Chapter 3

VECTORS

Physical Quantities

Vector Quantities

magnitude + direction

Follow certain rules of addition and multiplication

Scalar Quantities

+Ve Number

-Ve Number

Follow the rules of ordinary algebra

Vectors Addition

Adding Vectors Geometrically

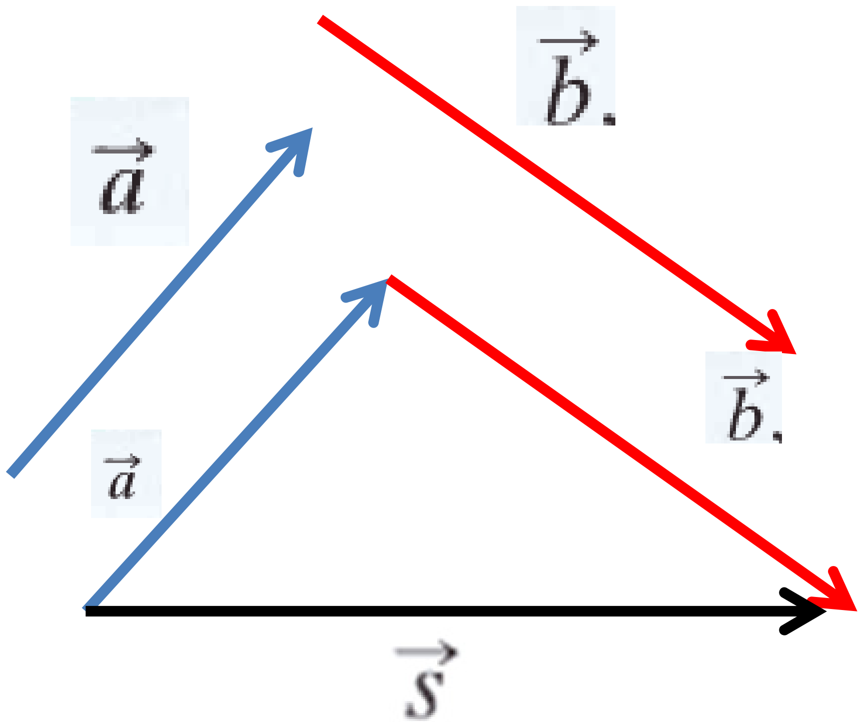
- Vector equation
- Commutative Law
- Associative Law
- Vector Subtraction

Adding Vectors by Components

- Components
- resolving the vector
- writing a vector in magnitude- angle notation

