CALCULUS 110

(1.2) Mathematical Models

A catalog of Essential Functions

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(1.2) Mathematical Models

Algebraic function

are functions constructed from polynomials using algebraic operations (such as $+, -, \times, \div, \sqrt[n]{\ }$) starting with polynomials.

(1) Polynomial (constant, linear, quadratic , cubic)

(2) Power x^a : $x^n, x^{\frac{1}{n}} = \sqrt[n]{x}$ $(root)$, x^{-n}

(3) Rational : $\frac{x^4-16x^2}{x+1}$

More examples

(4) $\frac{x^4-16x^2}{x+\sqrt{x}} + (x-2)\sqrt[3]{x+1}$

(5) $\sqrt[4]{x^2+1}$

Non-Algebraic function

(1) Trigonometric: $\sin(x)$, $\cos(x)$, ...

(2) Exponential : a^x, e^x, ...

(3) Logarithmic : $log_a(x), \ln(x)$

Polynomial functions

A function $f(x)$ is called **a polynomial** if

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

Leading coefficient (a_n); coefficients; x is a variable; constant (a_0)

n: non-negative integer

$a_n , a_{n-1} , \dots , a_1 , a_0 \in \mathbb{R}$

Domain of $f(x) = \mathbb{R} = (-\infty, \infty)$.

Degree of the polynomial $= n$.

