CHAPTER 25: CAPACITANCE

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Capacitors and Capacitance

What we will learn

- What is Capacitors and Capacitance?
- The Capacitance of different types of capacitors:
 - Parallel plates Cylindrical Spherical Isolated sphere
- How do we add the effect of capacitors?
- How do we find out the energy stored in a capacitor?
- Capacitors with dielectric materials.

Capacitors and Capacitance



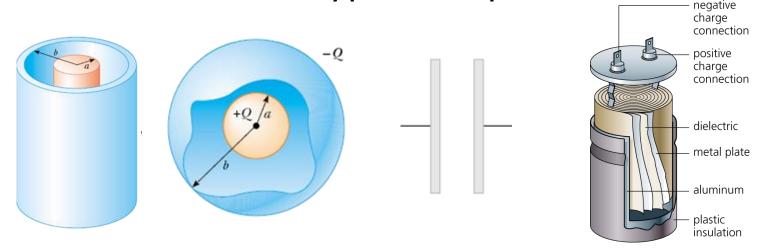
Explosions of airborne dust in grain storage bins (as above), coal mines, flour mills, and many powder industries are a common occurrence, often with loss of life and much property damage. Usually the explosions are due to sparking between charged objects or between a charged object and a grounded connection. Engineers cannot eliminate the possibility of sparking, but they can take measures to reduce the chance that a spark will set off an explosion.

What determines whether sparking will cause an explosion of airborne dust?

The answer is in this chapter.

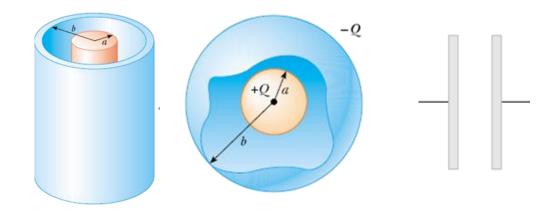
Capacitance

- Capacitors are important because they can provide a large supply of energy at a fast rate.
- □ A capacitor consists of two conducting objects (+Q,-Q)
- \square We refer to charge of the capacitor with Q (Q_{net}=0)
- The symbol of a capacitor in electric circuits
- There are different types of capacitors:



Capacitance

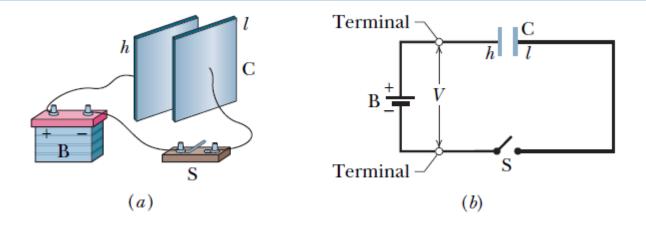
- When a capacitor is charged, the plates have equal and opposite charge (+Q,-Q)
- □ The plates of the conductors are equipotential surfaces.
- There is a potential difference between the plates V.
- Capacitance: How much charge can be stored in a capacitor to produce V. $Q \propto V \qquad Q = CV$



C depends on the geometry of the plates and the material between them and **NOT** on Q or V

Units (C/V \equiv Farad)

Charging a capacitor



The battery maintains V between its terminals. (+ , -)

When the switch is closed, charge (electrons) flow under the influence of E set by the battery.

 \square e moves from h \rightarrow + (h loses electrons \rightarrow + charged)

 \square e moves from - \rightarrow I (I gains electrons \rightarrow - charged)

 \square V on capacitor goes from 0 \rightarrow V_{battery} (fully charged E=0)

Charging a capacitor

CHECKPOINT 1 Does the capacitance C of a capacitor increase, decrease, or remain the same (a) when the charge q on it is doubled and (b) when the potential difference V across it is tripled?

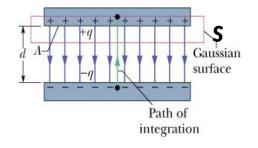
How to calculate C?