- Unit Vectors
- writing a vector in Unit vector notation

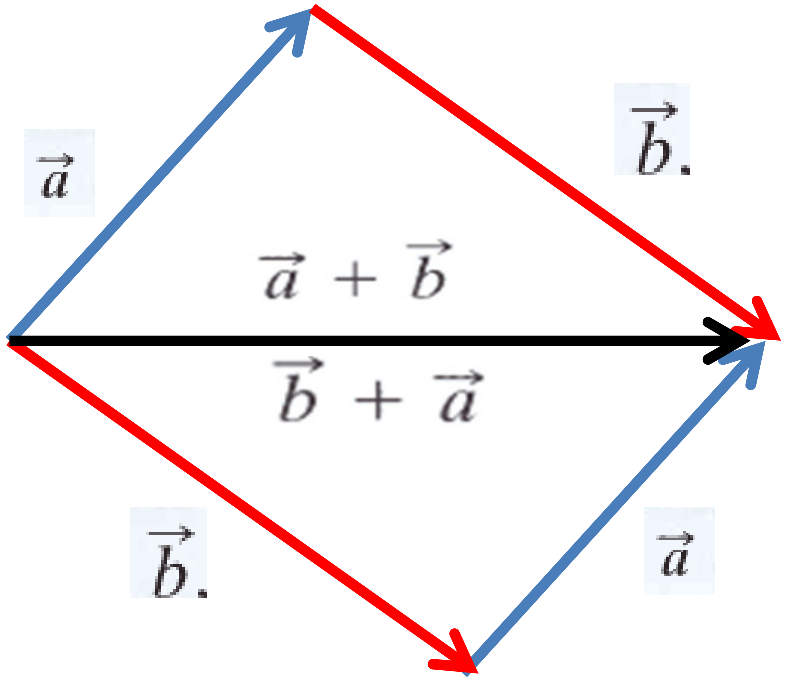
Adding Vectors Geometrically



- Vector equation

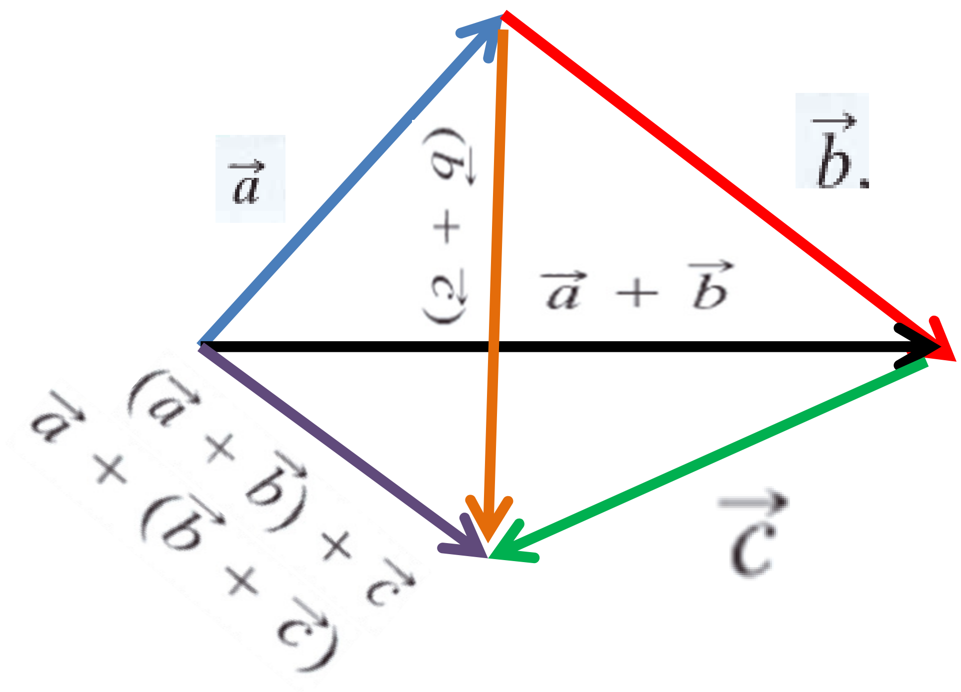
$$\vec{s} = \vec{a} + \vec{b},$$

• Commutative Law



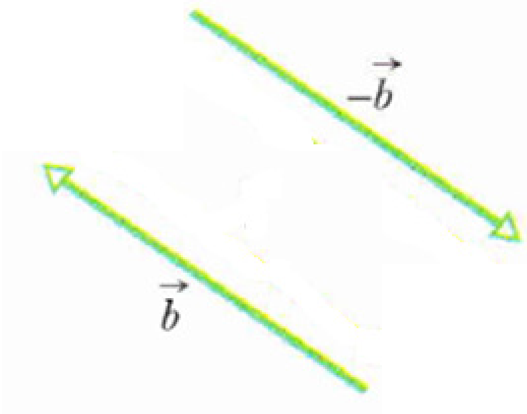
$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

