CALCULUS 110

(4.1) Maximum and Minimum values

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Maximum and Minimum values

Some of the most important applications of differential calculus are *optimization problems*, in which we are required to find the optimal (best) way of doing something. Here are examples of such problems that we will solve in this chapter:

- What is the shape of a can that minimizes manufacturing costs?
- What is the maximum acceleration of a space shuttle?
- What is the radius of a contracted windpipe that expels air most rapidly during a cough?
- At what angle should blood vessels branch so as to minimize the energy expended by the heart in pumping blood?

These problems can be reduced to finding the maximum or minimum values of a function. Let's first explain exactly what we mean by maximum and minimum values.

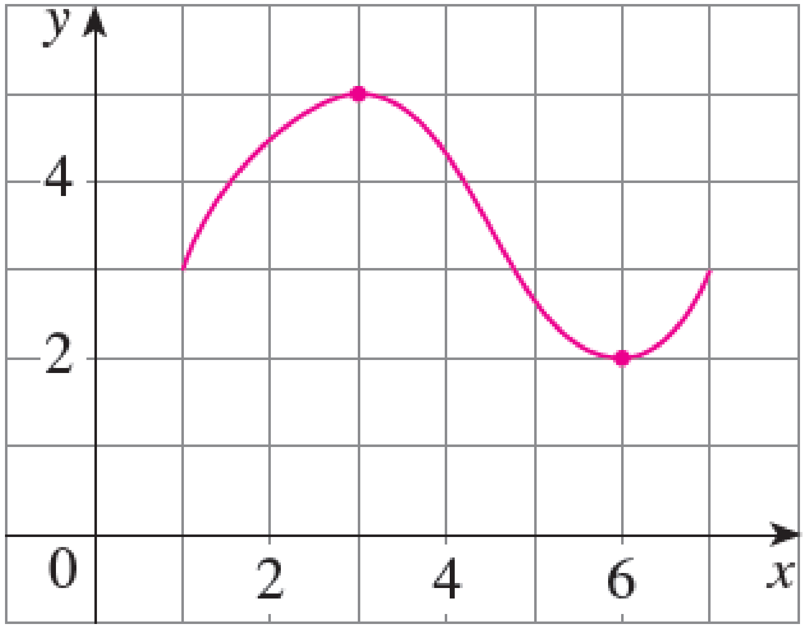
We see that the highest point on the graph of the function f shown in Figure 1 is the point $(3,5)$.

In other words, the largest value of f is $f(3) = 5$.

Likewise, the smallest value is $f(6) = 2$.

We say that $f(3) = 5$ is the absolute maximum of f

and $f(6) = 2$ is the absolute minimum.



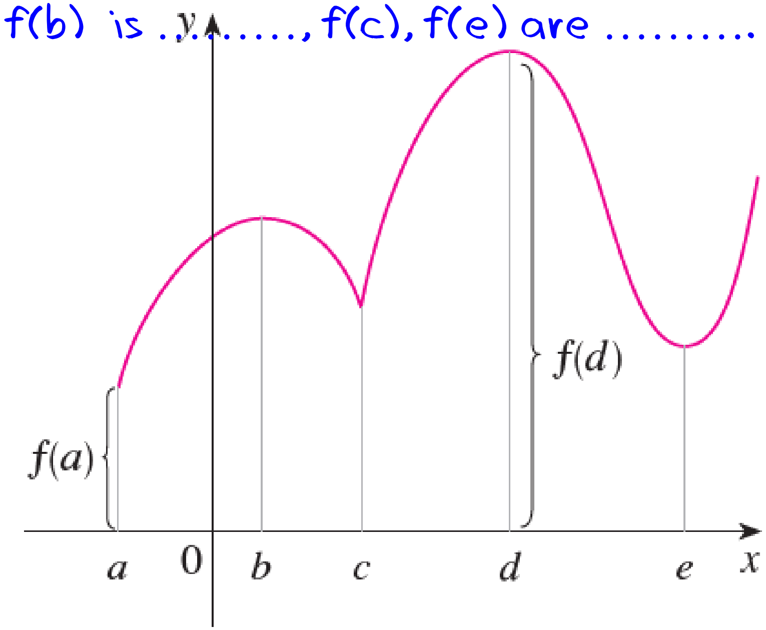
Absolute Maximum and Minimum

Let c be a number in the domain D of a function f. Then $f(c)$ is an:

- Absolute maximum value (Global max.) of f on D if $f(c) \geq f(x) \forall x \in D$.
- Absolute minimum value (Global min.) of f on D if $f(c) \leq f(x) \forall x \in D$.