- 1. Assume a charge q on the plates.
- 2. Find E between the plates in terms of q using Gauss' law $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{c}$
- 3. Calculate V from E $V = -\int_{i}^{f} \vec{E} d\vec{s}$
- 4. Calculate C from C=q/V

□ Assumption 1:

- E and dA are parallel
- **E.dA** = EA

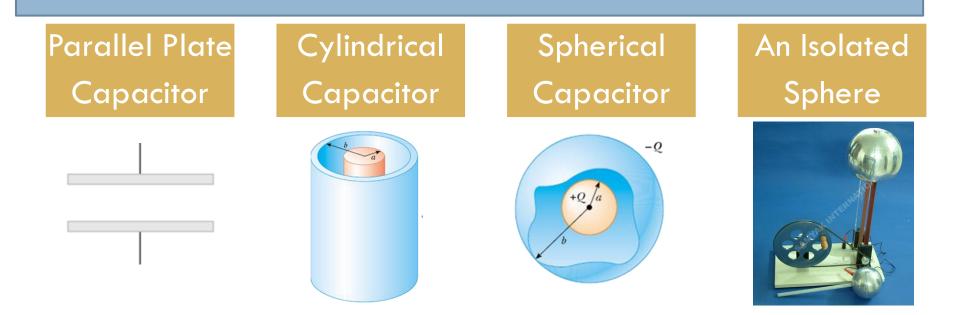
$$\varepsilon_o EA = q_{enc}$$
 (E const.)

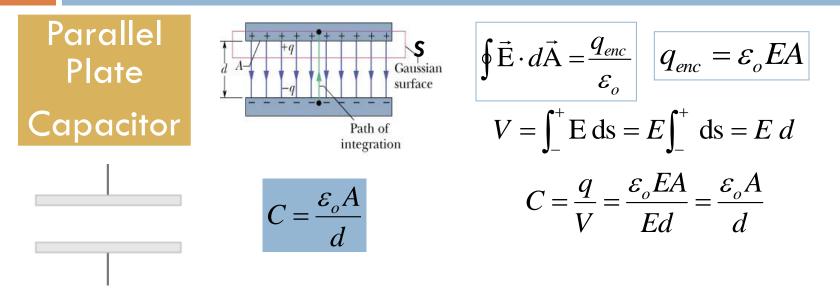


- □ Assumption 2:
 - Path starts from + along E
 - E and ds are anti-parallel

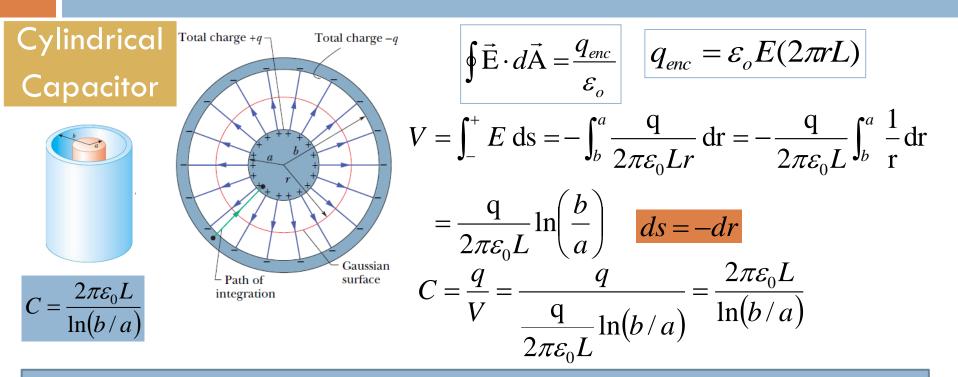
$$V = \int_{-}^{+} \mathbf{E} \, \mathrm{ds}$$