Examples

(1) $f(x) = x^5 + \frac{2}{3}x^2 + 1$ is a polynomial

deg.$(f(x)) =$

(2) $g(x) = \sqrt{x} + 2x^{-1} + x^{2/3}$

is not a polynomial

① Constant function

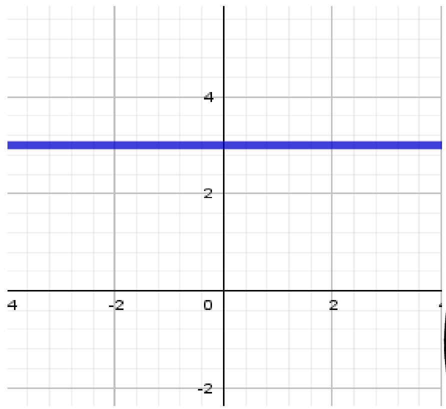
$f(x) = c$

Deg. $f(x) =$

$D_f =$

$Range =$

Algebraic function



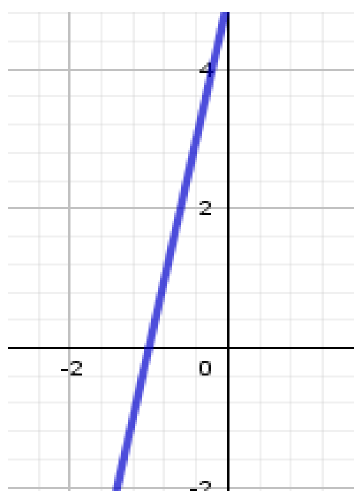
② Linear Function

$f(x) = ax + b$

Deg. $f(x) =$

Domain =

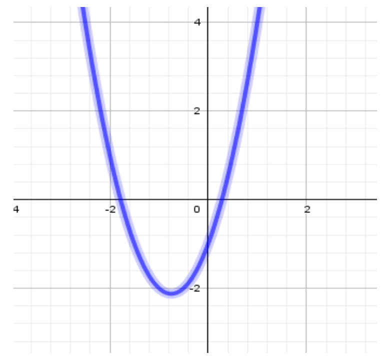
Range=



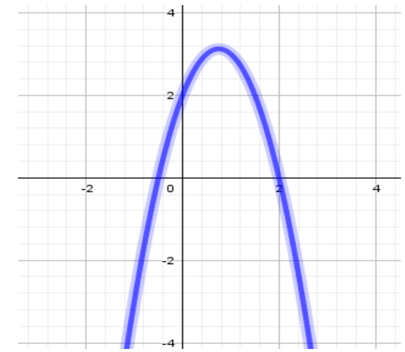
③ Quadratic function

$f(x) = ax^2 + bx + c$

Deg. $f(x) =$



$a > 0$



$a < 0$

④ Cubic Function

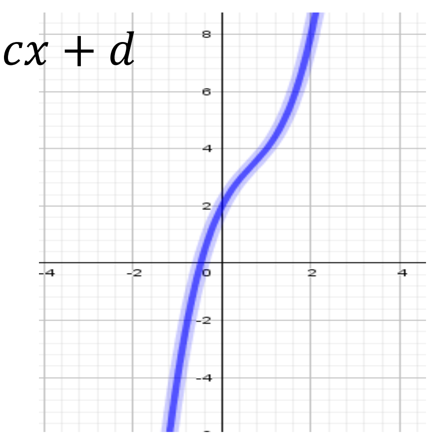
$f(x) = ax^3 + bx^2 + cx + d$

Deg. $f(x) =$

Domain =

Range=

Algebraic function



Power function is a function of the from $f(x) = x^a$,

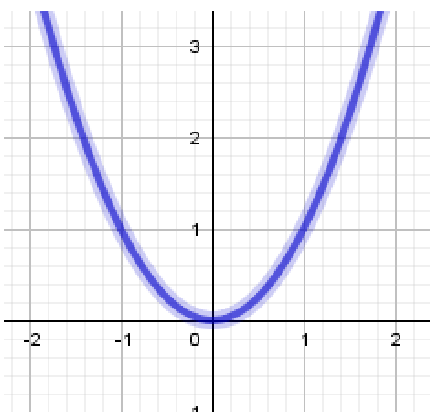
a is a constant

Examples $f(x) = \ldots\ldots.$

we have 3 cases:

① $a = n$ ($+ve$ $integer$ $example$: $x^1, x^2, x^5, \ldots$)

