Chapter (5): Multiples Integrals

Section (2) P. (206): 1, 2, 4, 5, 7, 19, 33, 37, 39, 41, 42, 44

PROBLEMS, SECTION 2

In Problems 1 to 6, evaluate the integrals.

1. $\int_{x=0}^{1}\int_{y=2}^{4} 3x\,dy\,dx$

2. $\int_{y=-2}^{1}\int_{x=1}^{2} 8xy\,dx\,dy$

4. $\int_{x=0}^{4}\int_{y=0}^{x/2} y\,dy\,dx$

5. $\int_{x=0}^{1}\int_{y=x}^{e^x} y\,dy\,dx$

In Problems 7 to 18 evaluate the double integrals over the areas described. To find the limits, sketch the area and compare Figures 2.5 to 2.7.

7. $\iint_A (2x - 3y)\,dx\,dy$, where A is the triangle with vertices (0, 0), (2, 1), (2, 0).

In Problems 19 to 24, use double integrals to find the indicated volumes.

19. Above the square with vertices at (0, 0), (2, 0), (0, 2), and (2, 2), and under the plane $z = 8 - x + y$.

In Problems 29 to 32, observe that the inside integral cannot be expressed in terms of elementary functions. As in Problems 25 to 28, change the order of integration and so evaluate the double integral.

33. A lamina covering the quarter circle $x^2 + y^2 \le 4$, $x > 0$, $y > 0$, has (area) density $x + y$. Find the mass of the lamina.

In Problems 37 to 40, evaluate the triple integrals.

37. $\int_{x=1}^{2}\int_{y=x}^{2x}\int_{z=0}^{y-x} dz\,dy\,dx$

39. $\int_{y=-2}^{3}\int_{z=1}^{2}\int_{x=y+z}^{2y+z} 6y\,dx\,dz\,dy$

41. Find the volume between the planes $z = 2x + 3y + 6$ and $z = 2x + 7y + 8$, and over the triangle with vertices (0, 0), (3, 0), and (2, 1).

42. Find the volume between the planes $z = 2x + 3y + 6$ and $z = 2x + 7y + 8$ and over the square in the (x, y) plane with vertices (0, 0), (1, 0), (0, 1), (1, 1).

44. Find the mass of the solid in Problem 42 if the density at (x, y, z) is proportional to y.

Supporting Materials (P203-204) of Section (2)'s Questions:

As the geometry would indicate, the results in (2.2) and (2.4) are the same; we have two methods of evaluating the double integral by using iterated integrals.

Often one of these two methods is more convenient than the other; we choose whichever method is easier. To see how to decide, study the following sketches of areas A over which we want to find $\iint_A f(x, y)\, dx\, dy$. In each case we think of combining little rectangles $dx\, dy$ to form strips (as shown) and then combining the strips to cover the whole area.

Areas shown in Figure 2.5: Integrate with respect to y first. Note that the top and bottom of area A are curves whose equations we know; the boundaries at $x = a$ and $x = b$ are either vertical straight lines or else points.

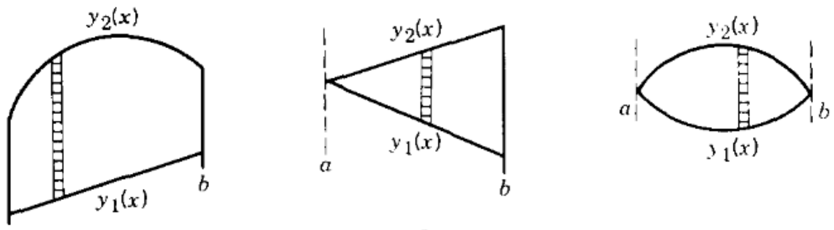

FIGURE 2.5

We find

(2.5) $$\iint_A f(x, y)\, dx\, dy = \int_{x=a}^{b} \left(\int_{y=y_1(x)}^{y_2(x)} f(x, y)\, dy \right) dx.$$

Areas shown in Figure 2.6: Integrate with respect to x first. Note that the sides of area A are curves whose equations we know; the boundaries at $y = c$ and $y = d$ are either horizontal straight lines or else points.

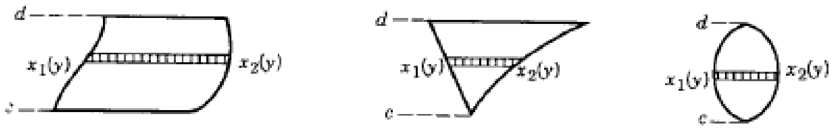

FIGURE 2.6

We find

(2.6) $$\iint_A f(x, y)\, dx\, dy = \int_{y=c}^{d} \left(\int_{x=x_1(y)}^{x_2(y)} f(x, y)\, dx \right) dy.$$

Areas shown in Figure 2.7: Integrate in either order. Note that these areas all satisfy the requirements for both (2.5) and (2.6).

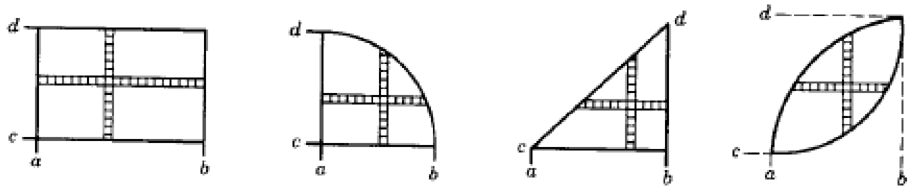

FIGURE 2.7

We find

(2.7) $$\iint_A f(x, y)\, dx\, dy = \int_{x=a}^{b} \int_{y=y_1(x)}^{y_2(x)} f(x, y)\, dy\, dx$$

$$= \int_{y=c}^{d} \int_{x=x_1(y)}^{x_2(y)} f(x, y)\, dx\, dy.$$

Section (3) P. (215): 7(a,b,c), 17

PROBLEMS, SECTION 3

The following notation is used in the problems:

M = mass,

$\bar{x}, \bar{y}, \bar{z}$ = coordinates of center of mass (or centroid if the density is constant),

I = moment of inertia (about axis stated),

I_x, I_y, I_z = moments of inertia about x, y, z axes,

I_m = moment of inertia (about axis stated) through the center of mass.

Note: Answers for I, I_m, I_x, etc., should always be given as multiples of M (for example, $I = \frac{1}{3}Ml^2$).

7. A rectangular lamina has vertices (0, 0), (0, 2), (3, 0), (3, 2) and density xy. Find
   (a) M,
   (b) $\bar{x}, \bar{y}$,
   (c) I_x, I_y,

In Problems 17 to 30, for the curve $y = \sqrt{x}$, between $x = 0$ and $x = 2$, find:

17. The area under the curve.

Section (4) P. (225): 3 (a), 11, 14, 15

PROBLEMS, SECTION 4

3. (a) Find the moment of inertia of a circular disk (uniform density) about an axis through its center and perpendicular to the plane of the disk.

11. Write a triple integral in cylindrical coordinates for the volume inside the cylinder $x^2 + y^2 = 4$ and between $z = 2x^2 + y^2$ and the (x, y) plane. Evaluate the integral.

14. Express the integral

$$I = \int_0^1 dx \int_0^{\sqrt{1-x^2}} e^{-x^2-y^2}\, dy$$

as an integral in polar coordinates (r, θ) and so evaluate it.

15. Find the cylindrical coordinate volume element by Jacobians.

Disclaimer:

All the problems and excerpts above have been borrowed from the book of "Mathematical Methods in the Physical Sciences" 2nd Edition by Mary L. Boas, and it was done only for the purpose of outlining the students' assignments.