5.4 Indefinite Integrals and the Net Change Theorem

(page402, 8th edition)

HOMEWORK 5 - 17 (ODD) , 21 - 45 (ODD)

students

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The Fundamental Theorem of Calculus, FTC 1

If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)dt \qquad a \le x \le b$$

is continuous on $[a, b]$, and differentiable on (a, b), and $g'(x) = f(x)$.

The Fundamental Theorem of Calculus, FTC 2

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where F is any antiderivative of f, that is , a function such that $F' = f$.

We saw in Section 5.3 that the second part of the Fundamental Theorem of Calculus provides a very powerful method for evaluating the definite integral of a function, assuming that we can find an antiderivative of the function. In this section we introduce a notation for antiderivatives, review the formulas for antiderivatives, and use them to evaluate indefinite integrals.

Indefinite Integrals

Because of the relation between antiderivatives and integrals given by the Fundamental Theorem, the notation $\int f(x)dx$ is traditionally used for an antiderivative of f and is called an **indefinite integral**. Thus

$$\int f(x)dx = F(x) \quad means\ F'(x) = f(x)$$

- A definite integral $\int_a^b f(x)dx$ is a
- An indefinite integral $\int f(x)dx$ is a

Table of Indefinite Integtrals

1. $$\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$$

2. $$\int cf(x)dx = c\int f(x)dx$$

3. $$\int k dx = \boxed{}$$

4. $$\int x^n dx = \boxed{}$$

4. $$\int e^x dx = \boxed{}$$

5. $$\int b^x dx = \boxed{}$$

6. $$\int \frac{1}{x} dx = \boxed{}$$

Table of Indefinite Integtrals

① $\int \sin x \; dx =$ ☐

② $\int sec^2x \; dx =$ ☐

③ $\int \sec x \tan x \; dx =$ ☐

④ $\int \frac{1}{x^2+1} dx =$ ☐

⑥ $\int \sinh x \; dx =$ ☐

① $\int cosx \; dx =$ ☐

② $\int csc^2x \; dx = -\cot x + C$

③ $\int \csc x \cot x \; dx = - \csc x + C$

④ $\int \frac{1}{\sqrt{1-x^2}} dx = sin^{-1}x + C$

⑥ $\int \cosh x \; dx =$ ☐

- $$\int \sin kx \ dx = -\frac{\cos kx}{k} + C$$

- $$\int \cos kx \ dx = \frac{\sin kx}{k} + C$$

- $$\int sec^2\ kx\ dx = \frac{\tan kx}{kx} + C$$

- $$\int \sinh kx \ dx = \frac{\cosh kx}{k} + C$$

- $$\int \cosh kx \ dx = \frac{\sinh kx}{k} + C$$

Example (1): Evaluate the following

$$\int (10x^4 - 2sec^2x\ )\ dx$$

Example (2):

$$\int \frac{\cos\theta}{sin^2\theta}\ d\theta$$

Example (3):

$$\int_0^3 (x^3 - 6x)dx$$

Example (4):

$$\int_0^3 \left(2x^3 - 6x + \frac{3}{x^2+1}\right) dx$$

Example (5):

$$\int_1^9 \frac{2t^2 + t^2\sqrt{t} - 1}{t^2}\, dt$$

Exercise (37):

$$\int_0^{\frac{\pi}{4}} \frac{1 + cos^2\theta}{cos^2\theta}\, d\theta$$

Recall That:

$$s(t) \xrightarrow[\text{المسافة}]{\text{اشتقاق}} s'(t) = v(t) \xrightarrow[\text{السرعة}]{\text{اشتقاق}} v'(t) = a(t)$$

velocity السرعة

acceleration التسارع

$$a(t) \xrightarrow{\text{antiderivative}} v(t) \xrightarrow{\text{antiderivative}} s(t)$$

اشتقاق المسافة

اشتقاق السرعة

Displacement → **Velocity** → **Acceleration**

$$s(t) = \int v(t)dt$$

antiderivative

Integration = $\int$

antiderivative

Integration = $\int$

$$v(t) = \int a(t)dt$$

Net Change Theorem

Application of the FTC 2

Recall That: FTC2

If f is continuous on $[a, b]$, then $\int_a^b f(x)dx = F(b) - F(a)$ Where $F' = f$

