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ANSWERS TO SELECTED PROBLEMS

Chapter 1

- | | | | | | |
|-------|---|-------|-------------------------|-------|-----------------------------|
| 1.1 | 0.0173 yd; 0.104 yd (compared to a total of 5 yd) | 1.11 | 9/37 | 1.14 | \$2000 |
| 1.3 | 5/9 | 1.5 | 7/12 | 1.9 | 5/27 |
| 5.2 | Test further | 5.4 | D | 5.5 | D |
| 5.6 | Test further | 5.8 | Test further | 5.9 | D |
| 6.5 | (b) D | 6.7 | D | 6.9 | C |
| 6.18 | D | 6.20 | C | 6.22 | C |
| 6.26 | C | 6.29 | D | 6.31 | D |
| 6.36 | D | | | 6.10 | C |
| 7.1 | C | 7.2 | D | 6.23 | D |
| 9.2 | D | 9.3 | C | 6.32 | D |
| 9.10 | D | 9.12 | C | | |
| 9.20 | C | 9.21 | C | 7.4 | C |
| | | | | 7.6 | D |
| | | | | 7.8 | C |
| | | | | 9.7 | D |
| | | | | 9.8 | C |
| | | | | 9.13 | C |
| | | | | 9.15 | D |
| | | | | 9.9 | D |
| | | | | 9.16 | C |
| 10.1 | $ x < 1$ | 10.3 | $ x \leq 1$ | 10.4 | $ x \leq \sqrt{2}$ |
| 10.5 | All x | 10.9 | $ x < 1$ | 10.10 | $ x \leq 1$ |
| 10.11 | $-5 \leq x < 5$ | 10.13 | $-1 < x \leq 1$ | 10.15 | $-1 < x < 5$ |
| 10.17 | $-2 < x \leq 0$ | 10.18 | $-3/4 \leq x \leq -1/4$ | 10.20 | All x |
| 10.21 | $0 \leq x \leq 1$ | 10.22 | No x | 10.24 | $ x < \frac{1}{2}\sqrt{5}$ |
| 10.25 | $n\pi - \pi/6 < x < n\pi + \pi/6$ | | | | |
| 13.1 | $-(x^3 + x^4/2 + x^5/3 + x^6/4 \cdots)$ | | | | |

- 13.2 $x + x^2 + x^3/3 - x^5/30 \dots$
 13.3 $x^2 + 2x^4/3 + 17x^6/45 \dots$
 13.6 $1 + 2x + 5x^2/2 + 8x^3/3 + 65x^4/24 \dots$
 13.8 $1 + x^2/2! + 5x^4/4! + 61x^6/6! \dots$
 13.9 $1 + x^2/2 + 3x^4/8 + 5x^6/16 \dots$
 13.11 $1 - x + x^2/3 - x^4/45 \dots$
 13.12 $1 + x^2/4 + 7x^4/96 \dots$
 13.14 $1 - x/3! + x^2/5! \dots$
 13.15 $1 + x + x^2/2 - x^4/8 - x^5/15 \dots$
 13.16 $x - x^2/2 + x^3/6 - x^5/12 \dots$
 13.18 $x - x^3/3 + x^5/(5 \cdot 2!) - x^7/(7 \cdot 3!) \dots$
 13.19 $x + x^3/3 + x^5/5 + x^7/7 \dots$
 13.20 $x + x^3/6 + 3x^5/40 + 5x^7/112 \dots$
 13.23 $1 + x/2 - 3x^2/8 + 17x^3/48 \dots$
 13.27 $1 + x^3/6 + x^4/6 + 19(x^5 + x^6)/5! \dots$
 13.28 $x - x^2 + x^3 - 13x^4/12 + 5x^5/4 \dots$
 13.29 $1 + x^2/3! + 7x^4/(3 \cdot 5!) + 31x^6/(3 \cdot 7!) \dots$
 13.32 $x - x^3/6 + 3x^5/40 - 5x^7/112 \dots$
 13.33 $-(x^2/2 + x^4/12 + x^6/45 \dots)$
 13.36 $1 - (x - 1) + (x - 1)^2 - (x - 1)^3 \dots$
 13.37 $e^3[1 + (x - 3) + (x - 3)^2/2! + (x - 3)^3/3! \dots]$
 13.39 $-[(x - \pi/2) + (x - \pi/2)^3/3 + 2(x - \pi/2)^5/15 \dots]$
 13.40 $5 + (x - 25)/10 - (x - 25)^2/10^3 + (x - 25)^3/(5 \times 10^4) \dots$

14.8 For $x < 0$, error < 0.001 ; for $x > 0$, error < 0.002 .

- 15.1 -8.64×10^{-14} 15.3 2.08×10^{-13} 15.5 -6.9
 15.7 $10!$ 15.8 0.09967 15.10 1.318 15.12 $\frac{1}{2}$
 15.14 -1 15.16 $1/3$ 15.17 -1 15.18 $\frac{1}{2}$
 15.20 $1/3$ 15.23 (b) $-\pi$ 15.23 (d) -1
 15.24 (a) 0 15.24 (c) 2 15.24 (e) 0
 15.27 (b) $1 - v/c \cong 5 \times 10^{-7}$ or $v \cong 0.9999995c$
 15.27 (d) $1 - v/c \cong 1.25 \times 10^{-11}$
 15.29 (b) $F/W = x/l + x^3/(2l^3) + 3x^5/(8l^5) \dots$
 15.30 (b) $T = \frac{1}{2}(F/\theta)(1 + \theta^2/6 + 7\theta^4/360 \dots)$
 15.31 (a) C 15.31 (b) D

- 16.6 C 16.7 D 16.9 $-1 \leq x < 1$
 16.10 $-4 < x < 4$ 16.13 $-5 < x \leq 1$
 16.15 $-x^2/6 - x^4/180 - x^6/2835 \dots$
 16.16 $1 - x/2 + 3x^2/8 - 11x^3/48 + 19x^4/128 \dots$
 16.19 $-(x - \pi) + (x - \pi)^3/3! - (x - \pi)^5/5! \dots$
 16.20 $2 + (x - 8)/12 - (x - 8)^2/(2^5 \cdot 3^2) + 5(x - 8)^3/(2^8 \cdot 3^4) \dots$
 16.26 $-1/3$ 16.28 1 16.31 1.728×10^{-13}
 16.32 5.56×10^{-11} 16.33 $6!$ 16.35 $\sinh 1 = 1.175$