• Associative Law

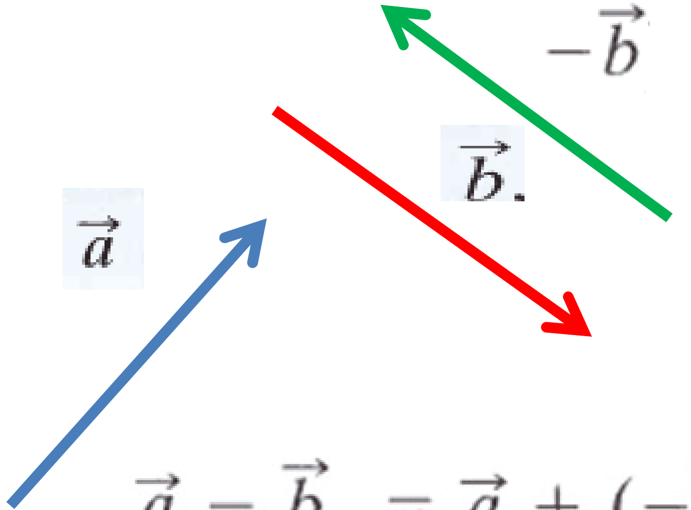


$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$$

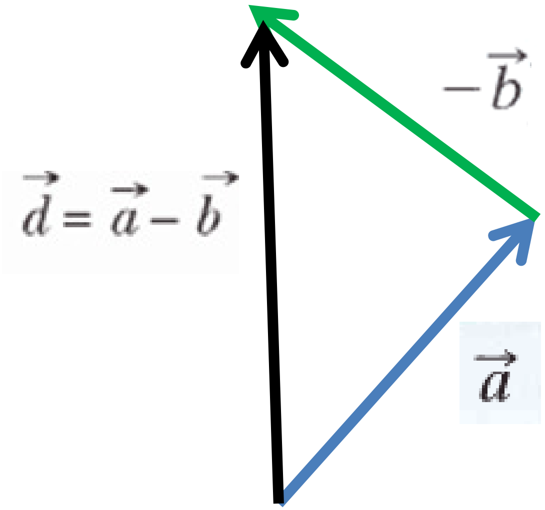
• Vector Subtraction



$$\vec{b} + (-\vec{b}) = 0$$



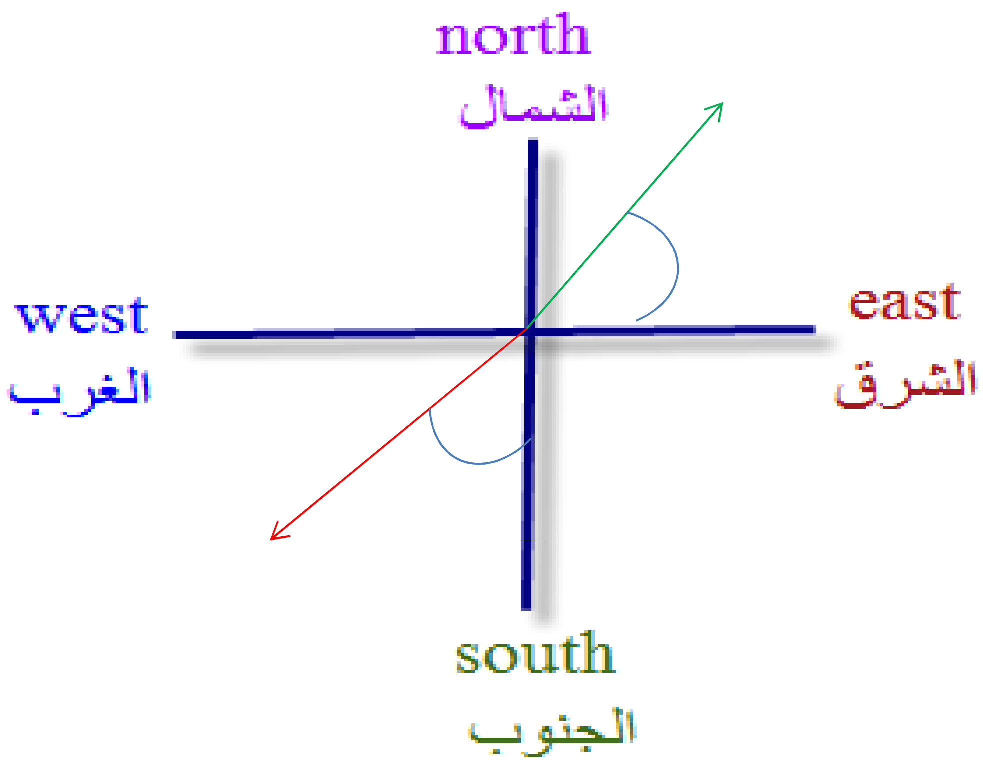
$$\vec{a} - \vec{b} = \vec{a} + (-\vec{b})$$



$$\boxed{\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})}$$

Sample Problem 3-1

In an orienteering class, you have the goal of moving as far (straight-line distance) from base camp as possible by making three straight-line moves. You may use the following displacements in any order: (a) $\vec{a}$, 2.0 km due east (directly toward the east); (b) $\vec{b}$, 2.0 km 30° north of east (at an angle of 30° toward the north from due east); (c) $\vec{c}$, 1.0 km due west. Alternatively, you may substitute either $-\vec{b}$ for $\vec{b}$ or $-\vec{c}$ for $\vec{c}$. What is the greatest distance you can be from base camp at the end of the third displacement?



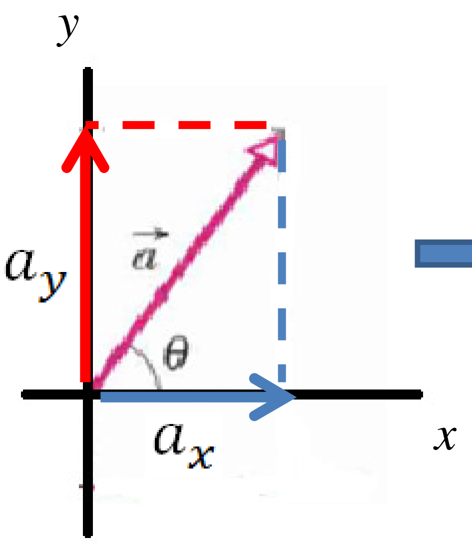
North of east = toward the north from due east

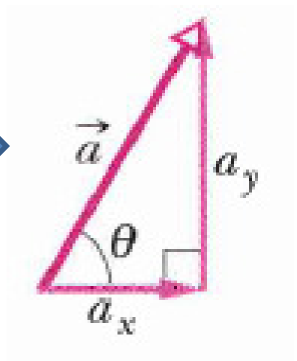
West of south= = toward the west from due south

Components of Vectors

• Resolving the vector is the process of finding the components

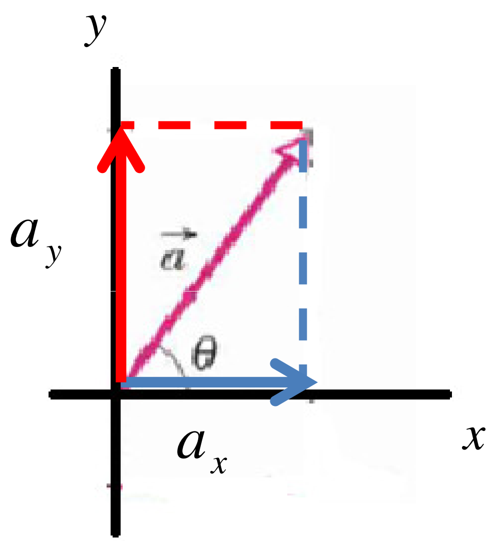
• Component is the projection of the vector on an axis