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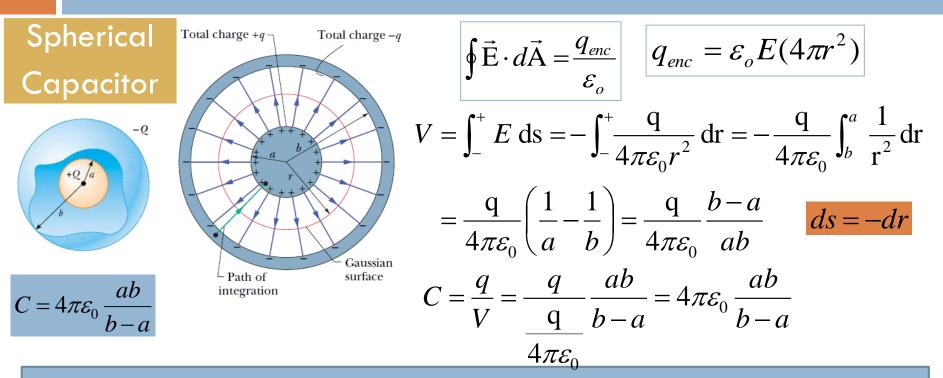




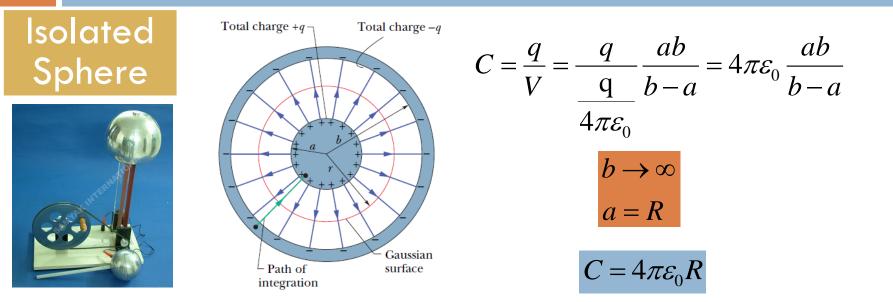
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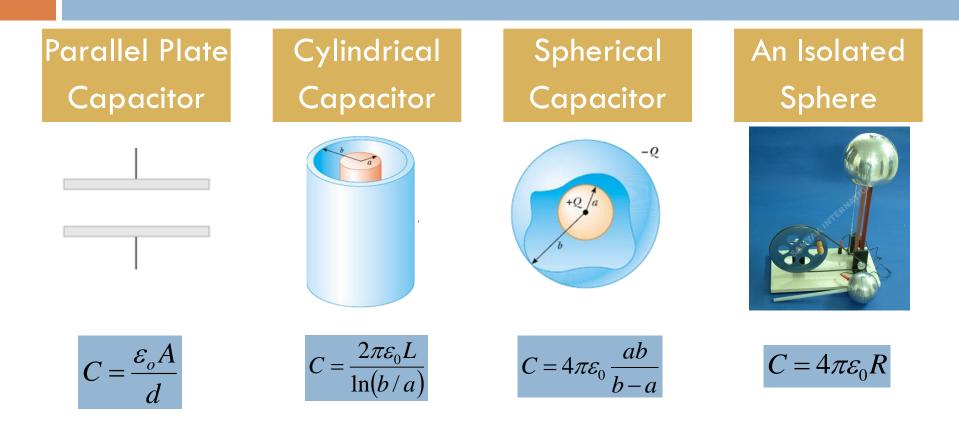


