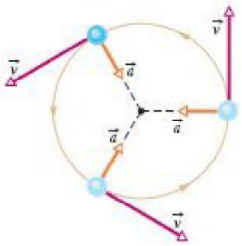
# CHAPTER(4) Motion in Two and Three Dimensions

| | 1D | 2/3 D |
|---|---|---|
| **1-Position** | In x-axis ➔$x$<br>In y-axis ➔$y$<br>In z-axis ➔$z$ | *Position vector:* $(\vec{r})$<br>$\vec{r} = x\,i + y\,j + z\,k$ |
| **2-Displacement** | $\Delta x = x_2 - x_1$<br>$\Delta y = y_2 - y_1$<br>$\Delta z = z_2 - z_1$ | $\Delta r = r_2 - r_1$<br>$\Delta r = \Delta x\,i + \Delta y\,j + \Delta z\,k$ |
| **3-Velocity:**<br>**-Average Velocity** | $\upsilon_{avg} = \frac{\Delta x}{\Delta t}$ | *Velocity Vector*<br>$\upsilon_{avg} = \frac{\Delta r}{\Delta t} = \frac{r_2 - r_1}{\Delta t}$<br>$= \frac{\Delta x}{\Delta t} i + \frac{\Delta y}{\Delta t} j + \frac{\Delta z}{\Delta t} k$ |
| **- Velocity** | $\upsilon = \frac{dx}{dt}$ | $v = \frac{dr}{dt} = \frac{dx}{dt} i + \frac{dy}{dt} j + \frac{dz}{dt} k$<br>$v = \frac{dr}{dt} = v_x i + v_y j + v_z k$<br>The direction of $v$ of a particle is always tangent to the particle's path at the particle's position. |
| **4- Acceleration Average acceleration:** | $a_{avg} = \frac{\Delta \upsilon}{\Delta t} = \frac{\Delta x}{(\Delta t)^2}$ | $a_{avg} = \frac{\Delta \upsilon}{\Delta t} = \frac{r_2 - r_1}{(\Delta t)^2} = \frac{\Delta x}{\Delta t^2} i + \frac{\Delta y}{\Delta t^2} j + \frac{\Delta z}{\Delta t^2} k$ |
| **Acceleration** | $a = \frac{d\upsilon}{dt} = \frac{d^2 x}{dt^2}$ | $a = \frac{dv}{dt} = \frac{dv_x}{dt} i + \frac{dv_y}{dt} j + \frac{dv_z}{dt} k$<br>$a = \frac{d^2 r}{dt^2} = \frac{d^2 x}{dt^2} i + \frac{d^2 y}{dt^2} j + \frac{d^2 z}{dt^2} k$ |
| **Magnitude & Direction** | $V = \underset{\text{direction}}{\pm}$ $\underset{\text{magnitude}}{no}$ | $\lvert V\rvert = V = \sqrt{{V_x}^2 + {V_y}^2}$<br>$\theta = \tan^{-1}\frac{V_y}{V_x}$ |
| **6-Uniform circular Motion**<br>  | // | $V = \underset{\text{variable (tangent)}}{(\text{dir.})}$ $\underset{\text{Const. (speed=}\lvert v\rvert)}{(\text{mag.})}$<br>$a = \underset{\text{variable (inward)}}{(\text{dir.})}$ $\underset{\text{Const. } (a = \frac{v^2}{r})}{(\text{mag.})}$<br>**Period=**$T = \frac{Circumference(distance)}{speed} = \frac{2\pi r}{\lvert v\rvert}$ |