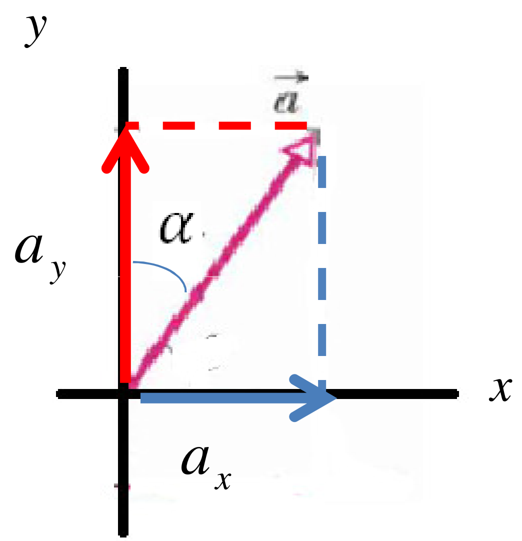


$$\cos\theta = \frac{a_x}{a} \;, \qquad \sin\theta = \frac{a_y}{a}$$

$$\boxed{a_x = a\ \cos\theta \ \ and\ \ a_y = a\ \sin\theta}$$



$$a_x = a\cos\theta \ \ and \ a_y = a\sin\theta$$



$$a_x = a \ \sin\alpha \ \ and \ a_y = a\cos\alpha$$

- Writing a vector in magnitude- angle notation

$$\vec{a} : \ a_x \ and \ a_y$$

Magnitude

$$a = |a| = \sqrt{a_x{}^2 + a_y{}^2}$$

Angle (Direction)

$$\tan\theta = \frac{a_y}{a_x} \ => \theta = \tan^{-1}\frac{a_y}{a_x}$$

$$\vec{a}$$

$$a \text{ and } \theta$$

- Finding the components.

$$a_x = a \cos\theta$$

$$a_y = a \sin\theta$$

Rem : When use these formulas to find the components, the angle must be measured from positive X-axis, if clockwise put θ -ve if counterclockwise put θ +ve.

$$a_x \text{ and } a_y$$

- Writing a vector in magnitude-angle notation

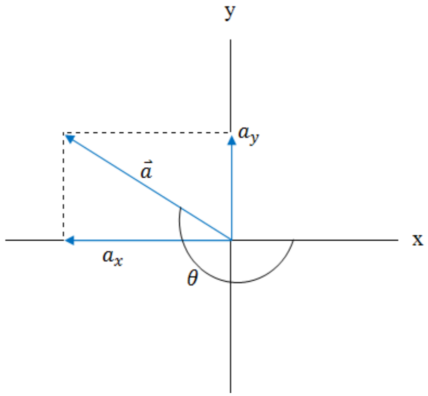
$$a = |a| = \sqrt{a_x{}^2 + a_y{}^2}$$

$$\tan\theta = \frac{a_y}{a_x}$$

How to find the components of a vector in different positions?