The function values at these points are called the **extreme values.**

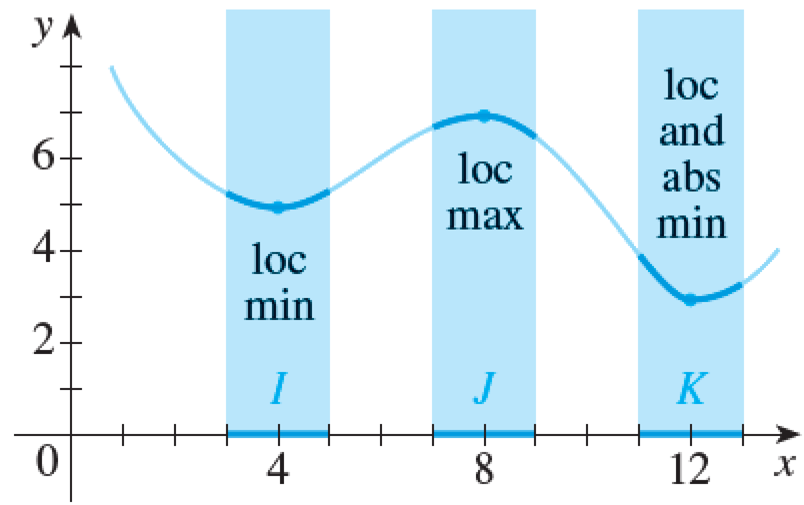
f(d) is .... + ...
f(a)) is an ....
f(b) is ...., f(c), f(e) are ....



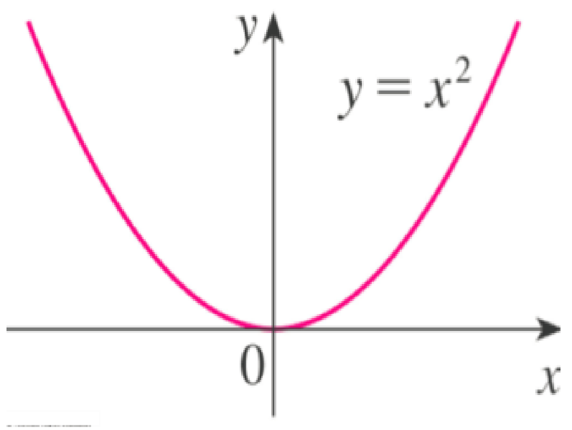
Local maximum and

The number $f(c)$ is a:

- Local maximum value of f on D if $f(c) \geq f(x)$ *when x near c.*
- Local minimum value of f on D if $f(c) \leq f(x)$ *when x near c.*



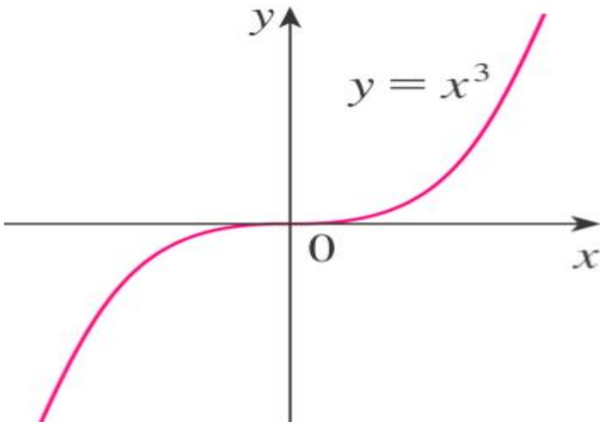
Example 2



$f(0) = 0$ is the ……………….