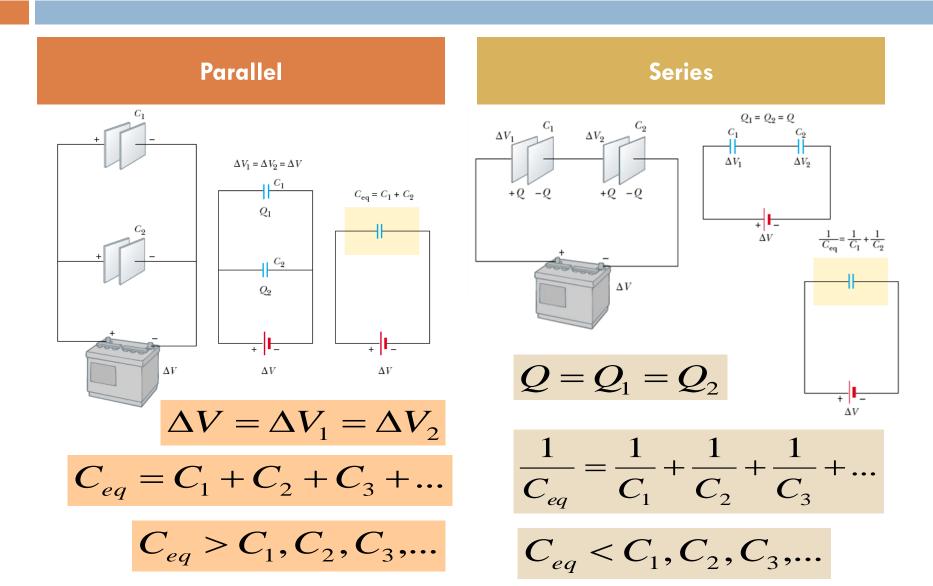
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CHECKPOINT 2 For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The radius of the inner cylinder of a cylindrical capacitor is increased.





CHECKPOINT 3 A battery of potential V stores charge q on a combination of two identical capacitors. What are the potential difference across and the charge on either capacitor if the capacitors are (a) in parallel and (b) in series?

(b) The potential difference applied to the input terminals in Fig. 25-10*a* is V = 12.5 V. What is the charge on C_1 ?

Next, we work

12.5 V

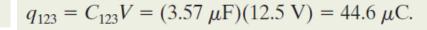
backwards to the

desired capacitor.

 $C_{123} =$

3.57 µF

(b)



$$q_{12} = q_{123} = 44.6 \ \mu\text{C}.$$

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6 \ \mu \text{C}}{17.3 \ \mu \text{F}} = 2.58 \text{ V}.$$
$$V_1 = V_{12} = 2.58 \text{ V},$$

$$q_1 = C_1 V_1 = (12.0 \ \mu \text{F})(2.58 \text{ V})$$

= 31.0 \mu C.

Series capacitors and their equivalent have the same q ("seri-q").

We first reduce the

 $C_2 =$ 5.30 µF

circuit to a single

 $C_3 =$

4.50 µF

(a)

capacitor.

 $C_{1} =$

12.0 µF

App the potential difference.

arallel capacitors and eir equivalent have the same V ("par-V").

 $C_{123} = V_{123} =$

3.57 µF 12.5 V

Applying q = CV

yields the charge.

 $q_{123} =$

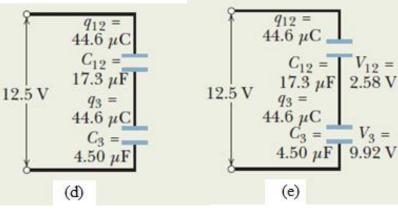
44.6 µC

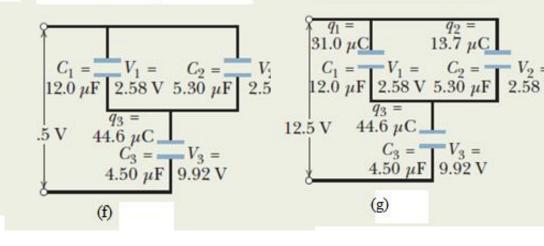
(c)

12.5 V

Applying q = CVyields the charge.