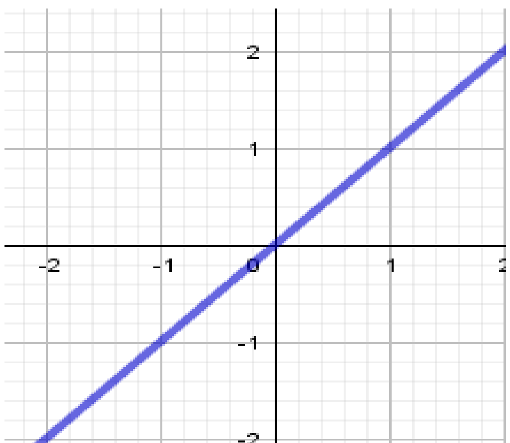
$n = is\ even,$
$ex. x^2, x^4, x^8$

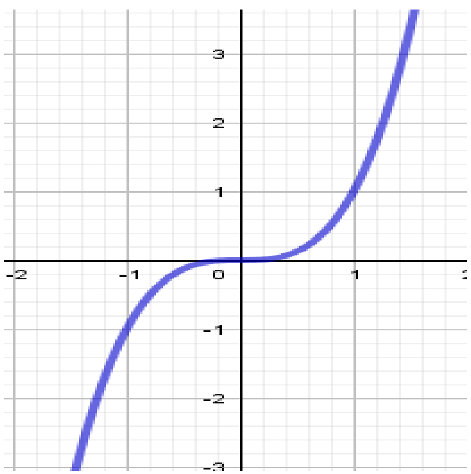


Domain=

Range =

$n = is\ odd, ex. x^1, x^3, x^5$





Domain=

Range =

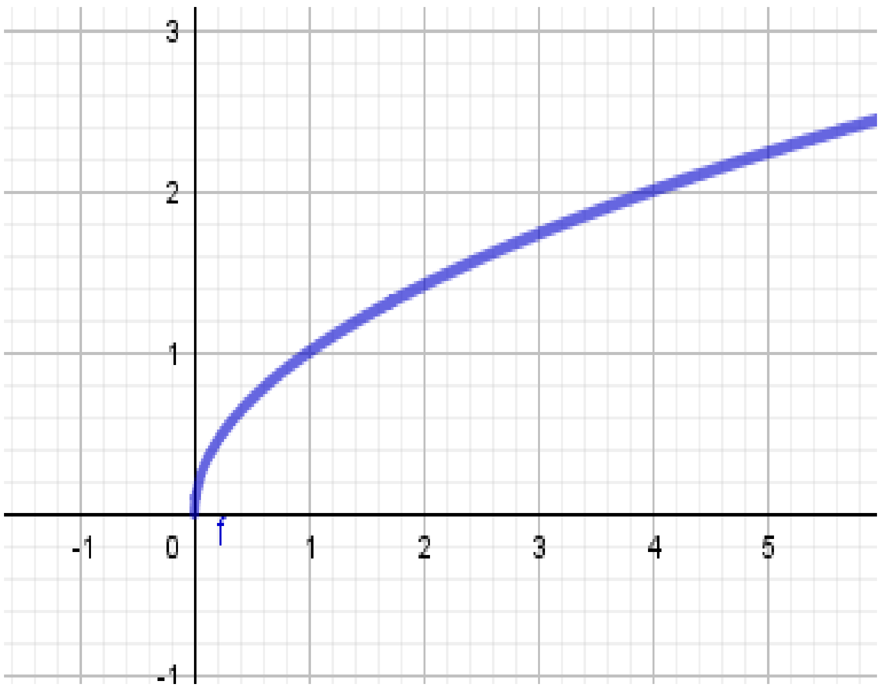
② $a = \frac{1}{n}$ $\quad (+ve\ integer\ example:\ x^{\frac{1}{2}}, x^{\frac{1}{3}}, \ldots)$