This function has **……………….**

Example 3

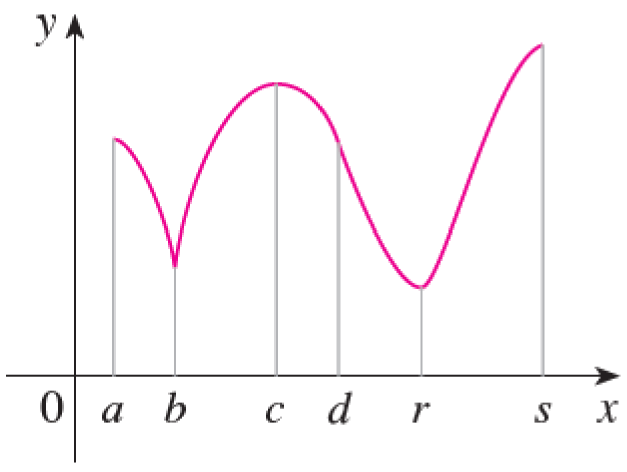


This function has neither an …………..

nor an ……………….

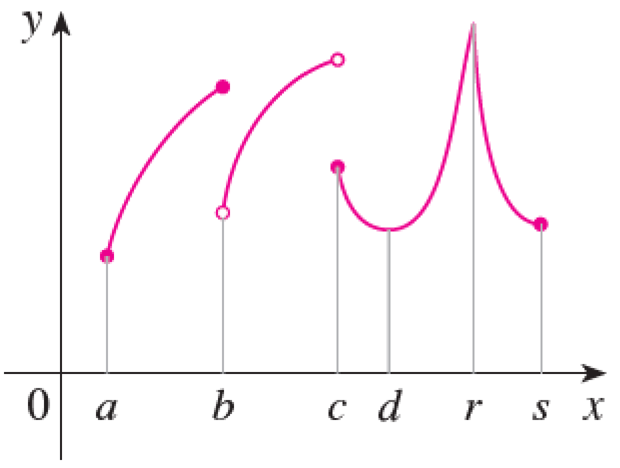
For each of the numbers $a, b, c, d, r, and\ s$, state whether the function whose graph is shown has an absolute maximum or minimum, a local maximum or minimum, or neither a maximum nor a minimum.

Exercise 3



$f(a)$	
$f(b)$	
$f(c)$	
$f(d)$	
$f(r)$	
$f(s)$	

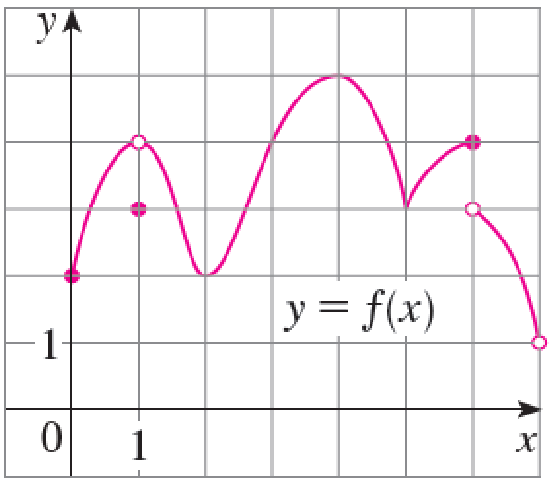
Exercise 4



$f(a)$	
$f(b)$	
$f(c)$	
$f(d)$	
$f(r)$	
$f(s)$	

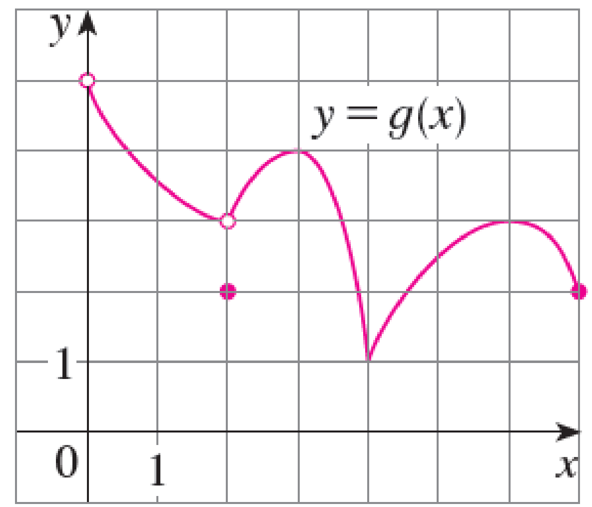
Use the graph to state the absolute and local maximum and minimum values of the function.

Exercise 5



$f(0)$	Not abs. minimum
$f(4)$	Absolute +local max.
$f(1)$	Local minimum
$f(2)$	Local minimum
$f(5)$	Local minimum
$f(6)$	Local maximum
$f(7)$)	Neither

Exercise 6



Note:

The endpoints are not local maxima or minima.

