CH.(9): Center of mass (COM) and Linear Momentum

	Single Particle	System of Particles
Position(1D) x-axis	x إحداثيات النقطة على محور x	$x_{com} = \dfrac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$ *Position of centre of mass* Where M(total mass)= $m_1 + m_2 + m_3 + \ldots$
y-axis	y إحداثيات النقطة على محور y	$y_{com} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$
z-axis	z إحداثيات النقطة على محور z	$z_{com} = \dfrac{m_1z_1 + m_2z_2 + m_3z_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$
Position vector (3D)	$r = x\,i + y\,j + z\,k$	$r_{com} = x_{com}\,i + y_{com}\,j + z_{com}\,k$ *Position vector of centre of mass* x_{com} is the x-component of the coordinate of the COM y_{com} is the y-component of the coordinate of the COM z_{com} is the z-component of the coordinate of the COM *The coordinate of the COM: (x_{com}, y_{com}, z_{com})*

Exp. (1): Three particles of masses m_1=1 kg, m_2=2 kg, and m_3=3 kg are located in *xy* plane as (3,2), (-1,1), and (3,-2), respectively. Find the coordinate of the center of mass.
The components of the coordinate of the center of mass are x_{COM} and y_{COM}

Particle	m	x	Y
1	1	3	2
2	2	-1	1
3	3	3	-2
		$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$	$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$
		$X_{com} = \frac{1*3+2*-1+3*3}{1+2+3}$ = 1.67	$y_{com} = \frac{1*2+2*1+3*-2}{1+2+3}$ = -0.33

The coordinate of the center of mass is(1.67,-0.33)

Exp.(2): Problem (1): (a) The x coordinates of the system's center of mass is

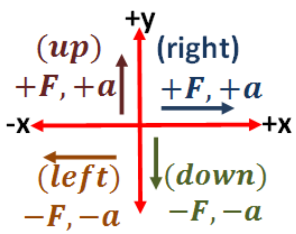
$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3} = \frac{2*(-1.2) + 4*6 + 3*x_3}{2+4+3} = -0.5$$

➔ x_3= - 1.5 m

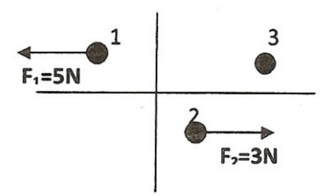
(b) The y coordinates of the system's center of mass is

$$y_{com} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3} = \frac{2*5 + 4*(-0.75) + 3*y_3}{2+4+3} = -0.7$$

➔ y_3= - 1.43 m

	Single Particle	*System of Particles*
Newton's 2^{nd} law 	$$\vec{F}_{net} = m\,\vec{a}$$ مع مراعاة أن القوة و التسارع كميات متجهه يعوض عنهما بمقدار واتجاه **If body is stationary ➔ v=0 ➔a=0 ➔ $F_{net} = 0$**	$$\vec{F}_{net} = M\,\vec{a}_{com}$$ ***Where a_{com} the acceleration of center of mass*** مع مراعاة أن القوة و التسارع كميات متجهه يعوض عنهما بمقدار و اتجاه **If body is stationary ➔ v_{COM} =0 ➔ a_{com}=0 ➔ $F_{net} = 0$**

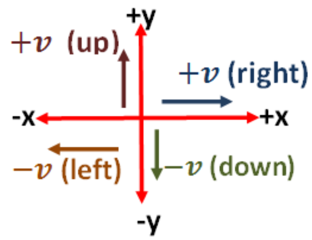
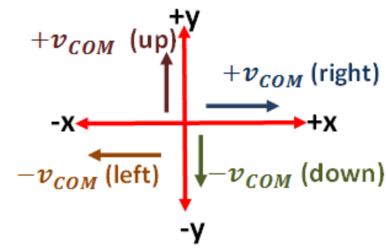
Exp.(3): In the figure, what is the magnitude of the force F_3 acting on particle 3 if the center of mass of system is stationary?