Chapter 2

	x	y	r	θ	
4.1	1	1	$\sqrt{2}$	$\pi/4$	See Fig. 5.1
4.2	-1	1	$\sqrt{2}$	$3\pi/4$	See Fig. 9.6
4.3	1	$-\sqrt{3}$	2	$-\pi/3$	
4.5	0	2	2	$\pi/2$	See Fig. 5.2
4.7	-1	0	1	π	See Fig. 9.2
4.9	-2	2	$2\sqrt{2}$	$3\pi/4$	
4.11	$\sqrt{3}$	1	2	$\pi/6$	See Fig. 9.1
4.14	$\sqrt{2}$	$\sqrt{2}$	2	$\pi/4$	
4.15	-1	0	1	$-\pi$ or π	See Fig. 9.2
4.17	1	-1	$\sqrt{2}$	$-\pi/4$	See Fig. 9.5
4.20	-2.39	-6.58	7	$-110^\circ =$ -1.92 radians	
5.2	-1/2	-1/2	$1/\sqrt{2}$	$-3\pi/4$ or $5\pi/4$	
5.4	0	2	2	$\pi/2$	
5.6	-1	0	1	π	
5.8	1.6	-2.7	3.14	-59.3°	
5.10	-25/17	19/17	$\sqrt{58/17}$	142.8°	
	$= -1.47$	$= 1.12$	$= 1.85$		
5.12	2.65	1.41	3	28°	
5.14	1.27	-2.5	2.8	-1.1 radians $= -63^\circ$	
5.16	1.53	-1.29	2	-40°	
5.17	-7.35	-10.9	13.1	-124°	
5.18	-0.94	-0.36	1	201° or -159°	
5.19	$(2 + 3i)/13; (x - yi)/(x^2 + y^2)$				
5.21	$(1 + i)/6; (x + 1 - iy)/[(x + 1)^2 + y^2]$				
5.23	$(-6 - 3i)/5; (1 - x^2 - y^2 + 2yi)/[(1 - x)^2 + y^2]$				
5.26	1		5.30 $3/2$	5.31 1	
5.32	169		5.34 1	5.35 $x = -4, y = 3$	
5.36	$x = -1/2, y = 3$				5.39 $x = y = \text{any real number}$
5.42	$x = -1/7, y = -10/7$				5.43 $(x, y) = (0, 0), \text{ or } (1, 1), \text{ or } (-1, 1)$
5.45	$x = 0, \text{ any real } y; \text{ or } y = 0, \text{ any real } x$				5.46 $y = -x$
5.48	$x = 36/13, y = 2/13$				5.49 $y = 0, x = 1/2$
5.53	Circle (Find center and radius.)				
5.55	Straight line (What is its equation?)				
5.56	Part of a straight line (Describe it.)				
5.57	Hyperbola (What is its equation?)				
5.60	Circle (Find center and radius.)				
5.62	Ellipse (Find its equation; where are the foci?)				
5.63	Two straight lines (What lines?)				
5.68	$v = 2, a = 4$				
6.2	D		6.3 C	6.4 D	
6.5	D		6.10 C	6.12 C	
7.1	All z		7.3 All z	7.6 $ z < 1/3$	
7.7	All z		7.10 $ z < 1$	7.12 $ z < 4$	

- 7.14 $|z - 2i| < 1$ 7.16 $|z + (i - 3)| < 1/\sqrt{2}$
- 8.3 For answers, see Chapter 1, Section 13A, Example 2, and Chapter 1, Problem 13.2
- 9.3 $-9i$ 9.4 $-e(1 + i\sqrt{3})/2$ 9.6 1
 9.7 $3e^2$ 9.8 $-\sqrt{3} + i$ 9.10 -2
 9.11 $-1 - i$ 9.13 $-4 + 4i$ 9.14 64
 9.17 $-(1 + i)/4$ 9.19 16 9.20 i
 9.21 1 9.24 $4i$ 9.26 $(1 + i\sqrt{3})/2$
 9.29 1 9.32 $3e^2$ 9.34 $4/e$
 9.35 21 9.38 $1/\sqrt{2}$
- 10.3 $\pm 1, \pm i$
 10.4 $\pm 2, \pm 2i$
 10.7 $\pm \sqrt{2}, \pm i\sqrt{2}, \pm 1 \pm i$
 10.9 $1, 0.309 \pm 0.951i, -0.809 \pm 0.588i$
 10.16 $\pm i, (\pm \sqrt{3} \pm i)/2$
 10.17 $-1, 0.809 \pm 0.588i, -0.309 \pm 0.951i$
 10.18 $\pm (1 \pm i)/\sqrt{2}$
 10.21 $\pm (\sqrt{3} + i)$
 10.22 $1 + i, -1.366 + 0.366i, 0.366 - 1.366i$
 10.24 $\pm (\sqrt{3} + i)/2, \pm (1 - i\sqrt{3})/2, \pm (0.259 + 0.966i), \pm (0.966 - 0.259i)$
 10.25 $0.758(1 + i), -0.487 + 0.955i, -1.059 - 0.168i, -0.168 - 1.059i, 0.955 - 0.487i$
- 11.3 $3(1 - i)/\sqrt{2}$ 11.5 $1 + i$ 11.8 $-41/9$ 11.9 $4i/3$
- 12.25 $\sin x \cosh y - i \cos x \sinh y, \sqrt{\sin^2 x + \sinh^2 y}$
 12.26 $\cosh 2 \cos 3 - i \sinh 2 \sin 3 = -3.72 - 0.54i, 3.76$
 12.28 $1.084 + 0.272i, 1.117$
 12.30 $-i$ 12.32 $-4i/3$
 12.33 $i \tanh 1 = 0.762i$ 12.35 $2.03 + 3.05i$
- 14.2 $-i\pi/2$ or $3i\pi/2$ 14.3 $\ln 2 + i\pi/6$
 14.5 $\ln 2 + 5i\pi/4$ 14.6 $-i\pi/4$ or $7i\pi/4$
 14.8 $-1, (1 \pm i\sqrt{3})/2$ 14.10 $e^{-\pi^2/4}$
 14.11 $\cos(\ln 2) + i \sin(\ln 2) = 0.769 + 0.639i$ 14.14 $0.3198i - 0.2657$
 14.15 $e^{-\pi \sinh 1} = 0.0249$ 14.18 -1
 14.20 1 14.23 $e^{\pi/2} = 4.81$
- 15.2 $\pi/2 + n\pi + (i \ln 3)/2$ 15.3 $i(\pm \pi/3 + 2n\pi)$
 15.4 $i(2n\pi + \pi/6), i(2n\pi + 5\pi/6)$ 15.5 $\pm [\pi/2 + 2n\pi - i \ln(3 + \sqrt{8})]$
 15.8 $\pi/2 + 2n\pi \pm i \ln 3$ 15.9 $i(\pi/3 + n\pi)$
 15.12 $i(2n\pi \pm \pi/6)$ 15.14 $2n\pi + i \ln 2, (2n + 1)\pi - i \ln 2$
 15.15 $n\pi + 3\pi/8 + (i/4) \ln 2$
- 16.3 $|z| = \sqrt{2}$; motion around a circle of radius $\sqrt{2}$, at constant speed $v = \sqrt{2}$, constant acceleration $a = \sqrt{2}$.
 16.5 $v = |z_1 - z_2|$; $a = 0$
 16.6 (a) Series: $3 - 2i$; parallel: $5 + i$
 (b) Series: $4/60^\circ$; parallel: $1.73/90^\circ$
 16.8 $[R - i(\omega CR^2 + \omega^3 L^2 C - \omega L)]/[(\omega CR)^2 + (\omega^2 LC - 1)^2]$; this simplifies to $L/(RC)$ at resonance.
 16.9 (b) $\omega = 1/\sqrt{LC}$ 16.12 $(1 + r^4 - 2r^2 \cos \theta)^{-1}$