Critical Number

A critical number of a function f is a number c in the domain of f such that either

$f'(c) = 0$ $(horizontal\ tangent)$ or

$f'(c)$ $does\ not\ exist$ $(vertical\ tangent)$

The Closed Interval Method

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

- Find the values of f at the critical numbers of f in (a, b).
- Find the values of f at the endpoints of the interval.

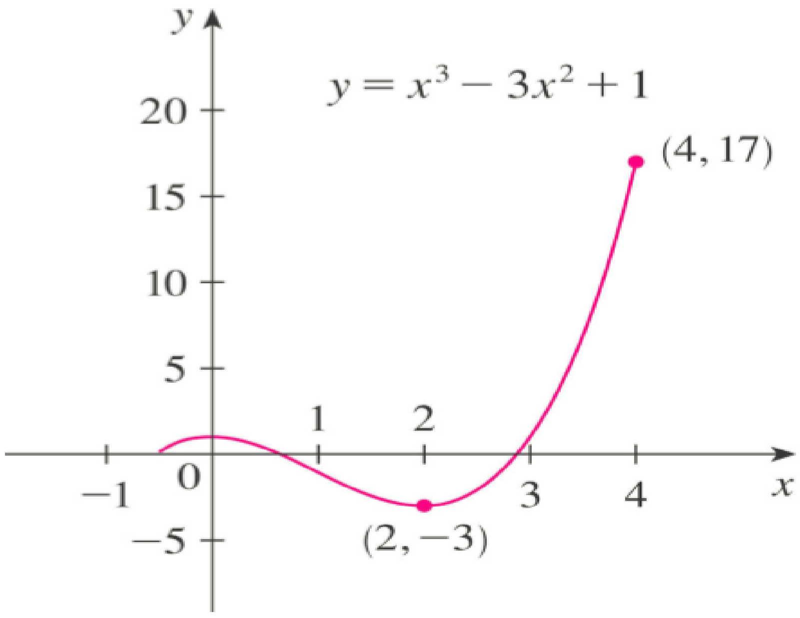
The largest of the values from steps 1 and 2 is the absolute maximum; the smallest of these values is the absolute minimum value.

Example 8

Find the absolute maximum and minimum values of the function

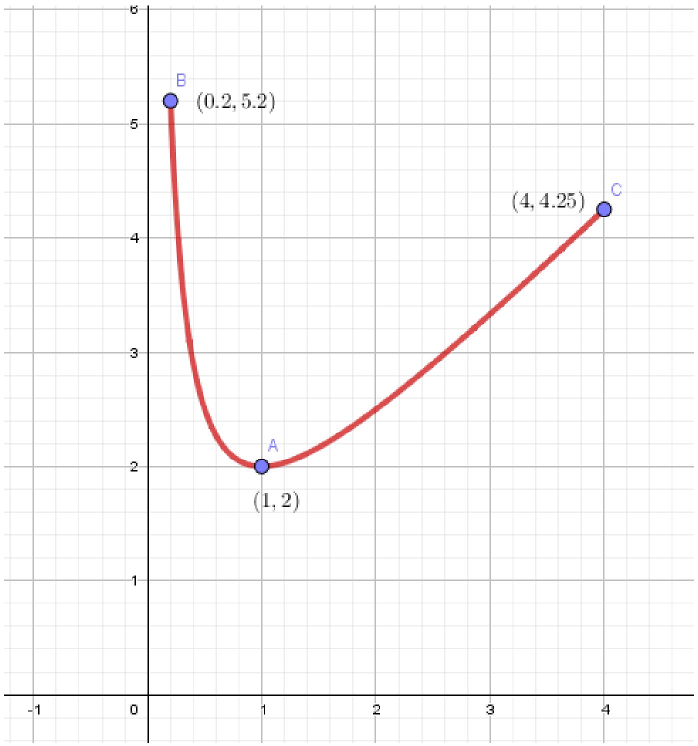
$y = x^3 - 3x^2 + 1 \quad -\frac{1}{2} \leq x \leq 4.$

Solution



Homework 53 Find the absolute maximum and minimum values of

$$f(x) = x + \frac{1}{x} \qquad [0.2, 4]$$



Solution