Stationary ➔ v_{COM} =0 ➔ a_{COM} =0

$\Sigma F_x = 0$

$F_{1x} + F_{2x} + F_{3x} = 0$ ➔ $F_3 = -F_1 - F_2$ =-(-5)-(+3)=5-3=2N

	Single Particle	System of Particles
Linear Momentum *The unit of P is kg m/s ***Linear momentum is vector quantity**	$\vec{P} = m\,\vec{v}$ مع مراعاة أن السرعة كمية متجهه يعوض عنها بمقدار واتجاه  **If body is stationary ➔ v=0 ➔P=0**	$\vec{P} = M\,\vec{v}_{com}$ مع مراعاة أن السرعة كمية متجهه يعوض عنها بمقدار واتجاه  **If body is stationary ➔ v=0 ➔P=0**

Single Particle	System of Particles
Newton's 2nd law $F_{net} = m\,a$ $\quad F_{net} = \frac{dP}{dt}$	***Newton's 2nd law*** $F_{net} = M\,a_{com}$ $\quad F_{net} = \frac{dP}{dt}$

The law of conservation of linear momentum: $P_{initial} = P_{final}$

$$((m_1v_1 + m_2v_2 + m_3v_3 + \ \dots)_i = (m_1v_1 + m_2v_2 + m_3v_3 + \ \dots)_f$$

Exp.(4): A 0.4 kg ball is dropped from a window and landed on the street with speed 35 m/s, and then rebound with a speed 25 m/s. What is the magnitude of the change of its momentum?

m= 0.4 kg v_i= -35 m/s, v_f = +25 m/s

$|\Delta P| = |P_f - P_i| = m\,|v_f - v_i| = 0.4\,|(+25) - (-35)| = 0.4\,|25+35| = 24$ kg.m/s

Exp.(5): A ballot box with mass m=6 kg slides with speed across a frictionless floor in positive direction of an x-axis. The box explodes (انشطر) into two pieces. One piece, with m_1= 2kg, moves in the positive direction of the x-axis at v_1=8m/s. The second piece, with m_2=4kg, rebounds (ارتد) with speed v_2 =2m/s. What is the velocity of the box?

m=6kg v=?? m1=2kg v1=+8m/s(positive x-axis (right)) m2=4kg v_2= -2m/s (rebounds in negative x-axis-to left)

$$P_{initial} = P_{final}$$

$((m\,v)_i = (m_1 v_1 + m_2 v_2)_f$

6 xv =2 x 8 + 4x -2= 16-8

V= 8/6= +1.33 m/s

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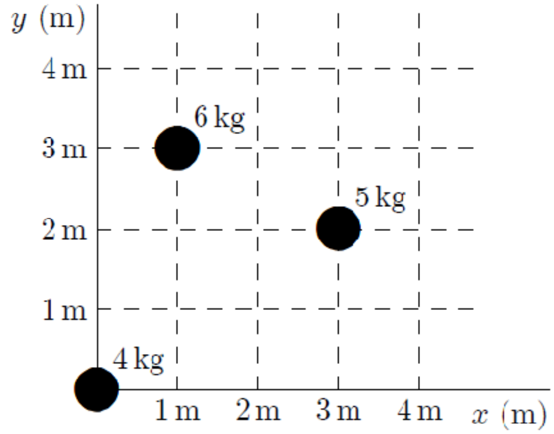
Problems:

1- The x and y coordinates of the center of mass of the three-particle system shown below are:



A. 0, 0
B. 1.3 m, 1.7 m
C. 1.4 m, 1.9 m
D. 1.9 m, 2.5 m
E. 1.4 m, 2.5 m

ans: C

2- The center of mass of a system of particles obeys an equation similar to Newton's second law $\vec{F} = m\vec{a}_{com}$, where:

A. $\vec{F}$ is the net internal force and m is the total mass of the system
B. $\vec{F}$ is the net internal force and m is the mass acting on the system
C. $\vec{F}$ is the net external force and m is the total mass of the system
D. $\vec{F}$ is the force of gravity and m is the mass of Earth
E. $\vec{F}$ is the force of gravity and m is the total mass of the system

ans: C

3- Momentum may be expressed in:

A. kg/m
B. gram·s
C. N·s
D. kg/(m·s)
E. N/s

ans: C

4- A 1.0-kg ball moving at 2.0 m/s perpendicular to a wall rebounds from the wall at 1.5 m/s. The change in the momentum of the ball is:

A. zero
B. 0.5 N · s away from wall
C. 0.5 N · s toward wall
D. 3.5 N · s away from wall
E. 3.5 N · s toward wall

ans: D

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5- If the total momentum of a system is changing:

A. particles of the system must be exerting forces on each other
B. the system must be under the influence of gravity
C. the center of mass must have constant velocity
D. a net external force must be acting on the system
E. none of the above

ans: D

6- A 2.5-kg stone is released from rest and falls toward Earth. After 4.0 s, the magnitude of its momentum is:

A. $98\,\mathrm{kg \cdot m/s}$
B. $78\,\mathrm{kg \cdot m/s}$
C. $39\,\mathrm{kg \cdot m/s}$
D. $24\,\mathrm{kg \cdot m/s}$
E. zero

ans: A