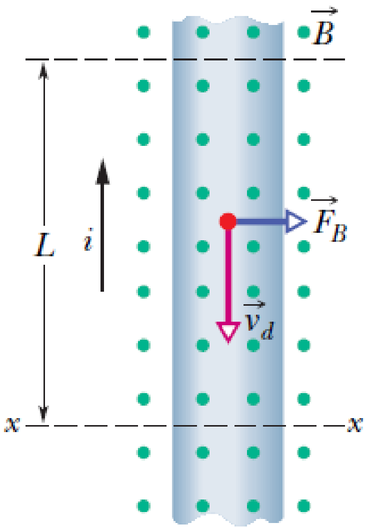
# Magnetic Force on a Current Carrying Wire

**Fig. 28-14** A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (*a*) Without current in the wire, the wire is straight. (*b*) With upward current, the wire is deflected rightward. (*c*) With downward current, the deflection is leftward. The connections for getting the current into the wire at one end and out of it at the other end are not shown.

A force acts on a current through a $B$ field.



# Magnetic Force on a Current Carrying Wire




Consider a length *L* of the wire in the figure. All the conduction electrons in this section of wire will drift past plane *xx* in a time $t = L/v_d$.

Thus, in that time a charge will pass through that plane that is given by

$$q = it = i\frac{L}{v_d}$$

$$F_B = qv_dB\sin\phi = \frac{iL}{v_d}v_dB\sin 90^\circ$$

$$F_B = iLB.$$

$$\vec{F}_B = i\vec{L} \times \vec{B} \qquad \text{(force on a current).}$$

Here ***L*** is a length vector that has magnitude *L* and is directed along the wire segment in the direction of the (conventional) current.

# Magnetic Force on a Current Carrying Wire

A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire—that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

# Magnetic Force on a Current Carrying Wire

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**KEY IDEAS**

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig. 28-17). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

***Calculations:*** Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 28-17) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$ (Eq. 28-27). Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg, \tag{28-29}$$

where $mg$ is the magnitude of $\vec{F}_g$ and $m$ is the mass of the wire.

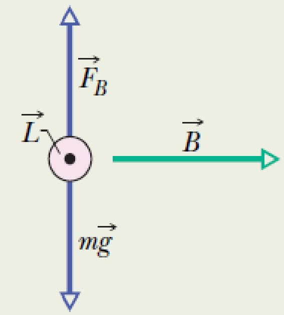



**Fig. 28-17** A wire (shown in cross section) carrying current out of the page.

We also want the minimal field magnitude $B$ for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin \phi$ in Eq. 28-29. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin \phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}. \tag{28-30}$$

We write the result this way because we know $m/L$, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3}\ \text{kg/m})(9.8\ \text{m/s}^2)}{28\ \text{A}}$$

$$= 1.6 \times 10^{-2}\ \text{T}. \qquad \text{(Answer)}$$

This is about 160 times the strength of Earth's magnetic field.

# Torque on a current loop

The two magnetic forces ***F*** and –***F*** produce a torque on the loop, tending to rotate it about its central axis.