 $q_2 =$





$$\begin{array}{c} \mathsf{Pa} \\ \mathsf{lying} \ V = q/C \ \mathsf{yields} \end{array} \begin{array}{c} \mathsf{Pa} \\ \mathsf{the} \end{array}$$

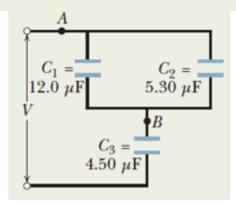
 $V_{q} =$

 $V_{123} =$

12.5 V

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

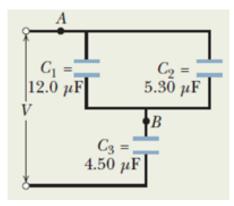
$$C_1 = 12.0 \ \mu\text{F}, \quad C_2 = 5.30 \ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50 \ \mu\text{F}.$$



 $C_{12} = C_1 + C_2 = 12.0 \ \mu\text{F} + 5.30 \ \mu\text{F} = 17.3 \ \mu\text{F}.$ (a) Find the equivalent capacitance for the combination of $\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$ capacitances shown in Fig. 25-10a, across which potential difference V is applied. Assume $C_1 = 12.0 \ \mu\text{F}, \quad C_2 = 5.30 \ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50 \ \mu\text{F}.$ $= \frac{1}{17.3 \,\mu\text{F}} + \frac{1}{4.50 \,\mu\text{F}} = 0.280 \,\mu\text{F}^{-1},$ $C_2 = 5.30 \ \mu F$ $C_1 = 12.0 \ \mu F$ $C_{123} = \frac{1}{0.280 \ \mu \text{F}^{-1}} = 3.57 \ \mu \text{F}.$ (Answer) 4.50 µl The equivalent of The equivalent of We first reduce the Next, we work series capacitors parallel capacitors circuit to a single backwards to the is smaller. is larger. capacitor. desired capacitor. $C_{12} =$ 17.3 µF $C_2 = - 5.30 \ \mu F$ $C_{123} =$ 3.57 μ F 12.5 V $C_{123} = 3.57 \ \mu F$ $C_3 =$ $C_3 = 4.50 \ \mu F$ $4.50 \ \mu F$

 $V_{123} = 12.5 V$

(b) The potential difference applied to the input terminals in Fig. 25-10*a* is V = 12.5 V. What is the charge on C_1 ?

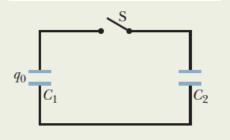


(b) The potential difference applied to the input terminals in Fig. 25-10*a* is V = 12.5 V. What is the charge on C_1 ? $q_{123} = C_{123}V = (3.57 \ \mu\text{F})(12.5 \ \text{V}) = 44.6 \ \mu\text{C}.$ We first reduce the $q_{12} = q_{123} = 44.6 \,\mu\text{C}.$ Next, we work circuit to a single backwards to the Applying q = CVcapacitor. desired capacitor. yields the charge. $V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6 \ \mu \text{C}}{17.3 \ \mu \text{F}} = 2.58 \text{ V}.$ A $q_{123} =$ $C_1 =$ $C_2 = 5.30 \ \mu F$ $44.6 \ \mu C$ 12.0 µF 12.5 V12.5 V $V_1 = V_{12} = 2.58 \text{ V},$ $\begin{array}{ccc} C_{123} = & V_{123} = \\ 3.57 \ \mu \mathrm{F} & 12.5 \ \mathrm{V} \end{array}$ $C_{123} = V_{123} =$ $C_3 =$ 3.57 µF 12.5 V 4.50 µF $q_1 = C_1 V_1 = (12.0 \ \mu \text{F})(2.58 \text{ V})$ (a) (b) (c) $= 31.0 \ \mu C.$ Parallel capacitors and Series capacitors and Applying q = CVtheir equivalent have Applying V = q/C yields their equivalent have yields the charge. the potential difference. the same V ("par-V"). the same q ("seri-q"). $q_1 = 31.0 \ \mu C$ $q_{12} =$ $q_2 = 13.7 \ \mu C$ $q_{12} =$ 44.6 µC 44.6 µC $\begin{array}{c} C_1 = & V_1 = & C_2 = & V_2 = \\ 12.0 \ \mu \text{F} & 2.58 \ \text{V} & 5.30 \ \mu \text{F} & 2.58 \end{array}$ $C_1 = V_1 = C_2 = V_1$ $C_{12} = 0$ $C_{12} = V_{12} =$ 12.0 µF 2.58 V 5.30 µF 2.5 $17.3 \,\mu F$ 17.3 µF 2.58 V 12.5 V12.5 V $q_3 =$ $q_3 =$ 44.6 µC. 44.6 µC 12.5 V 44.6 µC .5 V 44.6 µC $C_{2} = V_{2} =$ $C_3 = C_3 = - V_3 =$ $C_3 = V_3 =$ 4.50 µF 9.92 V $4.50 \mu F$ 4.50 µF 9.92 V 4.50 µF 9.92 V (d) (e) (g) (f)