When the angle is from the +ve x-axis

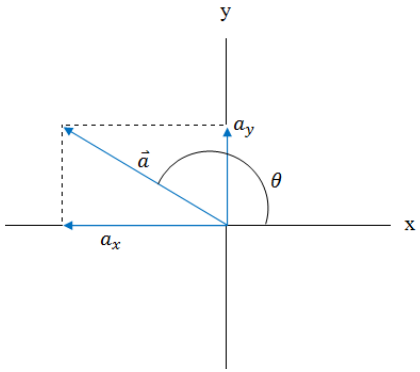
Clockwise



$$a_x = a \cos -\theta$$

$$a_y = a \sin -\theta$$

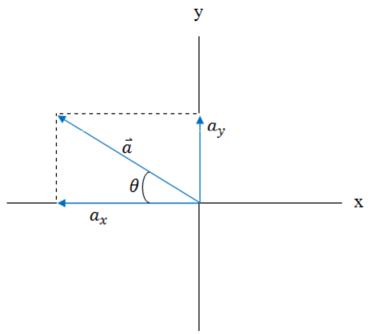
Counter-clockwise



$$a_x = a \cos \theta$$

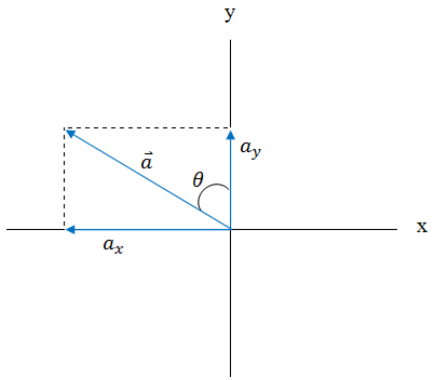
$$a_y = a \sin \theta$$

When the angle is from any different axis



$$a_x = -a \cos \theta$$

$$a_y = +a \sin \theta$$

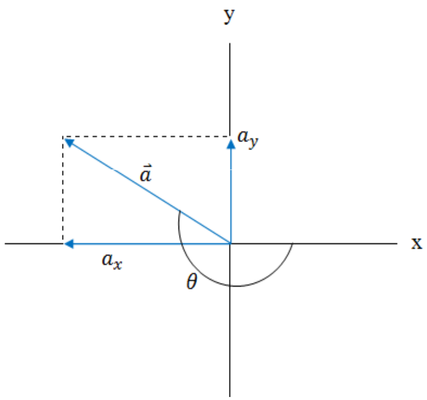


$$a_x = -a \sin \theta$$

$$a_y = +a \cos \theta$$

To find the components, when the angle is measured from the +ve x-axis use

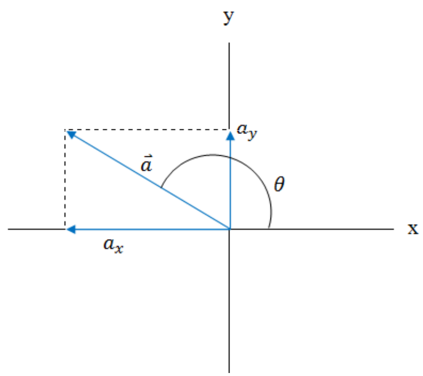
$$\boxed{\begin{aligned} a_x &= a \cos\theta \\ a_y &= a \sin\theta \end{aligned}}$$



Put **<u>θ = –ve</u>**, **When the angle is measured <u>clockwise</u> from the +ve x-axis**

$$a_x = a \cos -\theta$$

$$a_y = a \sin -\theta$$



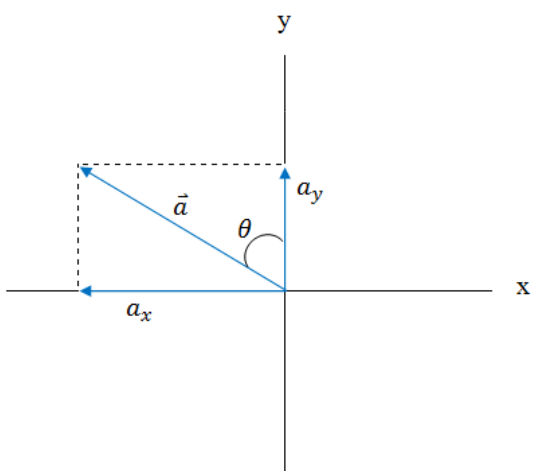
Put **<u>θ = +ve</u>**, **When the angle is measured <u>Counter-clockwise</u> from the +ve x-axis**

$$a_x = a \cos\theta$$

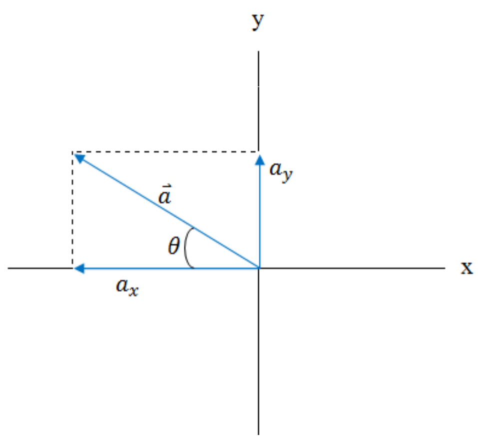
$$a_y = a \sin\theta$$

To find the components , when the angle is measured from any axis even the +ve x-axis

1- Take the given angle with the axis
2- put the <u>signs</u> of the components according to their positions on the axes
3- put <u>sine</u> or <u>cosine</u> according to the angle position.



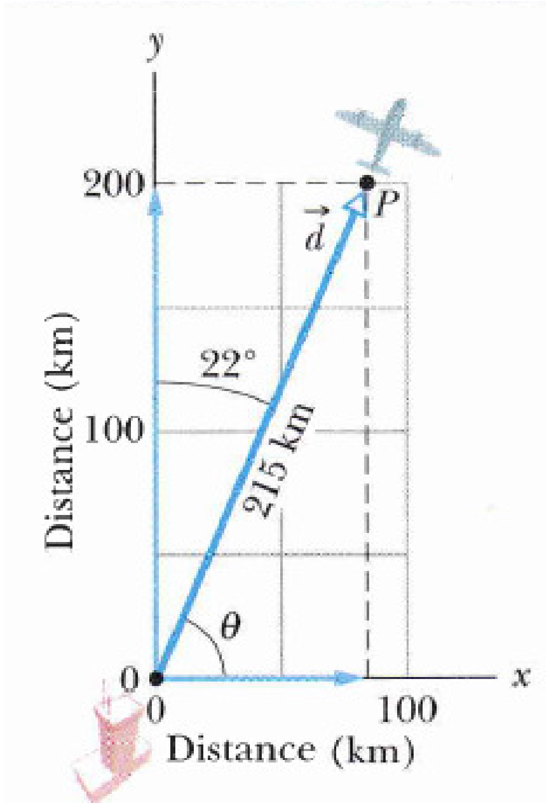
$$a_x = -\,a\,\sin\theta$$
$$a_y = +\,a\,\cos\theta$$