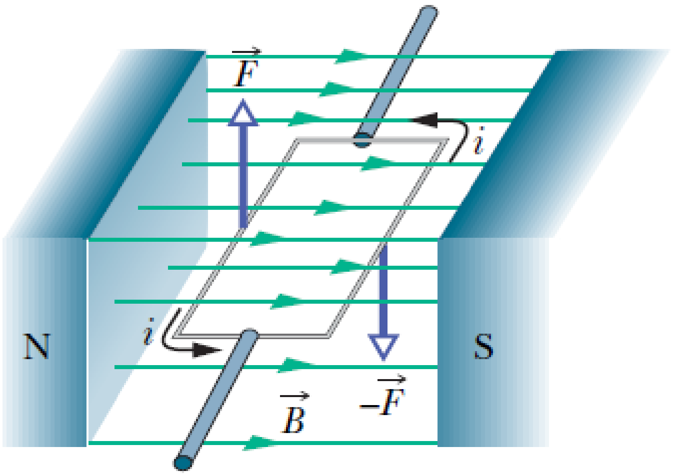
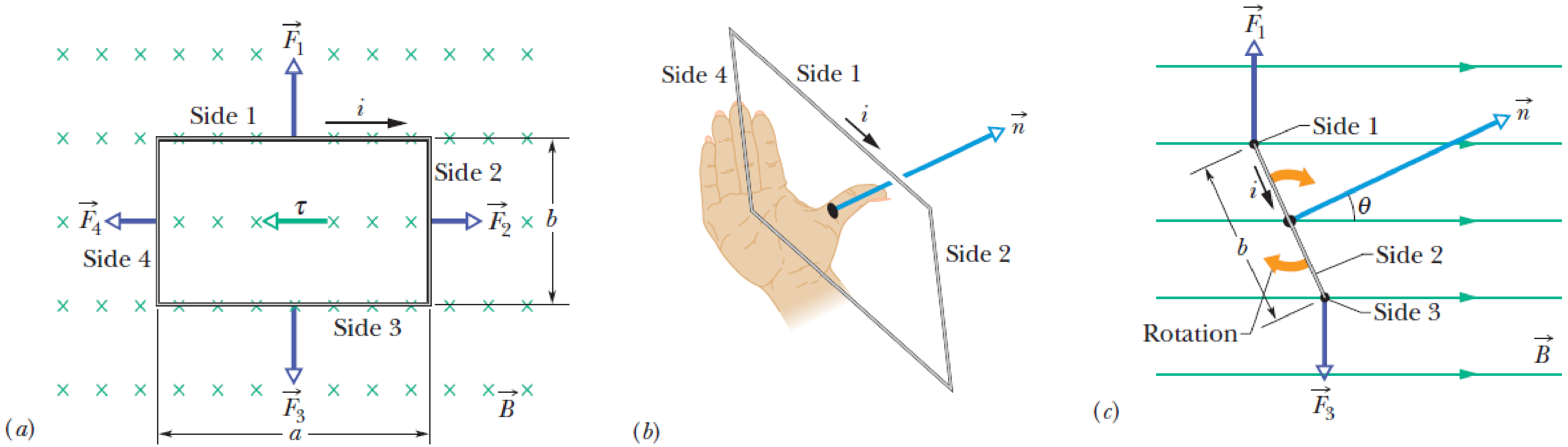