د.هناء فرحان

# Projectiles المقذوفات

## 1D

### Const. acceleration
الحركة الأفقية بتسارع ثابت

**Horizontal motion(x-axis)**

1- $\upsilon = \upsilon_0 + a\,t$

2- $x - x_0 = \upsilon_0 t + \frac{1}{2} a t^2$

3- $\upsilon^2 = \upsilon_0^2 + 2\,a(x - x_0)$

4- $x - x_0 = \frac{1}{2}\,(\upsilon + \upsilon_0)\,t$

5- $x - x_0 = \upsilon t - \frac{1}{2} a t^2$

### Free Falling
الحركة العمودية بتسارع ثابت

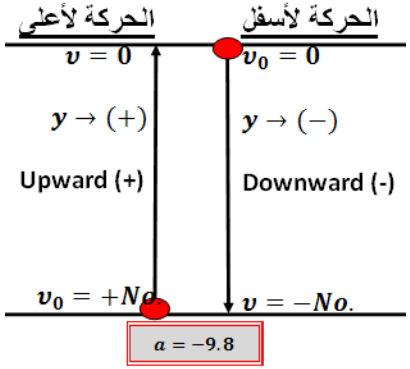
**Vertical motion(y-axis)**

1- $\upsilon = \upsilon_0 - g\,t$

2- $y - y_0 = \upsilon_0 t - \frac{1}{2} g t^2$

3- $\upsilon^2 = \upsilon_0^2 - 2\,g(y - y_0)$

4- $y - y_0 = \frac{1}{2}\,(\upsilon + \upsilon_0)\,t$




## 2D

### Projectile motion

$\upsilon_0 = \upsilon_{0x} i + \upsilon_{0y} j$ $\quad a = a_x i + a_y j$

**Horizontal motion**

$a_x = 0,\ \upsilon_{0x} = \upsilon_0 cos\theta_0$

1- $\upsilon_x = \upsilon_{0x}$

2- $x = \upsilon_{0x} t$

**Horizontal range (R)=**

$$R = \frac{\upsilon_0^2}{g} \sin 2\theta_0$$

**Maximum range**

$\theta_0 = 45^0 \rightarrow R_{max} = \frac{\upsilon_0^2}{g}$

**Vertical motion**

$a_y = -g,\ \upsilon_{0y} = \upsilon_0 \sin\theta_0$

1- $\upsilon_y = \upsilon_{0y} - g\,t$

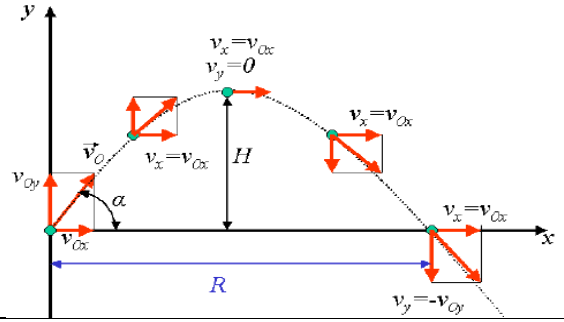
2- $y = \upsilon_{0y} t - \frac{1}{2} g t^2$

3- $\upsilon_y^2 = \upsilon_{0y}^2 - 2\,gy$

**Maximum height (H)**

$$H = \frac{(\upsilon_0 \sin\theta_0)^2}{2g}$$

, where $\upsilon_y = 0$



# Problems:

1- A particle goes from $x = -2\,\text{m}$, $y = 3\,\text{m}$, $z = 1\,\text{m}$ to $x = 3\,\text{m}$, $y = -1\,\text{m}$, $z = 4\,\text{m}$. Its displacement is:

A. $(1\,\text{m})\,\hat{\text{i}} + (2\,\text{m})\,\hat{\text{j}} + (5\,\text{m})\,\hat{\text{k}}$
B. $(5\,\text{m})\,\hat{\text{i}} - (4\,\text{m})\,\hat{\text{j}} + (3\,\text{m})\,\hat{\text{k}}$
C. $-(5\,\text{m})\,\hat{\text{i}} + (4\,\text{m})\,\hat{\text{j}} - (3\,\text{m})\,\hat{\text{k}}$
D. $-(1\,\text{m})\,\hat{\text{i}} - (2\,\text{m})\,\hat{\text{j}} - (5\,\text{m})\,\hat{\text{k}}$
E. $-(5\,\text{m})\,\hat{\text{i}} - (2\,\text{m})\,\hat{\text{j}} + (3\,\text{m})\,\hat{\text{k}}$