Charging a capacitor

Capacitor 1, with $C_1 = 3.55 \ \mu\text{F}$, is charged to a potential difference $V_0 = 6.30 \text{ V}$, using a 6.30 V battery. The battery is then removed, and the capacitor is connected as in Fig. 25-11 to an uncharged capacitor 2, with $C_2 = 8.95 \ \mu\text{F}$. When switch S is closed, charge flows between the capacitors. Find the charge on each capacitor when equilibrium is reached.

After the switch is closed, charge is transferred until the potential differences match.



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After the switch is closed, charge is transferred until the potential differences match.

Fig. 25-11 A potential difference V_0 is applied to capacitor 1 and the charging battery is removed. Switch S is then closed so that the charge on capacitor 1 is shared with capacitor 2.

Calculations: Initially, when capacitor 1 is connected to the battery, the charge it acquires is, from Eq. 25-1.

 $q_0 = C_1 V_0 = (3.55 \times 10^{-6} \,\mathrm{F}) \,(6.30 \,\mathrm{V})$ = 22.365 × 10⁻⁶ C.

When switch S in Fig. 25-11 is closed and capacitor 1 begins to charge capacitor 2, the electric potential and charge on capacitor 1 decrease and those on capacitor 2 increase until

 $V_1 = V_2$ (equilibrium).

From Eq. 25-1, we can rewrite this as

thus

 $\frac{q_1}{C_1} = \frac{q_2}{C_2} \qquad \text{(equilibrium)}.$

Because the total charge cannot magically change, the total after the transfer must be

$$q_1 + q_2 = q_0$$
 (charge conservation);
 $q_2 = q_0 - q_1.$

We can now rewrite the second equilibrium equation as

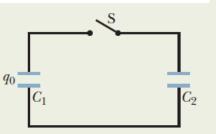
$$\frac{q_1}{C_1} = \frac{q_0 - q_1}{C_2}$$

Solving this for q_1 and substituting given data, we find

$$q_1 = 6.35 \,\mu\text{C.}$$
 (Answer)

The rest of the initial charge $(q_0 = 22.365 \ \mu\text{C})$ must be on capacitor 2: $q_0 = 16.0 \ \mu\text{C}$ (Answer)

$$q_2 = 16.0 \,\mu\text{C.} \qquad (\text{Answer})$$



Energy Stored in an Electric Field

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other. The potential difference V' between the plates at that instant will be q'/C. If an extra increment of charge dq' is then transferred, the increment of work required will be,

$$dW = V' \, dq' = \frac{q'}{C} \, dq'.$$

The work required to bring the total capacitor charge up to a final value q is

$$W = \int dW = \frac{1}{C} \int_{0}^{q} q' \, dq' = \frac{q^2}{2C}.$$

This work is stored as potential energy U in the capacitor, so that,

$$U = \frac{q^2}{2C}$$
 (potential energy). $U = \frac{1}{2}CV^2$ (potential energy).

Energy Density

In a parallel-plate capacitor, neglecting fringing, the electric field has the same value at all points between the plates. Thus, the **energy density u**—that is, the potential energy per unit volume between the plates—should also be uniform.