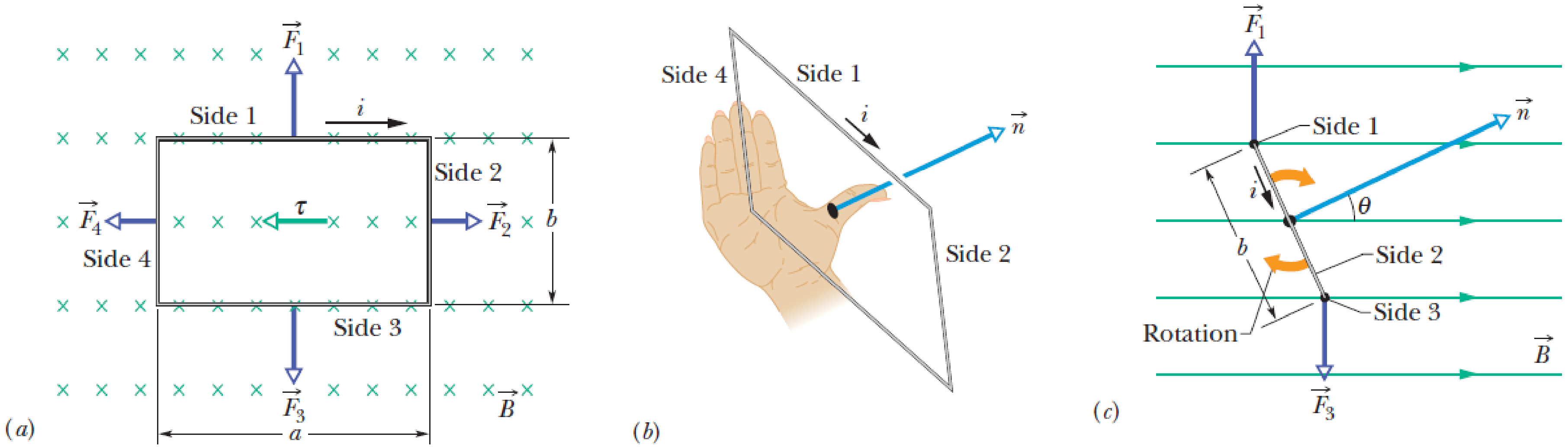

**Fig. 28-18** The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.

# Torque on a current loop




To define the orientation of the loop in the magnetic field, we use a normal vector ***n*** that is perpendicular to the plane of the loop. Figure *b* shows a right-hand rule for finding the direction of ***n***. In Fig. *c,* the normal vector of the loop is shown at an arbitrary angle $\theta$ to the direction of the magnetic field.

# Torque on a current loop




For side 2 the magnitude of the force acting on this side is
$\boldsymbol{F_2=ibB\sin(90°-\theta)=ibB\cos\theta=F_4}$. → $\boldsymbol{F_2}$ and $\boldsymbol{F_4}$ cancel out exactly.
Forces $\boldsymbol{F_1}$ and $\boldsymbol{F_3}$ have the common magnitude *iaB.* As Fig. 28-19*c* shows, these two forces do not share the same line of action; so they produce a net torque.

$$\tau' = \left(iaB\,\frac{b}{2}\sin\theta\right) + \left(iaB\,\frac{b}{2}\sin\theta\right) = iabB\sin\theta.$$

For *N* loops, when *A=ab*, the area of the loop, the total torque is:

$$\tau = N\tau' = NiabB\sin\theta = (NiA)B\sin\theta,$$

# Magnetic Dipole Moment μ

***Definition:***

Here, $N$ is the number of turns in the coil, $i$ is the current through the coil, and $A$ is the area enclosed by each turn of the coil.

$$\mu = NiA \qquad \text{(magnetic moment)},$$

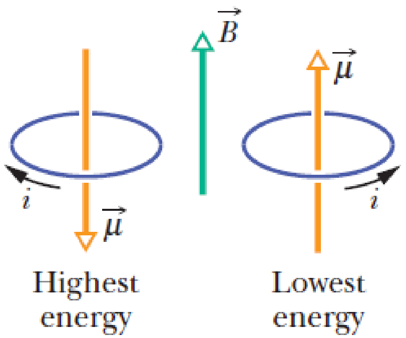
***Direction:*** The direction of $\mu$ is that of the normal vector to the plane of the coil.

$$\tau = \mu B \sin\theta,$$

The definition of torque can be rewritten as:

$$\vec{\tau} = \vec{\mu} \times \vec{B},$$

The magnetic moment vector attempts to align with the magnetic field.




Just as in the electric case, the magnetic dipole in an external magnetic field has an energy that depends on the dipole's orientation in the field:

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

A magnetic dipole has its lowest energy (*-$\mu B$ cos 0=-$\mu B$)* when its dipole moment $\mu$ is lined up with the magnetic field. It has its highest energy (*-$\mu B$ cos 180°=+$\mu B$)* when $\mu$ is directed opposite the field.

# Magnetic Dipole Moment μ

$$\mu = NiA \qquad \text{(magnetic moment)},$$

$$U(\theta) = -\vec{\mu} \cdot \vec{B}.$$

From the above equations, one can see that the unit of $\mu$ can be the joule per tesla (J/T), or the ampere–square meter.

**Table 28-2**

**Some Magnetic Dipole Moments**

| | |
|---|---|
| Small bar magnet | 5 J/T |
| Earth | $8.0 \times 10^{22}$ J/T |
| Proton | $1.4 \times 10^{-26}$ J/T |
| Electron | $9.3 \times 10^{-24}$ J/T |

# What have we learnt

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- How can we define the magnetic field and the magnetic field lines?
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