(4.9) Antiderivatives

Home work 1 – 47 (odd)

Dr. Rola Asaad Hijazi

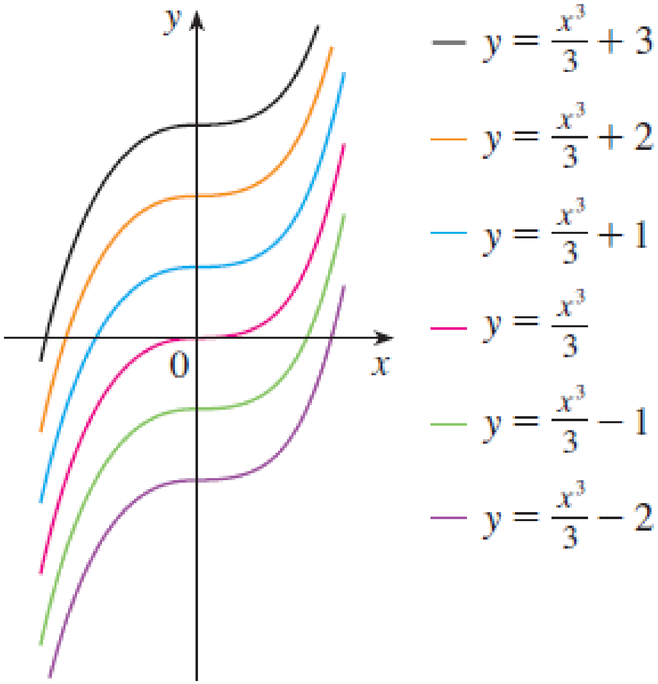
Antidervative

Definition

A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem

If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + C$ where C is an arbitrary constant.



Members of the family of antiderivatives of $f(x) = x^2$

Table of Antidifferentiation Formulas

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\dfrac{1}{1+x^2}$	$\tan^{-1} x$
b^x	$\dfrac{b^x}{\ln b}$	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$

To obtain the most general antiderivative from the particular ones in Table 2, we have to add a constant (or constants), as in Example 1.

Example (1):

Find the most general antiderivative of each of the following functions.

(a) $f(x) = \sin x$

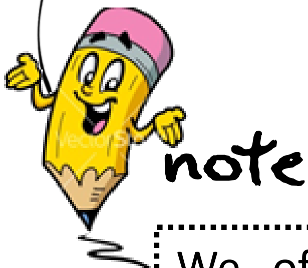
(b) $f(x) = \dfrac{1}{x}$

(c) $f(x) = x^n,\ n \neq 1$

Example (2):

Find all functions g such that

$$g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}$$



We often use a capital letter F to represent an antiderivative of a function f. If we begin with derivative notation, f', an antiderivative is f.

Example (3): Find f if

$$f'(x) = e^x + 20(1 + x^2)^{-1} \text{ and } f(0) = -2$$

Example (4): Find f if

$$f''(x) = 12x^2 + 6x - 4$$

$$f(0) = 4 \quad \text{and } f(1) = 1$$

Rectilienear Motion

Antidifferentiation is particularly useful in analyzing the motion of an object moving in a straight line.

Recall that if the object has position function $s = f(t)$, then the velocity function is $v(t) = s'(t)$. This means that the position function is an antiderivative of the velocity function.

Likewise, the acceleration function $a(t) = v'(t)$, so the velocity function is an antiderivative of the acceleration.

If the acceleration and the initial values $s(0)$ and $v(0)$ are known, then the position function can be found by antidifferentiating twice.

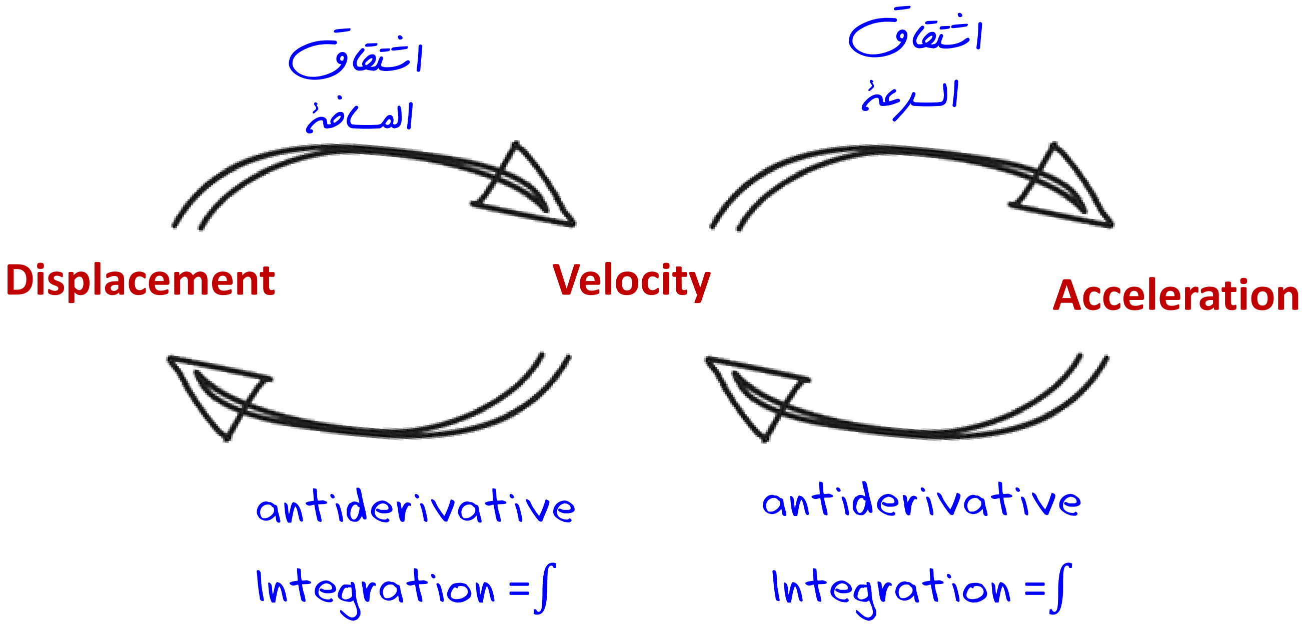
Recall That:

$$s(t) \xrightarrow[\text{المسافة}]{\text{اشتقاق}} \frac{ds}{dt} = v(t) \xrightarrow[\text{السرعة}]{\text{اشتقاق}} \frac{dv}{dt} = a(t)$$

velocity السرعة

acceleration التسارع

$$a(t) \xrightarrow{\text{antiderivative}} v(t) \xrightarrow{\text{antiderivative}} s(t)$$



Example (6):

A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4.$
Its initial velocity is $v(0) = -6\ cm/s$ and its initial displacement is $s(0) = 9cm.$
Find its position function $s(t)$.

Find the most general antiderivative of the function.
(Check your answer by differentiation.)

Exercise (5):

$$f(x) = x(12x + 8)$$

Exercise (9):

$$f(x) = \sqrt{2}$$

Exercise (15):

$$g(t) = \frac{1 + t + t^2}{\sqrt{t}}$$