- 17.2 $(\sqrt{3} + i)/2$ 17.4 $i \cosh 1 = 1.54i$
 17.6 $-e^{-\pi^2} = -5.17 \times 10^{-5}$ 17.7 $e^{\pi/2} = 4.81$
 17.9 $\pi/2 \pm 2n\pi$ 17.11 i
 17.13 $x = 0, y = 4$ 17.15 $|z| < 1/e$
 17.26 1 17.27 (c) $e^{-2(x-i)^2}$
 17.28 $1 + (a^2 + b^2)^2(2ab)^{-2} \sinh^2 b$
 17.30 $e^x \cos x = \sum_{n=0}^{\infty} (2^{n/2}/n!) x^n \cos n\pi/4$
 $e^x \sin x = \sum_{n=0}^{\infty} (2^{n/2}/n!) x^n \sin n\pi/4$

Chapter 3

- 2.3 $(3, -1, 1)$ 2.4 $(4, -2, 1)$
 2.5 $(x, y, z, w) = (2, -1, 1, 3)$ 2.8 $(-2, 1, 1)$
 2.10 $(p, q, r, s, t) = (1, 0, -1, \frac{3}{2}, \frac{1}{2})$ 2.11 $(u, v, w) = (1 + i, 1 - i, i)$
 2.13 $A = (pa^2 + qa + r)/(b - a)^2, B = [(b^2 - 2ab)p - aq - r]/(b - a)^2,$
 $C = (pb^2 + qb + r)/(b - a)$
 2.14 $I_1 = \frac{8}{11}, I_2 = -\frac{23}{11}, I_3 = \frac{31}{11}$
 2.15 $I_1 = \frac{12}{23}, I_2 = \frac{6}{23}, I_3 = \frac{3}{23}, I_4 = \frac{21}{23}$
- 3.1 -11 3.4 2140 3.5 -544 3.12 16
 3.15 $D = 22, (x, y, z) = (\frac{3}{2}, -1, 1)$
 3.17 $I_2 - I_3 = D^{-1}V(R_2R_3 - R_1R_4)$ where
 $D = R_1R_2(R_3 + R_4) + R_3R_4(R_1 + R_2) + R_5(R_1 + R_2)(R_3 + R_4)$
 3.19 $x = \gamma(x' + vt')$, $t = \gamma(t' + vx'/c^2)$
 3.21 $x = 33, y = 49$
 3.23 Bonus = \$30,000, tax = \$80,000
 3.24 $A = 1024, B = 640$
- 4.12 $\arccos(-2^{-1/2}) = 3\pi/4$ 4.14 (a) $\arccos \frac{1}{3} = 70.5^\circ$
 4.14 (c) $\arccos \sqrt{\frac{2}{3}} = 35.3^\circ$ 4.15 (b) $8i - 4j + 8k$
 4.18 $2i - 8j - 3k$ 4.19 $i + j + k$
 4.22 Law of cosines 4.24 A^2B^2
- 5.1 $\mathbf{r} = (2i - 3j) + (4i + 3j)t$ [Note that $2i - 3j$ may be replaced by *any* point on the line; $4i + 3j$ may be replaced by *any* vector along the line. Thus, for example, $\mathbf{r} = 6i - (8i + 6j)t$ is just as good an answer, and similarly for all such problems.]
 5.4 $\mathbf{r} = i + (2i + j)t$
 5.6 $(x - 1)/1 = (y + 1)/(-2) = (z + 5)/2$, or $\mathbf{r} = i - j - 5k + (i - 2j + 2k)t$
 5.8 $x/3 = (z - 4)/(-5), y = -2$; or $\mathbf{r} = -2j + 4k + (3i - 5k)t$
 5.9 $x = -1, z = 7$; or $\mathbf{r} = -i + 7k + jt$
 5.11 $(x - 4)/1 = (z - 3)/(-2), y = -1$; or $\mathbf{r} = 4i - j + 3k + (i - 2k)t$
 5.12 $(x - 5)/5 = (y + 4)/(-2) = (z - 2)/1$; or $\mathbf{r} = 5i - 4j + 2k + (5i - 2j + k)t$
 5.14 $36x - 3y - 22z = 23$
 5.16 $5x - 2y + z = 35$
 5.18 $x + 6y + 7z + 5 = 0$
 5.20 $x - 4y - z + 5 = 0$
 5.21 $\cos \theta = 25/(7\sqrt{30}) = 0.652, \theta = 49.3^\circ$
 5.22 $\cos \theta = 2/\sqrt{6}, \theta = 35.3^\circ$

