



CALCULUS IIO

(3.1) Derivatives of Polynomials and Exponential functions

(3.2) The product and quotient rules

Dr. Rola Asaad Hijazi

3.1 Derivatives of Polynomials and Exponential functions

In this section we learn how to differentiate constant functions, power functions, polynomials, and exponential functions and the product, quotient rules.

Rules for Derivative:

$$(1) \quad \frac{d}{dx}(c) = 0 \quad \frac{d}{dx}(x) = 1$$

$$(2) \quad \frac{d}{dx}(x^n) = nx^{n-1}$$

$$(3) \quad \frac{d}{dx}\sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$(4) \quad \frac{d}{dx}(a^x) = a^x \ln a$$

$$(5) \quad \frac{d}{dx}(e^x) = e^x$$

$$(6) \quad \frac{d}{dx}(cf(x)) = c \frac{d}{dx}(f(x))$$

$$(7) \quad \frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

$$(8) \quad (f(x)g(x))' = fg' + gf'$$

$$(9) \quad \left(\frac{f}{g}\right)' = \frac{gf' - g'f}{g^2}$$

note



Examples 1+4

Differentiate the following

(a) $f(x) = x^6$ (b) $f(x) = x^{1000}$

(c) $y = t^4$ (d) $\frac{d}{dr}(r^3)$

(e) $\frac{d}{dx} 3x^4$ (f) $\frac{d}{dx}(-x) =$

solution

Example 2

Differentiate the following

(a) $f(x) = \frac{1}{x^2}$ (b) $y = \sqrt[3]{x^2}$

solution

Example 5

$$\frac{d}{dx}(x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5)$$

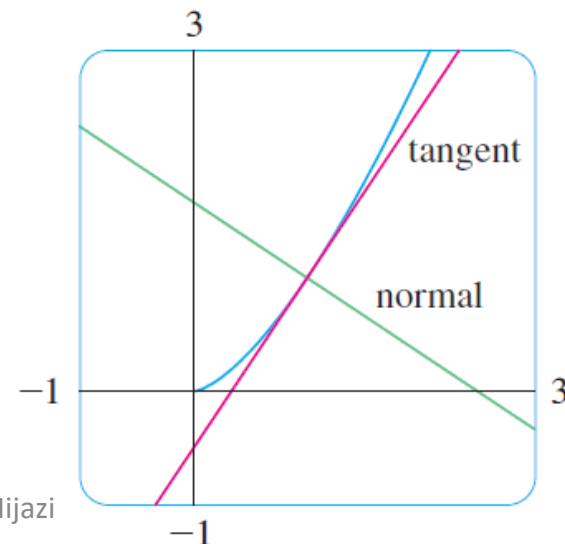
Examples 3

Find equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at $(1,1)$.

solution

Normal Line

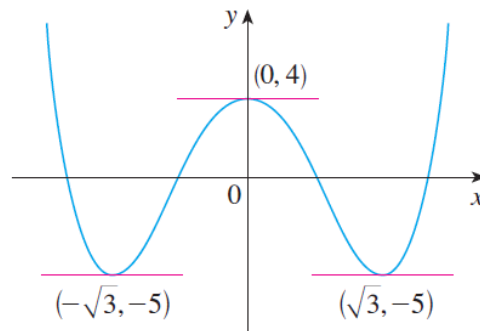
The normal line to a curve c at a point P is the line through P that is perpendicular to the tangent line at P . i.e. $m_1 = \frac{-1}{m_2}$.



Examples 6

Find the points on the curve
 $y = x^4 - 6x^2 + 4$ where the tangent line
is horizontal.

solution



Examples 8

If $f(x) = e^x - x$, find f' , f'' .

Exercise 23

Differentiate

$$y = \frac{x^2 + 4x + 3}{\sqrt{x}}$$

solution

(3.2) The product rule and Quotient



note

$$(fg)' \neq f'g', \left(\frac{f}{g}\right)' \neq \frac{f'}{g'}$$

Example 1

(a) $f(x) = xe^x$, Find $f'(x)$

(b) Find $f^{(n)}(x)$

solution

Example 3

If $f(x) = \sqrt{x}g(x)$, where $g(4) = 2$ and $g'(4) = 3$. Find $f'(4)$.

solution

Example 4

$$y = \frac{x^2 + x - 2}{x^3 + 6}$$

solution

Example 5

Find an equation of the tangent line to the curve

$$y = \frac{e^x}{1 + x^2} \quad , \text{ at } \left(1, \frac{1}{2}e\right)$$

solution



$$(3.1) \quad 3 - 31(\text{odd}) + 37$$

$$(3.2) \quad 3 - 33(\text{odd})$$