$$a_x = -\,a\,\cos\theta$$
$$a_y = +\,a\,\sin\theta$$

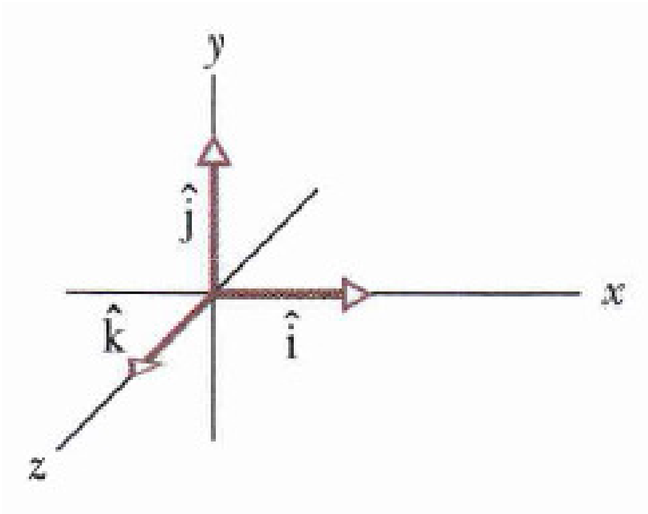
Sample Problem 3-2

A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?



Unit Vectors

• Unit vector is a vector of magnitude 1 and points in a particular direction



• **Writing a vector in Unit vector notation**

$$\vec{a} = a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}$$

Vector Components

$$\vec{a} = \boxed{a_x\hat{\mathrm{i}}} + \boxed{a_y\hat{\mathrm{j}}} + \boxed{a_z\hat{\mathrm{k}}}$$

Scalar components

Adding vectors by Components

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\boxed{\vec{a} + \vec{b} = (a_x + b_x)\hat{\imath} + (a_y + b_y)\hat{\jmath} + (a_z + b_z)\hat{k}}$$

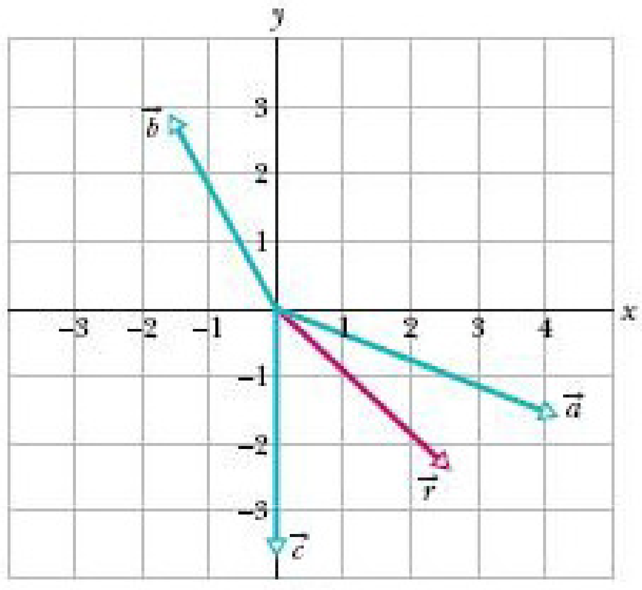
Sample Problem 3-4

Figure 3-16*a* shows the following three vectors:

$$\vec{a} = (4.2\text{ m})\hat{\text{i}} - (1.5\text{ m})\hat{\text{j}},$$
$$\vec{b} = (-1.6\text{ m})\hat{\text{i}} + (2.9\text{ m})\hat{\text{j}},$$

and $$\vec{c} = (-3.7\text{ m})\hat{\text{j}}.$$

What is their vector sum $\vec{r}$ which is also shown?



Vectors Multiplication

Multiplying a vector by a scalar

- +ve scalar
  - will produce a new vector in the same direction as the started vector

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$

$$2\vec{a} = 4\hat{\imath} + 6\hat{\jmath}$$

- -ve scalar
  - will produce a new vector in the opposite direction of the started vector

$$\vec{a} = 2\hat{\imath} + 3\hat{\jmath}$$

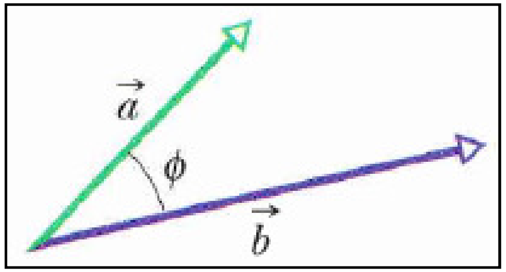
$$-2\vec{a} = -4\hat{\imath} - 6\hat{\jmath}$$

Multiplying a vector by a vector

- **Scalar product (or Dot product)**
  - will produce a scalar
- **Vector product (or cross product)**
  - will produce a new vector

Scalar (or Dot product)

If the two vectors are given in magnitude and the angle between them



$$\vec{a} \cdot \vec{b} = ab \cos\phi$$

If the two vectors are given in unit vector notation

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

$$\boxed{\vec{a} \cdot \vec{b} = ab \cos\phi}$$

1- The scalar product is commutative $\Rightarrow$ $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

2- If the two vectors are parallel $\Rightarrow$ $\theta = 0 \Rightarrow \vec{a} \,.\, \vec{b} = a\, b$