5.24 $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + (\mathbf{j} + 2\mathbf{k})t$, $d = 2\sqrt{\frac{6}{5}}$

5.25 $\mathbf{r} = \mathbf{i} - 2\mathbf{j} + (4\mathbf{i} + 9\mathbf{j} - \mathbf{k})t$, $d = \frac{3}{2}\sqrt{3}$

5.29 $2/6^{1/2} = 0.816$

5.31 $\frac{5}{7}$

5.34 $\sqrt{\frac{11}{10}}$

5.36 3

5.39 Intersect at (3, 2, 0); $\cos \theta = 5/\sqrt{60}$, $\theta = 49.8^\circ$

5.42 $1/\sqrt{5} = 0.447$

5.43 $20/\sqrt{21} = 4.36$

5.33 $\sqrt{\frac{43}{15}} = 1.69$

5.38 $\arccos \sqrt{\frac{21}{22}} = 12.3^\circ$

5.45 $d = \sqrt{2}$, $t = -1$

6.2 $AB = \begin{pmatrix} -2 & -2 \\ 1 & 2 \end{pmatrix}$, $BA = \begin{pmatrix} -6 & 17 \\ -2 & 6 \end{pmatrix}$,

$$A + B = \begin{pmatrix} 1 & -1 \\ -1 & 5 \end{pmatrix}, \quad 5A - 3B = \begin{pmatrix} 13 & -37 \\ -5 & 9 \end{pmatrix},$$

$$\det AB = -2, \det(A + B) = 4, \det(5A) = 25, \det(3B) = -18.$$

6.3 $AB = \begin{pmatrix} 7 & -1 & 0 \\ 3 & 1 & -1 \\ 3 & 9 & 5 \end{pmatrix}$, $BA = \begin{pmatrix} 4 & -1 & 2 \\ 6 & 3 & 1 \\ 0 & 1 & 6 \end{pmatrix}$,

$$A + B = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 1 & 1 \\ 3 & 4 & 1 \end{pmatrix}, \quad 5A - 3B = \begin{pmatrix} 2 & -3 & 10 \\ 15 & -11 & -3 \\ -9 & 28 & 5 \end{pmatrix},$$

$$\det AB = 116, \det(A + B) = 12, \det(5A) = 3625, \det(3B) = 108.$$

6.4 You should have found BA , C^2 , CB , C^3 , C^2B , and CBA ; all others are meaningless.

$$C^2B = \begin{pmatrix} 32 & 12 \\ 53 & 7 \\ -13 & -9 \end{pmatrix}, \quad CBA = \begin{pmatrix} 36 & 46 & 14 & -36 \\ 40 & 22 & 1 & 91 \\ -8 & -2 & 1 & -29 \end{pmatrix}.$$

6.10 $x'' = -x$, $y'' = -y$, that is, a rotation of $180^\circ = 60^\circ + 120^\circ$.6.11 $x'' = y$, $y'' = -x$, $z'' = z$, that is, a 90° rotation about the z axis.

6.13 (a) 6 (b) $5x^2 + 3y^2 = 30$

6.14 $\frac{1}{f} = (n-1) \left[\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)d}{nR_1R_2} \right]$

6.18 $\begin{pmatrix} \frac{5}{3} & -3 \\ -1 & 2 \end{pmatrix}$

6.20 $-\frac{1}{2} \begin{pmatrix} 4 & 5 & -8 \\ -2 & -2 & -2 \\ 2 & 3 & 4 \end{pmatrix}$

6.21 $\frac{1}{8} \begin{pmatrix} -2 & 1 & 1 \\ 6 & -3 & 5 \\ 4 & 2 & 2 \end{pmatrix}$

6.24 $(x, y) = (5, 0)$

6.27 $(x, y, z) = (1, -1, 2)$

6.29 $A^{-1} = A^T$

7.1 Not linear

7.4 Linear

7.6 Not linear

7.8 Not linear

7.11 Not linear

7.12 Linear

7.14 Not linear

7.15 Linear

8.2 $y = 1$, $x = (z + 1)/2$, or $\mathbf{r} = (\frac{1}{2}, 1, 0) + (\frac{1}{2}, 0, 1)z$

8.4 $z = 7$, $x = y - 11$, or $\mathbf{r} = (-11, 0, 7) + (1, 1, 0)y$

8.5 Inconsistent

8.8 $x = y = z = w = 0$

8.9 $x = -2$, $y = 2z + \frac{5}{2}$ or $\mathbf{r} = (-2, \frac{5}{2}, 0) + (0, 2, 1)z$

8.11 $x = -z$, $y = z$, or $\mathbf{r} = (-1, 1, 1)z$

8.12 $x = 3 - z$, $y = 1 + z$, or $\mathbf{r} = (3, 1, 0) + (-1, 1, 1)z$

8.13 $(x, y, z) = (-5, 2, -3)$

8.18 $\mathbf{r} = (-3, 1, 2)y$

8.19 $u = -1 - 2v$, $z = 2 + v$, $x = 5 + 3y - v$

8.21 $R = 2$

8.23 $R = 2$

8.25 $R = 3$

8.26 $(1, 4, -5) = \frac{1}{5}(9, 0, 7) - \frac{2}{5}(0, -9, 13)$

$$(5, 2, 1) = \frac{5}{3}(9, 0, 7) - \frac{2}{3}(0, -9, 13), \text{ etc.}$$

8.28 Basis: $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$, that is, $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$; the given vectors are already written using this basis!8.30 For $\lambda = 2$: $x = 0$, $y = -3z$; for $\lambda = -3$: $x = -5y$, $z = 3y$; for $\lambda = 4$: $z = 3y$, $x = 2y$.