We can find u by dividing the total potential energy by the volume Ad of the space between the plates.

$$u = \frac{U}{Ad} = \frac{CV^2}{2Ad}.$$

But since (C = $\varepsilon_0 A/d$), this result becomes

$$u = \frac{1}{2}\varepsilon_0 \left(\frac{V}{d}\right)^2.$$

However, $(E=-\Delta V/\Delta s)$, V/d equals the electric field magnitude E. Therefore.

2	$u = \frac{1}{2}\varepsilon_0 E^2$ (energy density).
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Energy Density

An isolated conducting sphere whose radius R is 6.85 cm has a charge q = 1.25 nC.

(a) How much potential energy is stored in the electric field of this charged conductor?

(b) What is the energy density at the surface of the sphere?

Energy Density

An isolated conducting sphere whose radius R is 6.85 cm has a charge q = 1.25 nC.

(a) How much potential energy is stored in the electric field of this charged conductor?

KEY IDEAS

(1) An isolated sphere has capacitance given by Eq. 25-18 $(C = 4\pi\varepsilon_0 R)$. (2) The energy *U* stored in a capacitor depends on the capacitor's charge *q* and capacitance *C* according to Eq. 25-21 $(U = q^2/2C)$.

Calculation: Substituting $C = 4\pi\varepsilon_0 R$ into Eq. 25-21 gives us

$$J = \frac{q^2}{2C} = \frac{q^2}{8\pi\epsilon_0 R}$$

= $\frac{(1.25 \times 10^{-9} \text{ C})^2}{(8\pi)(8.85 \times 10^{-12} \text{ F/m})(0.0685 \text{ m})}$
= $1.03 \times 10^{-7} \text{ J} = 103 \text{ nJ}.$ (Answer)

(b) What is the energy density at the surface of the sphere?

KEY IDEA

The density *u* of the energy stored in an electric field depends on the magnitude *E* of the field, according to Eq. 25-25 $(u = \frac{1}{2}\varepsilon_0 E^2)$.

Calculations: Here we must first find *E* at the surface of the sphere, as given by Eq. 23-15:

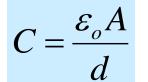
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2}.$$

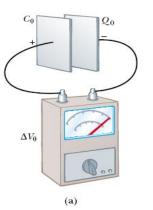
The energy density is then

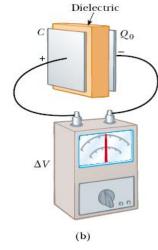
$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{32\pi^2\varepsilon_0 R^4}$$

= $\frac{(1.25 \times 10^{-9} \text{ C})^2}{(32\pi^2)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0685 \text{ m})^4}$
= 2.54 × 10⁻⁵ J/m³ = 25.4 µJ/m³. (Answer)

In a region completely filled by a dielectric material of dielectric constant κ , all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa \varepsilon_0$.

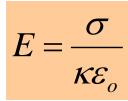






 $C = \kappa \frac{\varepsilon_o A}{2}$

E = $4\pi\kappa\epsilon_{o}$



$$E = \frac{\sigma}{\varepsilon_o}$$

 $4\pi\varepsilon$

E

A dielectric, is an insulating material such as mineral oil or plastic, and is characterized by a numerical factor κ , called the dielectric constant of the material.

The introduction of a dielectric also limits the potential difference that can be applied between the plates to a certain value *Vmax*, called the breakdown potential. Every dielectric material has a characteristic *dielectric strength*, which is the maximum value of the electric field that it can tolerate without breakdown.

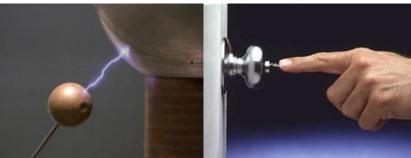




Table 25-1

Material	Dielectric Constant <i>κ</i>	
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer		
oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Fitania		
ceramic	130	
trontium		
titanate	310	8

For a vacuum, κ = unity.