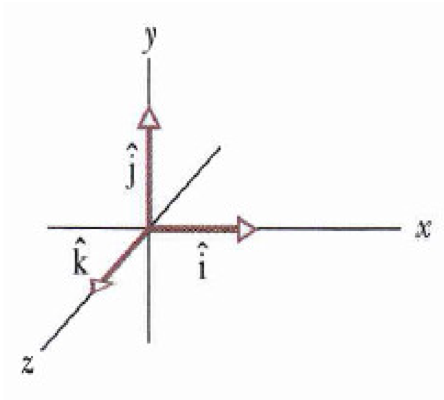
3- If the two vectors are perpendicular $\Rightarrow$ $\theta = 90 \Rightarrow \vec{a} \,.\, \vec{b} = 0$

4- If the two vectors are Antiparallel $\Rightarrow$ $\theta = 180 \Rightarrow \vec{a} \,.\, \vec{b} = -\, a\, b$

5- Multiplying Unit vectors

$$\hat{i} . \hat{i} = (1)(1)\cos 0 = 1 \Rightarrow \hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{i} . \hat{j} = (1)(1)\cos 90 = 0 \Rightarrow \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0,$$



Properties Of the scalar product

The scalar product is commutative

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$$

the angle between two vectors can be found

$$\vec{a} \cdot \vec{b} = ab \cos\theta$$

any two similar unit vectors

$$\hat{i} \cdot \hat{i} = \hat{j} \cdot \hat{j} = \hat{k} \cdot \hat{k} = 1$$

any two different unit vectors

$$\hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0,$$

If $\theta = 0 \Rightarrow \vec{a} . \vec{b} = a\,b$ ⇨ vectors are parallel

$\theta = 180 \Rightarrow \vec{a} . \vec{b} = -\,a\,b$ ⇨ vectors are anti parallel

$\theta = 90 \Rightarrow \vec{a} . \vec{b} = 0$ ⇨ vectors are perpendicular

Sample Problem 3-7

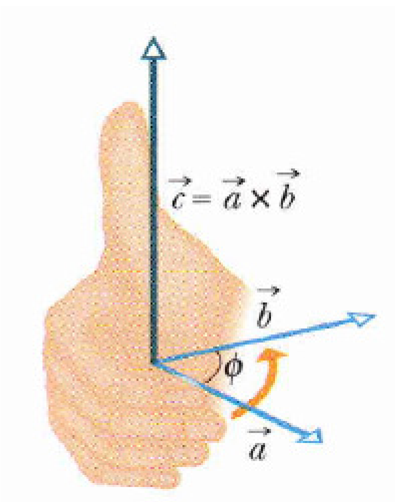
What is the angle ϕ between $\vec{a} = 3.0\hat{\mathrm{i}} - 4.0\hat{\mathrm{j}}$ and $\vec{b} = -2.0\hat{\mathrm{i}} + 3.0\hat{\mathrm{k}}$?

Vector (or Cross product)

If the two vectors are given in magnitude and angle between them

$$|\vec{a} \times \vec{b}| = |c| = ab \sin\phi$$

The direction of the result vector



If the two vectors are given in unit vector notation

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= \hat{\mathrm{i}} \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \hat{\mathrm{j}} \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \hat{\mathrm{k}} \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$= (a_y b_z - b_y a_z)\hat{\mathrm{i}} + (a_z b_x - b_z a_x)\hat{\mathrm{j}}$$

$$+ (a_x b_y - b_x a_y)\hat{\mathrm{k}}$$

$$\boxed{|\vec{a} \times \vec{b}| = |c| = ab \sin\phi}$$

1- The vector product is Anti commutative ⟹ $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

2- If the two vectors are parallel ⟹ $\theta = 0 \Rightarrow \vec{a} \times \vec{b} = 0$

3- If the two vectors are perpendicular ⟹ $\theta = 90 \Rightarrow |\vec{a} \times \vec{b}| = a\,b$

4- If the two vectors are Anti-parallel ⟹ $\theta = 180 \Rightarrow \vec{a} \times \vec{b} = 0$