8.40 $\mathbf{V} = 3\mathbf{A} - \mathbf{B}$

8.42 For Problem 8.11: Row space basis $(1, 0, 1)$, $(0, 1, -1)$; solution space basis, $(-1, 1, 1)$.

9.1 $AB = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$, $BA = \begin{pmatrix} 22 & 44 \\ -11 & -22 \end{pmatrix}$, $A \neq 0$, but $\det A = 0$.

9.4 $A^\dagger = \begin{pmatrix} 0 & i & 3 \\ -2i & 2 & 0 \\ -1 & 0 & 0 \end{pmatrix}$, $\text{adj } A = C^T = \begin{pmatrix} 0 & 0 & 2 \\ 0 & 3 & i \\ -6 & 6i & -2 \end{pmatrix}$, $A^{-1} = \frac{1}{6}C^T$

10.2 inconsistent

10.4 $x = 2z + 4$, $y = 3$, or $\mathbf{r} = (4, 3, 0) + (2, 0, 1)z$

10.6 $(x, y, z) = (-2, 0, 3)$

10.8 $(x, y, z, w) = (-3, 1, -2, 2)$

10.13 (a) $\mathbf{r} = (4, -1, 2) + (1, -2, -2)t$; or $(x - 4)/1 = (y + 1)/(-2) = (z - 2)/(-2)$

(b) $x - 5y + 3z = 0$

(c) $\frac{5}{7}$

(d) $\frac{5}{3}\sqrt{2} = 2.36$

(e) $\arcsin \frac{9}{11} = 64.8^\circ$

10.15 (a) $y = 7$, $(x - 2)/3 = (z + 1)/4$; or $\mathbf{r} = (2, 7, -1) + (3, 0, 4)t$

(b) $x - 4y - 9z = 0$

(c) $\arcsin(\frac{33}{70}\sqrt{2}) = 41.8^\circ$

(d) $12/\sqrt{98} = 1.21$

(e) $\frac{1}{5}\sqrt{29} = 1.08$

10.17 You should have found all except $A^T B^T$, BA^T , ABC , $AB^T C$, $B^{-1}C$, and CB^T , which are meaningless.

$$B^T AC = \begin{pmatrix} 2 & 2 \\ 1 - 3i & 1 \\ -1 - 5i & -1 \end{pmatrix}, \quad C^{-1}A = \begin{pmatrix} 0 & -i \\ 1 & -1 \end{pmatrix}$$

10.19 $x'' = -y$, $y'' = x$, 90° clockwise rotation

10.23 Area = $\frac{1}{2}|\vec{PQ} \times \vec{PR}| = 7/2$

Chapter 4

1.1 $\partial u / \partial x = 2xy^2/(x^2 + y^2)^2$, $\partial u / \partial y = -2x^2y/(x^2 + y^2)^2$