So the equation can be rewritten as: $\int_a^b F'(x)dx = F(b) - F(a)$

$F(b) - F(a)$ is the change in y when x changes from a to b

$F'(x)$ represents the rate of change of y=f(x) w.r.t. x

Net Change Theorem The integral of a rate of change is the net change:

$$\int_a^b F'(x)dx = F(b) - F(a)$$

$F(b) - F(a)$: Net change in quantity

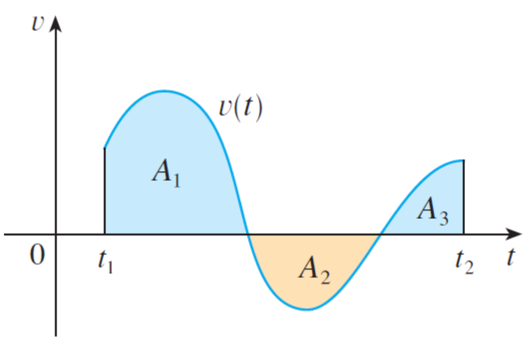
Displacement verses Distance traveled

- If an object moves along a straight line with position function $s(t)$, then its velocity is $v(t) = s'(t)$, so

$$\int_a^b v(t)dt \;= \int_a^b s'(t)dt = s(b) - s(a)$$

is the net change of position, or displacement, of the particle during the time period from a to b.

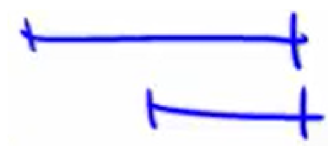
الإزاحة: أقصر مسافة من نقطة بداية الحركة إلى نهايتها

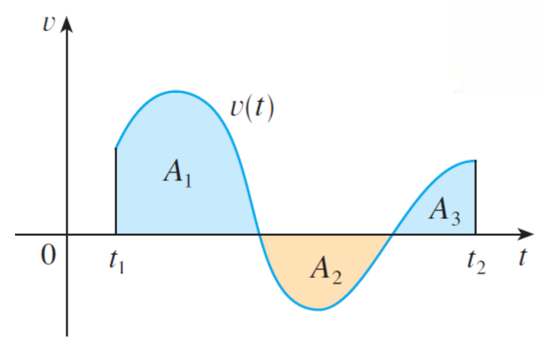


Displacement $= A_1 - A_2 + A_3$

- The distance of the object travels during the time interval, we have to consider the intervals when $v(t) \geq 0$(the particle moves to the right) and intervals when $v(t) \leq 0$(the particle moves to the left). In both cases the distance is computed by integrating $|v(t)|$,

$$\int_a^b |v(t)|dt \;= total\ distance\ traveled$$





distance $= A_1 + A_2 + A_3$

Recall That:

تكامل السرعة = المسافة

position $$s(t) = \int v(t)dt$$

تكامل التسارع = السرعة

$$v(t) = \int a(t)dt$$

- في السابق فصل 4.9 يطلب Velocity function or position function
- هنا مفهومين جديدين . الفرق بينهما وبين السابق إنها تكامل محدود ناتجها يطلع عدد

الإزاحة displacement

$$\int_a^b v(t)dt = s(t)\Big|_a^b = s(b) - s(a)$$

- الناتج عدد
- إذا كان موجب معناها الحركة بإتجاه اليمين
- إذا كان سالب معناها الحركة بإتجاه اليسار

$$\int_a^b |v(t)|dt = T.D.T$$

- الناتج عدد موجب
- نوجد أصفار ما بداخل القيمة المطلقة
- نوجد الفترات الموجبة والسالبة
- نقسم التكامل حسب الفترات واختلاف الإشارات

- The acceleration of the object is $a(t) = v'(t)$, so

$$\int_a^b a(t)dt = \int_a^b v'(t)dt = v(b) - v(a)$$

is the change in velocity from time a to time b .

- $v(t) = \int a(t)dt$
- $s(t) = \int v(t)dt$
- D.T.= $\int |v(t)|dt$

Example (6): A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 6$ (measured in meters per second).

(a) Find the displacement of the particle during the time period $1 \leq t \leq 4$.

(b) Find the distance traveled during this time period.

Exercise (61):

The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find

(a) The velocity at time t.

(b) The distance traveled during the given time interval.

$$a(t) = 2t + 4 \quad v(0) = 5, \qquad 0 \le t \le 10$$

Recall That:

- $v(t) = \int a(t)dt$
- $s(t) = \int v(t)dt$
- D.T.= $\int |v(t)|dt$