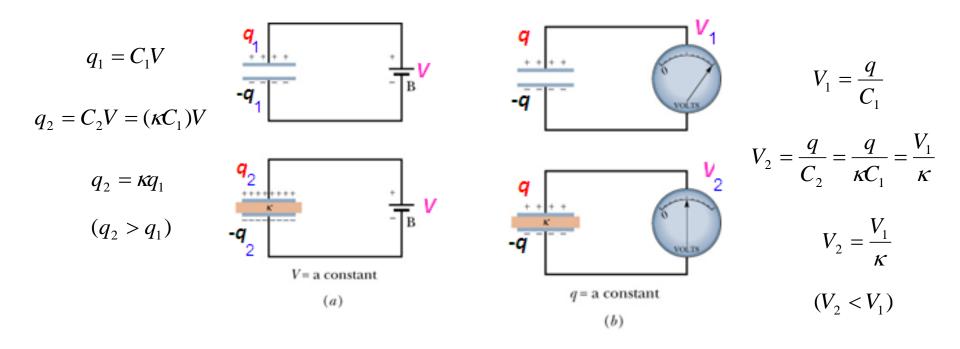
^aMeasured at room temperature, except for the water.

Fig. a: Capacitor voltage V remains constant

This is because the battery remains connected to the plates. Before the dielectric is inserted between the capacitor plates

Fig. b: Capacitor charge q remains constant

This is because the plates are isolated. Before the dielectric is inserted between the capacitor plates



A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference V = 12.5 V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

A parallel-plate capacitor whose capacitance C is 13.5 pF is charged by a battery to a potential difference V = 12.5 V between its plates. The charging battery is now disconnected, and a porcelain slab ($\kappa = 6.50$) is slipped between the plates.

(a) What is the potential energy of the capacitor before the slab is inserted?

KEY IDEA

We can relate the potential energy U_i of the capacitor to the capacitance C and either the potential V (with Eq. 25-22) or the charge q (with Eq. 25-21):

$$U_i = \frac{1}{2}CV^2 = \frac{q^2}{2C}.$$

Calculation: Because we are given the initial potential V (= 12.5 V), we use Eq. 25-22 to find the initial stored energy:

$$U_i = \frac{1}{2}CV^2 = \frac{1}{2}(13.5 \times 10^{-12} \text{ F})(12.5 \text{ V})^2$$

= 1.055 × 10⁻⁹ J = 1055 pJ ≈ 1100 pJ. (Answer)

(b) What is the potential energy of the capacitor–slab device after the slab is inserted?

KEY IDEA

Because the battery has been disconnected, the charge on the capacitor cannot change when the dielectric is inserted. However, the potential *does* change.

Calculations: Thus, we must now use Eq. 25-21 to write the final potential energy U_f , but now that the slab is within the capacitor, the capacitance is κC . We then have

$$U_f = \frac{q^2}{2\kappa C} = \frac{U_i}{\kappa} = \frac{1055 \text{ pJ}}{6.50}$$

= 162 pJ \approx 160 pJ. (Answer)

When the slab is introduced, the potential energy decreases by a factor of κ .

The "missing" energy, in principle, would be apparent to the person who introduced the slab. The capacitor would exert a tiny tug on the slab and would do work on it, in amount

$$W = U_i - U_f = (1055 - 162) \text{ pJ} = 893 \text{ pJ}.$$

If the slab were allowed to slide between the plates with no restraint and if there were no friction, the slab would oscillate back and forth between the plates with a (constant) mechanical energy of 893 pJ, and this system energy would transfer back and forth between kinetic energy of the moving slab and potential energy stored in the electric field.

What have we learnt

- What is Capacitors and Capacitance?
- The Capacitance of different types of capacitors:
 - Parallel plates Cylindrical Spherical Isolated sphere
- How do we add the effect of capacitors?
- How do we find out the energy stored in a capacitor?
- Capacitors with dielectric materials.