1.3 $\partial z / \partial u = u/(u^2 + v^2 + w^2)$

1.4 At $(0, 0)$, both = 0; at $(-\frac{2}{3}, \frac{2}{3})$, both = -4

1.7 $2x$

1.9 $2x(1 + 2 \tan^2 \theta)$

1.11 $2y$

1.13 $4r^2 \tan \theta$

1.15 $r^2 \sin 2\theta$

1.17 $4r$

1.19 0

1.21 $-4x \csc^2 \theta$

1.23 $2r \sin 2\theta$

- 1.8' $-2r^4/x^3$ 1.10' $2y + 4y^3x^{-2}$ 1.12' $2y \sec^2 \theta$
 1.14' $2y^2 \sec^2 \theta \tan \theta$ 1.16' $2r \tan^2 \theta$ 1.18' $-2ry^4/(r^2 - y^2)^2$
 1.20' $4x(\tan \theta \sec^2 \theta)(\tan^2 \theta + \sec^2 \theta)$
 1.22' $-8r^3/x^3$
 1.24' $-8y^3/x^3$
- 2.1 $y + \frac{1}{6}y^3 - \frac{1}{2}x^2y + \frac{1}{24}x^4y - \frac{1}{12}x^2y^3 + \frac{1}{120}y^5 \dots$
 2.3 $x - \frac{1}{2}x^2 - xy + \frac{1}{3}x^3 + \frac{1}{2}x^2y + xy^2 \dots$
 2.5 $1 + \frac{1}{2}xy - \frac{1}{8}x^2y^2 + \frac{1}{16}x^3y^3 - \frac{5}{128}x^4y^4 \dots$
 2.8 $e^x \cos y = 1 + x + \frac{1}{2}(x^2 - y^2) + \frac{1}{6}(x^3 - 3xy^2) \dots$
- 4.2 2.5×10^{-6} 4.4 12.2
 4.6 9% 4.8 5%
 4.10 4.28 nt 4.11 3.95
 4.15 8×10^{23}
- 5.1 $e^{-y} \sinh t + z \sin t$ 5.3 $2r(q^2 - p^2)$
 5.7 $(1 - 2b - e^{2a}) \cos(a - b)$
 6.2 $y' = 1, y'' = 0$ 6.3 $y' = 4(\ln 2 - 1)/(2 \ln 2 - 1)$
 6.5 $2x + 11y - 24 = 0$ 6.6 $1800/11^3$
 6.10 $x + y = 0$ 6.11 $y'' = 4$
- 7.1 $dx/dy = z - y + \tan(y + z), d^2x/dy^2 = \frac{1}{2} \sec^3(y + z) + \frac{1}{2} \sec(y + z) - 2$
 7.4 $\partial w/\partial u = -2(rv + s)w, \partial w/\partial v = -2(ru + 2s)w$
 7.7 $(\partial y/\partial \theta)_r = x, (\partial y/\partial \theta)_x = r^2/x, (\partial \theta/\partial y)_x = x/r^2$
 7.8 $\partial x/\partial s = -\frac{19}{13}, \partial x/\partial t = -\frac{21}{13}, \partial y/\partial s = \frac{24}{13}, \partial y/\partial t = \frac{6}{13}$
 7.10 $\partial x/\partial s = \frac{1}{6}, \partial x/\partial t = \frac{13}{6}, \partial y/\partial s = \frac{7}{6}, \partial y/\partial t = -\frac{11}{6}$
 7.13 $(\partial p/\partial q)_m = -p/q, (\partial p/\partial q)_a = 1/(a \cos p - 1),$
 $(\partial p/\partial q)_b = 1 - b \sin q, (\partial b/\partial a)_p = (\sin p)(b \sin q - 1)/\cos q,$
 $(\partial a/\partial q)_m = [q + p(a \cos p - 1)]/(q \sin p)$
 7.15 $(\partial x/\partial u)_v = (2yv^2 - x^2)/(2yv + 2xu), (\partial x/\partial u)_y = (x^2u + y^2v)/(y^2 - 2xu^2)$
 7.17 $(\partial p/\partial s)_t = -\frac{9}{2}, (\partial p/\partial s)_q = \frac{3}{2}$
 7.19 $(\partial x/\partial z)_s = \frac{7}{2}, (\partial x/\partial z)_r = 4, (\partial x/\partial z)_y = 3$
- 8.3 $(-1, 2)$ is a minimum point 8.4 $(-1, -2)$ is a saddle point
 8.8 $\theta = \pi/3$; bend up 8 cm on each side
 8.9 $l = w = 2h$ 8.11 $\theta = 30^\circ, x = y\sqrt{3} = z/2$
 8.13 $(\frac{4}{3}, \frac{5}{3})$ 8.16 $m = \frac{5}{2}, b = \frac{1}{3}$
 9.2 $r : l : s = \sqrt{5} : (1 + \sqrt{5}) : 3$ 9.4 $4/\sqrt{3}$ by $6/\sqrt{3}$ by $10/\sqrt{3}$
 9.6 $V = \frac{1}{3}$ 9.8 $(8/13, 12/13)$
 9.12 Let legs of right triangle be a and b , height of prism $= h$; then $a = b, h = (2 - \sqrt{2})a$.
- 10.2 4, 2 10.4 $d = 1$
 10.6 $d = 2$ 10.7 $\frac{1}{2}\sqrt{11}$
 10.10 (a) $\max T = \frac{1}{2}, \min T = -\frac{1}{2}$ 10.12 Largest sum $= 180^\circ$
 (b) $\max T = 1, \min T = -\frac{1}{2}$ Smallest sum $= 3 \arccos(3^{-1/2})$
 (c) $\max T = 1, \min T = -\frac{1}{2}$ $= 164.2^\circ$
 10.13 Largest sum $= 3 \arccos(3^{-1/2}) = 105.8^\circ$
 Smallest sum $= 90^\circ$
- 11.1 $z = f(y + 2x) + g(y + 3x)$ 11.6 $d^2y/dz^2 + dy/dz - 5y = 0$
 11.11 $H = p\dot{q} - L$

- 12.1 $\frac{1}{2}x^{-1/2} \sin x$
 12.3 $dz/dx = -\sin(\cos x) \tan x - \sin(\sin x) \cot x$
 12.4 $\frac{1}{2} \sin 2$
 12.7 $(\partial u/\partial x)_y = -e^4, (\partial u/\partial y)_x = e^4/\ln 2, (\partial y/\partial x)_u = \ln 2$
 12.10 $dy/dx = (e^x - 1)/x$
 12.12 $(2x + 1)/\ln(x + x^2) - 2/\ln(2x)$
 12.14 $\pi/(4y^3)$
- 13.2 (a) and (b) $d = 4/\sqrt{13}$
 13.4 $-\csc \theta \cot \theta$
 13.5 $-6x, 2x^2 \tan \theta \sec^2 \theta, 4x \tan \theta \sec^2 \theta$
 13.9 $dz/dt = 1 + (t/z)(2 - x - y), z \neq 0$
 13.10 $[x \ln x - (y^2/x)]x^y$ where $x = r \cos \theta, y = r \sin \theta$
 13.13 -1
 13.14 $(\partial w/\partial x)_y = (\partial f/\partial s)_{s,t} + 2(\partial f/\partial s)_{x,t} + 2(\partial f/\partial t)_{x,s} = f_1 + 2f_2 + 2f_3$
 13.18 $\sqrt{\frac{26}{3}}$ 13.21 $T(2) = 4, T(5) = -5$
 13.23 $t \cot t$ 13.25 $-e^x/x$
 13.29 $dt = 3.9$