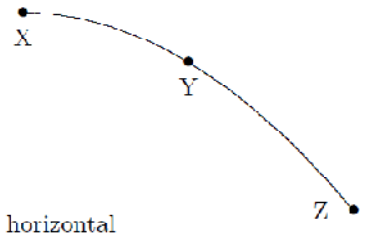
ans: B

2- A stone thrown from the top of a tall building follows a path that is:

A. circular
B. made of two straight line segments
C. hyperbolic
D. parabolic
E. a straight line

ans: D

3- A stone is thrown horizontally and follows the path XYZ shown. The direction of the acceleration of the stone at point Y is:




A. ↓
B. →
C. ↘
D. ↙
E. ↗

ans: A

4- A large cannon is fired from ground level over level ground at an angle of 30° above the horizontal. The muzzle speed is 980 m/s. Neglecting air resistance, the projectile will travel what horizontal distance before striking the ground?

A. 4.3 km
B. 8.5 km
C. 43 km
D. 85 km
E. 170 km

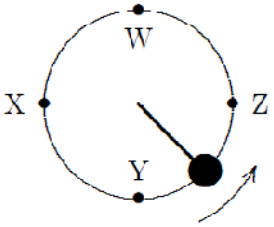
ans: D

5- A projectile is fired from ground level over level ground with an initial velocity that has a vertical component of 20 m/s and a horizontal component of 30 m/s. Using $g = 10\,\text{m/s}^2$, the distance from launching to landing points is:

A. 40 m
B. 60 m
C. 80 m
D. 120 m
E. 180 m

ans: D

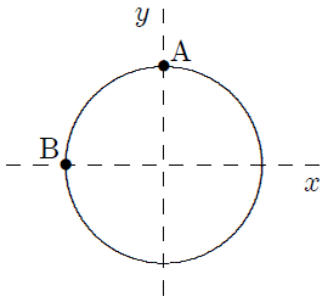
6- An object, tied to a string, moves in a circle at constant speed on a horizontal surface as shown. The direction of the displacement of this object, as it travels from W to X is:




A. ←
B. ↓
C. ↑
D. ↗
E. ↙

ans: E

7- A toy racing car moves with constant speed around the circle shown below. When it is at point A its coordinates are $x = 0$, $y = 3\,\text{m}$ and its velocity is $(6\,\text{m/s})\,\hat{\text{i}}$. When it is at point B its velocity and acceleration are:




A. $-(6\,\text{m/s})\,\hat{\text{j}}$ and $(12\,\text{m/s}^2)\,\hat{\text{i}}$, respectively
B. $(6\,\text{m/s})\,\hat{\text{i}}$ and $-(12\,\text{m/s}^2)\,\hat{\text{i}}$, respectively
C. $(6\,\text{m/s})\,\hat{\text{j}}$ and $(12\,\text{m/s}^2)\,\hat{\text{i}}$, respectively
D. $(6\,\text{m/s})\,\hat{\text{i}}$ and $(2\,\text{m/s}^2)\,\hat{\text{j}}$, respectively
E. $(6\,\text{m/s})\,\hat{\text{j}}$ and 0, respectively

ans: C

8- An object is moving on a circular path of radius $\pi$ meters at a constant speed of $1.0\,\text{m/s}$. The time required for one revolution is:

A. $2/\pi^2\,\text{s}$
B. $\pi^2/2\,\text{s}$
C. $\pi/2\,\text{s}$
D. $\pi^2/4$
E. $2/\pi\,\text{s}$

ans: B

9- A particle moves at constant speed in a circular path. The instantaneous velocity and instantaneous acceleration vectors are:

A. both tangent to the circular path
B. both perpendicular to the circular path
C. perpendicular to each other
D. opposite to each other
E. none of the above

ans: C

10 A car rounds a 20-m radius curve at $10\,\text{m/s}$. The magnitude of its acceleration is:

A. 0
B. $0.20\,\text{m/s}^2$
C. $5.0\,\text{m/s}^2$
D. $40\,\text{m/s}^2$
E. $400\,\text{m/s}^2$

ans: C