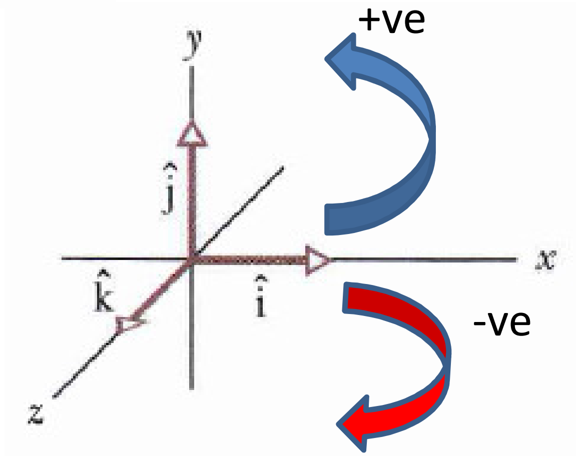
5- Multiplying Unit vectors

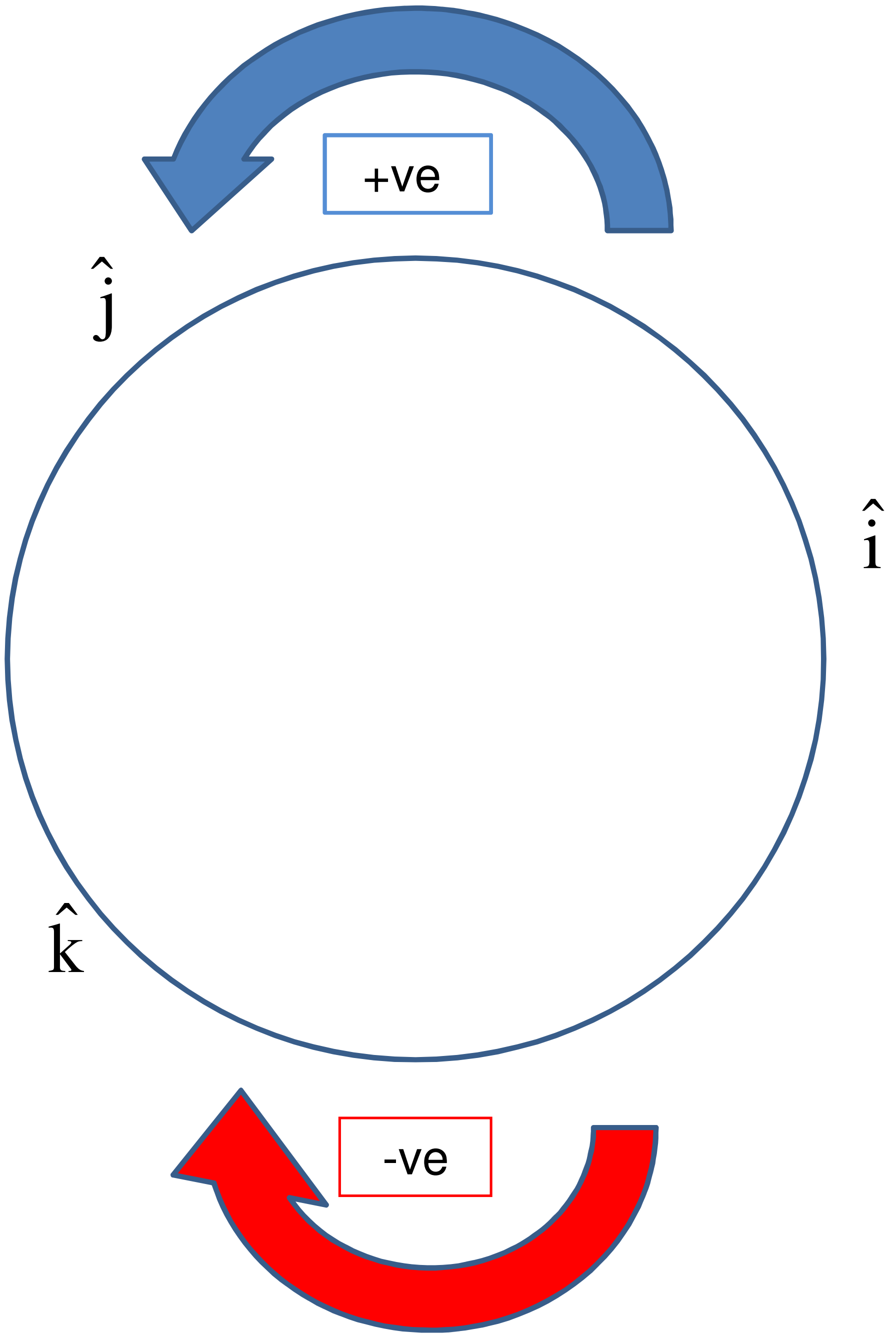
$$\left|\hat{i} \times \hat{i}\right| = (1)(1)\sin 0 = 0 \Longrightarrow \hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$\left|\hat{i} \times \hat{j}\right| = (1)(1)\sin 90 = 1 \Longrightarrow \hat{i} \times \hat{j} = \hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i}, \quad \hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k} \quad \hat{k} \times \hat{j} = -\hat{i} \quad \hat{i} \times \hat{k} = -\hat{j}$$





+ve
$\hat{j}$
$\hat{i}$
$\hat{k}$
-ve

Properties of the Vector product

any two different unit vectors

$$\hat{i} \times \hat{j} = \hat{k}, \quad \hat{j} \times \hat{k} = \hat{i},$$

$$\hat{k} \times \hat{i} = \hat{j}$$

Anti- commutative

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

any two similar unit vectors

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

The small angle between the two vectors must be used because the odd property of the sin function

$$|\vec{a} \times \vec{b}| = |c| = ab \sin\phi$$

If $\theta = 0 \Rightarrow \vec{a} \times \vec{b} = 0$ ➡ vectors are parallel

$\theta = 180 \Rightarrow \vec{a} \times \vec{b} = 0$ ➡ vectors are anti parallel

$\theta = 90 \Rightarrow |\vec{a} \times \vec{b}| = a\, b$ ➡ vectors are perpendicular

Sample Problem 3-9

If $\vec{a} = 3\hat{\mathrm{i}} - 4\hat{\mathrm{j}}$ and $\vec{b} = -2\hat{\mathrm{i}} + 3\hat{\mathrm{k}}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

The End