Chapter 5

- 2.1 3 2.3 4 2.5 $\frac{1}{4}e^2 - \frac{5}{12}$ 2.7 5/3 2.9 6
 2.11 36 2.13 7/4 2.15 3/2 2.17 $\frac{1}{2} \ln 2$ 2.19 32
 2.21 $\frac{131}{6}$ 2.23 9/8 2.25 3/2 2.27 32/5 2.29 2
 2.31 6 2.33 16/3 2.36 1/6 2.37 7/6 2.39 70
 2.41 5 2.43 9/2 2.45 $46k/15$ 2.47 16/3 2.49 1/3
- 3.2 (b) $MI^2/12$ (c) $MI^2/3$
 3.3 (a) $M = 140$ (b) $\bar{x} = 130/21$ (c) $I_m = 6.92M$ (d) $I = 150M/7$
 3.5 (a) $\frac{1}{3}Ma^2$ (b) $\frac{1}{12}Ma^2$ (c) $\frac{2}{3}Ma^2$
 3.7 (a) $M = 9$ (b) $(\bar{x}, \bar{y}) = (2, \frac{4}{3})$ (c) $I_x = 2M, I_y = \frac{9}{2}M$ (d) $I_m = \frac{13}{18}M$
 3.9 (a) $\frac{1}{6}$ (b) $(\frac{1}{4}, \frac{1}{4}, \frac{1}{4})$ (c) $M = \frac{1}{24}, \bar{z} = \frac{2}{3}$
 3.11 (a) $M = \frac{1}{6}(5\sqrt{5} - 1) = 1.7$
 (b) $\bar{x} = 0, \bar{y} = (313 + 15\sqrt{5})/620 = 0.56$
 3.14 $V = 2\pi^2 a^2 b, A = 4\pi^2 ab$, where a = radius of revolving circle, b = distance to axis from center of this circle.
 3.15 For area, $(\bar{x}, \bar{y}) = (0, \frac{4}{3}\pi/\pi)$; for arc, $(\bar{x}, \bar{y}) = (0, 2r/\pi)$
 3.18 $s = \frac{1}{2}[3\sqrt{2} + \ln(1 + \sqrt{2})]$
 3.20 $13\pi/3$
 3.21 $s\bar{x} = \frac{1}{32}[51\sqrt{2} - \ln(1 + \sqrt{2})], s\bar{y} = \frac{13}{6}, s$ as in Problem 3.18
 3.23 $(149/130, 0, 0)$
 3.25 I/M has the same numerical value as $\bar{x}$ in Problem 3.21
 3.26 $2M/3$ 3.27 $149M/130$ 3.29 2 3.30 $32/5$
- 4.1 (b) $\bar{x} = \bar{y} = \frac{4}{3}a/\pi$
 (c) $I = Ma^2/4$
 (e) $\bar{x} = \bar{y} = 2a/\pi$
 4.2 (c) $\bar{y} = \frac{4}{3}a/\pi$
 (d) $I_x = Ma^2/4, I_y = 5Ma^2/4, I_z = 3Ma^2/2$
 (e) $\bar{y} = 2a/\pi$
 (f) $\bar{x} = 6a/5, I_x = 48Ma^2/175, I_y = 288Ma^2/175, I_z = 48Ma^2/25$
 (g) $A = (\frac{2}{3}\pi - \frac{1}{2}\sqrt{3})a^2$

- 4.4 (c) $(0, 0, 3a/8)$ (d) $(0, 0, a/2)$ (e) $2Ma^2/3$
 4.5 $7\pi/3$
 4.11 12π
 4.12 (c) $M = (16\rho/9)(3\pi - 4) = 9.64\rho$
 $I = (128\rho/15^2)(15\pi - 26) = 12.02\rho = 1.25M$
 4.14 $\pi(1 - e^{-1})/4$ 4.16 $u^2 + v^2$
 4.22 $12(1 + 36\pi^2)^{1/2}$ 4.24 $\rho G\pi a/2$
 4.27 $2\pi ah$ (where h = distance between parallel planes)

- 5.1 $\frac{9}{5}\pi\sqrt{30}$
 5.5 8π for each nappe
 5.8 $\frac{3}{16}\sqrt{6} + \frac{9}{16}\ln(\sqrt{2} + \sqrt{3})$
 5.12 $M = \frac{1}{6}\sqrt{3}$, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{1}{2}, \frac{1}{4}, \frac{1}{4})$
 5.16 $\bar{x} = 0$, $\bar{y} = 1$, $\bar{z} = (\frac{32}{9}\pi^{-1})\sqrt{2/5} = 0.716$

- 6.2 $45(2 + \sqrt{2})/112$
 6.4 (a) $\frac{1}{2}MR^2$ (b) $\frac{3}{2}MR^2$
 6.7 $(8\pi - 3\sqrt{3})(4\pi - 3\sqrt{3})^{-1}M$
 6.10 (a) $(\bar{x}, \bar{y}) = (\pi/2, \pi/8)$
 6.12 $(abc)^2/6$
 6.15 $I_x = \frac{8}{15}Ma^2$, $I_y = \frac{7}{15}Ma^2$
 6.18 $(0, 0, \frac{2}{3}h)$
 6.19 $I_x = I_y = 20Mh^2/21$, $I_z = 10Mh^2/21$, $I_m = 65Mh^2/252$
 6.21 $\pi G\rho h(2 - \sqrt{2})$
 6.26 $\frac{1}{2}\sinh 1$

- 4.19 $\pi/4$
 4.26 (a) $\frac{7}{3}Ma^2$

- 5.3 $\pi(37^{3/2} - 1)/6$
 5.6 4
 5.9 $\pi\sqrt{2}$
 5.14 $M = \frac{1}{2}\pi - \frac{4}{3}$

- 6.3 $15\pi/8$
 6.6 (a) $(4\pi - 3\sqrt{3})/6$
 6.8 (b) $\frac{27}{20}$
 6.10 (c) $3M/8$
 6.14 $16a^3/3$
 6.16 $\bar{x} = \bar{y} = 2a/5$

