



CALCULUS IIO

(3.5) Implicit Differentiation

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Implicit Differentiation

Some functions are defined implicitly by a relation between x and y such as:

$$\begin{aligned}x^2 + y^2 &= 25 \\x^3 + y^3 &= 6xy\end{aligned}$$

Example 1

$$x^2 + y^2 = 25, \text{ find } \frac{dy}{dx}$$

Solution

Example 2

(a) $x^3 + y^3 = 6xy$, find y' .

(b) Find an equation of the tangent line to the circle $x^3 + y^3 = 6xy$ at $(3,3)$.

Solution

Example 3

Find y' , if $\sin(x + y) = y^2 \cos x$.

Solution

Example 4

If $x^4 + y^4 = 16$, find y'' .

Solution

Exercise 12

Find y' , if $1 + x = \sin(xy^2)$.

Exercise 25

Find an equation of the tangent line to the curve $y \sin 2x = x \cos 2y$, at $(\pi/2, \pi/4)$

Derivatives of Trigonometric Functions

- ① $y = \sin x$ $y' = \cos x$
- ② $y = \cos x$ $y' = -\sin x$
- ③ $y = \tan x$ $y' = \sec^2 x$
- ④ $y = \csc x$ $y' = -\csc x \cot x$
- ⑤ $y = \sec x$ $y' = \sec x \tan x$
- ⑥ $y = \cot x$ $y' = -\csc^2 x$

Derivatives of Inverse Trigonometric Functions

- ① $y = \sin^{-1}x$ $y' = \frac{1}{\sqrt{1-x^2}}$
- ② $y = \cos^{-1}x$ $y' = -\frac{1}{\sqrt{1-x^2}}$
- ③ $y = \tan^{-1}x$ $y' = \frac{1}{1+x^2}$
- ④ $y = \csc^{-1}x$ $y' = -\frac{1}{x\sqrt{x^2-1}}$
- ⑤ $y = \sec^{-1}x$ $y' = \frac{1}{x\sqrt{x^2-1}}$
- ⑥ $y = \cot^{-1}x$ $y' = -\frac{1}{1+x^2}$

Extra Example Differentiate

- ① $f(x) = \cos^{-1}(\sqrt{x})$.
- ② $f(x) = x^3 \csc^{-1}(x)$
- ③ $f(x) = x \sec^{-1}(x^2)$.
- ④ $f(x) = \cot^{-1}(x) + \cot^{-1}(1/x)$.



homework

5 – 11 (odd), 35, 37, 49, 55

Example 5

Differentiate

$$(a) y = \frac{1}{\sin^{-1}x}$$

$$(b) f(x) = x \tan^{-1}(\sqrt{x})$$

Solution