- 6.24 $(0, 0, 2c/3)$
 6.27 $e^2 - e - 1$

Chapter 6

- 3.1 $(\mathbf{A} \cdot \mathbf{B})\mathbf{C} = 6\mathbf{C}$, $(\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C} = \mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = -8$,
 $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = -4(\mathbf{i} + 2\mathbf{k})$
 3.3 -5
 3.6 $\mathbf{v} = (2/\sqrt{6})(\mathbf{A} \times \mathbf{B}) = (2/\sqrt{6})(\mathbf{i} - 7\mathbf{j} - 3\mathbf{k})$,
 $\mathbf{r} \times \mathbf{F} = (\mathbf{A} - \mathbf{C}) \times \mathbf{B} = 3\mathbf{i} + 3\mathbf{j} - \mathbf{k}$,
 $\mathbf{n} \cdot (\mathbf{r} \times \mathbf{F}) = [(\mathbf{A} - \mathbf{C}) \times \mathbf{B}] \cdot \mathbf{C}/|\mathbf{C}| = 8/\sqrt{26}$
 3.7 (a) $11\mathbf{i} + 3\mathbf{j} - 13\mathbf{k}$, (b) 3 , (c) 17
 3.9 $-9\mathbf{i} - 23\mathbf{j} + \mathbf{k}$, $1/\sqrt{21}$
 3.15 $\mathbf{u}_1 \cdot \mathbf{u} = -\mathbf{u}_3 \cdot \mathbf{u}$, $n_1 \mathbf{u}_1 \times \mathbf{u} = n_2 \mathbf{u}_2 \times \mathbf{u}$
 3.17 $\mathbf{a} = (\boldsymbol{\omega} \cdot \mathbf{r})\boldsymbol{\omega} - \omega^2\mathbf{r}$; for $\mathbf{r} \perp \boldsymbol{\omega}$, $\mathbf{a} = -\omega^2\mathbf{r}$, $|\mathbf{a}| = v^2/r$.
 3.19 (a) $16\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}$ (b) $8/\sqrt{6}$
 3.20 (b) 12
 4.2 (a) $t = 2$
 (b) $\mathbf{v} = 4\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}$, $|\mathbf{v}| = 2\sqrt{14}$
 (c) $(x - 4)/4 = (y + 4)/(-2) = (z - 8)/6$, $2x - y + 3z = 36$
 4.5 $|d\mathbf{r}/dt| = \sqrt{2}$; $|d^2\mathbf{r}/dt^2| = 1$; path is a helix.
 4.8 $d\mathbf{r}/dt = \mathbf{e}_r(dr/dt) + \mathbf{e}_\theta(r d\theta/dt)$;
 $d^2\mathbf{r}/dt^2 = \mathbf{e}_r[d^2r/dt^2 - r(d\theta/dt)^2] + \mathbf{e}_\theta[r d^2\theta/dt^2 + 2(dr/dt)(d\theta/dt)]$

- 6.2 $-\mathbf{i}$
 6.4 $\pi e/(3\sqrt{5})$
 6.6 $6x + 8y - z = 25$, $(x - 3)/6 = (y - 4)/8 = (z - 25)/(-1)$
 6.7 $5x - 3y + 2z + 3 = 0$, $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + (5\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})t$
 6.9 (a) $2\mathbf{i} - 2\mathbf{j} - \mathbf{k}$ (b) $5/\sqrt{6}$ (c) $\mathbf{r} = (1, 1, 1) + (2, -2, -1)t$
 6.12 (a) $2\sqrt{5}$, $-2\mathbf{i} + \mathbf{j}$ (b) $3\mathbf{i} + 2\mathbf{j}$ (c) $\sqrt{10}$
 6.14 (b) Down, at the rate $11\sqrt{2}$
 6.17 $\mathbf{e}_r$
 6.19 $\mathbf{j}$
 7.1 $\nabla \cdot \mathbf{r} = 3$, $\nabla \times \mathbf{r} = 0$
 7.2 $\nabla \cdot \mathbf{r} = 2$, $\nabla \times \mathbf{r} = 0$
 7.4 $\nabla \cdot \mathbf{V} = 0$, $\nabla \times \mathbf{V} = -(\mathbf{i} + \mathbf{j} + \mathbf{k})$
 7.6 $\nabla \cdot \mathbf{V} = 5xy$, $\nabla \times \mathbf{V} = xz\mathbf{i} - yz\mathbf{j} + k(y^2 - x^2)$
 7.7 $\nabla \cdot \mathbf{V} = 0$, $\nabla \times \mathbf{V} = x\mathbf{i} - y\mathbf{j} - x \cos y \mathbf{k}$
 7.10 0 7.11 $-(x^2 + y^2)/(x^2 - y^2)^{3/2}$
 7.13 $2xy$ 7.14 0
 7.16 $2(x^2 + y^2 + z^2)^{-1}$ 7.19 $2/r$
 8.1 $-\frac{11}{3}$ 8.2 (a) -4π (b) -16 (c) -8
 8.3 (a) $\frac{5}{3}$ (b) 1 (c) $\frac{2}{3}$ 8.4 (a) 3 , (b) $\frac{8}{3}$
 8.7 (b) 0 (d) 2π 8.8 $yz - x$
 8.9 $3xy - x^3yz - z^2$ 8.11 $-y \sin^2 x$
 8.14 $-\arcsin xy$ 8.18 (a) $\pi + \pi^2/2$ (b) $\pi^2/2$
 9.2 40 9.4 $-\frac{3}{2}$ 9.7 πab
 9.8 24π 9.10 -20 9.11 2
 10.2 3 10.4 36π 10.5 $4\pi \cdot 5^5$ 10.7 48π 10.9 16π
 10.12 $\phi = \begin{cases} 0, & r \leq R_1; \\ (k/2\pi\epsilon) \ln(R_1/r), & R_1 \leq r \leq R_2; \\ (k/2\pi\epsilon) \ln(R_1/R_2), & r \geq R_2. \end{cases}$
 11.2 $2ab^2$ 11.3 0 11.4 -12
 11.5 36 11.6 45π 11.7 0
 11.10 -6π 11.12 18π 11.15 $-2\pi\sqrt{2}$
 11.18 $\mathbf{A} = (xz - yz^2 - y^2/2)\mathbf{i} + (x^2/2 - x^2z + yz^2/2 - yz)\mathbf{j} + \nabla u$, any u
 11.20 $\mathbf{A} = \mathbf{i} \sin zx + \mathbf{j} \cos zx + k e^{zy} + \nabla u$, any u
 12.1 $(\sin \theta \cos \theta)\mathbf{C}$
 12.7 (a) $9\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}$ (b) $29/3$
 12.9 24
 12.11 (a) $\text{grad } \phi = -3y\mathbf{i} - 3x\mathbf{j} + 2z\mathbf{k}$
 (b) $-\sqrt{3}$
 (c) $2x + y - 2z + 2 = 0$, $\mathbf{r} = (1, 2, 3) + (2, 1, -2)t$
 12.13 (a) $6\mathbf{i} - \mathbf{j} - 4\mathbf{k}$ (b) $53^{-1/2}(6\mathbf{i} - \mathbf{j} - 4\mathbf{k})$
 (c) same as (a) (d) $53^{1/2}$ (e) $53^{1/2}$
 12.15 $\phi = -y^2 \cosh^2 xz$
 12.17 6
 12.18 Not conservative (a) $1/2$ (b) $4/3$
 12.21 4 12.23 192π 12.25 -18π
 12.27 4 12.29 10 12.31 $29/3$