Root function

$n\ is\ even\ ex:\ \sqrt{x}$

Domain =

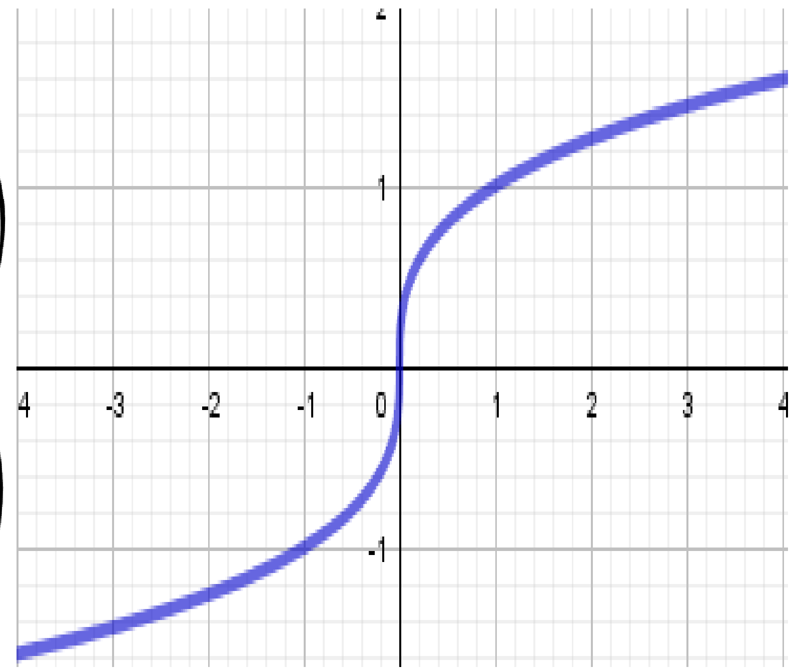
Range =



$n\ is\ odd\ ex:\ \sqrt[3]{x}$

Domain=

Range =

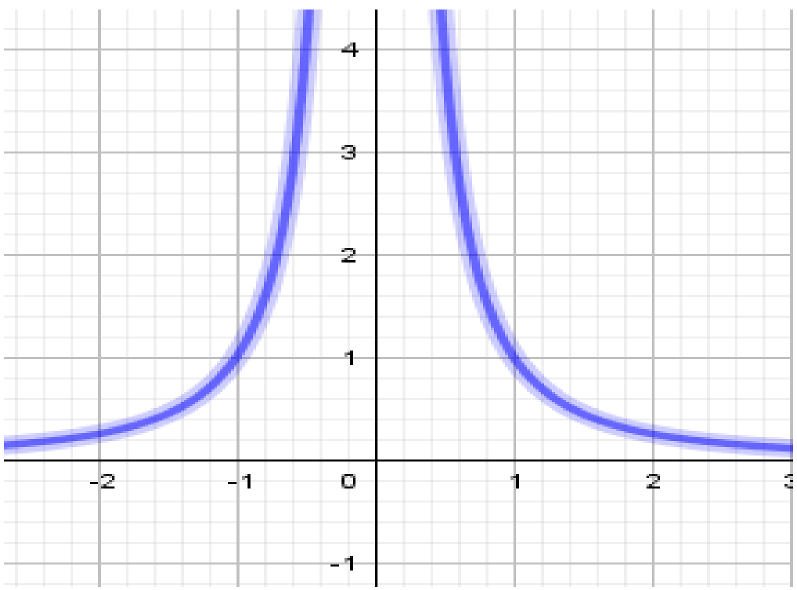


③ $a = -n \quad (example: \frac{1}{x}, \frac{1}{x^2}, \dots .)$ ← reciprocal

$n \text{ is even ex}: \frac{1}{x^2}$

Domain =

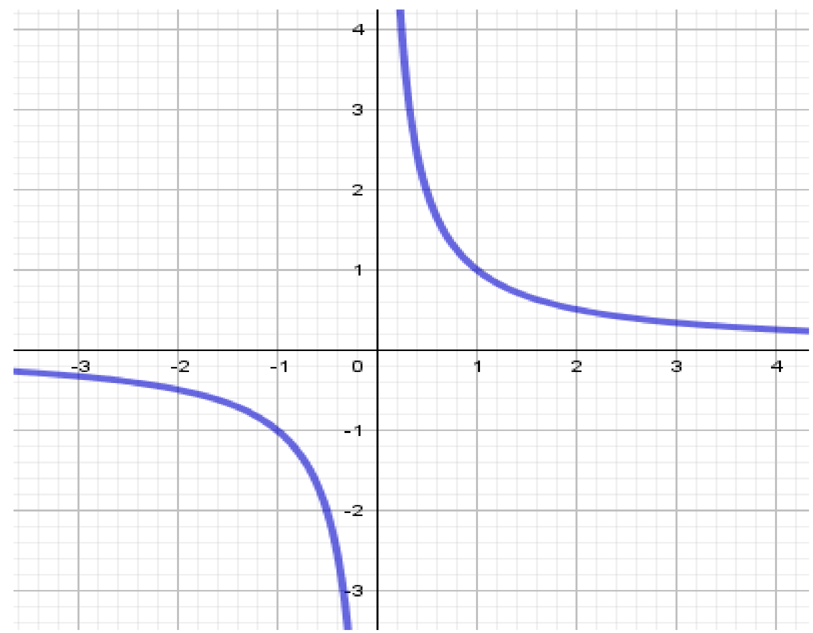
Range =



$n \text{ is odd ex}: \frac{1}{x}, \quad \frac{1}{x^3}, x^{-5}$

Domain =

Range =



Rational Functions

A rational function is a ratio of two polynomial functions:

$$f(x) = \frac{p(x)}{q(x)}, \qquad q(x) \neq 0$$

Examples

① $\frac{3x+3}{2x}, \frac{1}{x}, 5x^2+3x+5$

are all ……………….

② $\frac{3\sqrt{x}+2}{x+2}, \sqrt{\frac{3x+2}{5x}}$

are ………………………………….

The Domain of the rational function:

$$D_f = \{x \in \mathbb{R}: q(x) \neq 0\}$$
$$= \mathbb{R} - \{zeros\ of\ q(x)\}$$

Example

$$f(x) = \frac{2x^4 - x^2 + 1}{x^2 - 4}$$

$D_f =$

Algebraic Functions

Are functions constructed from polynomials using algebraic operations ($\times, \pm, \div, \sqrt{\ }$ ...)

Examples $\sqrt{x^3+2}$, $\frac{x^4-2\sqrt{x}}{x+\sqrt{x}}$, ...

remark

Rational, Power, Polynomails are also algebraic functions

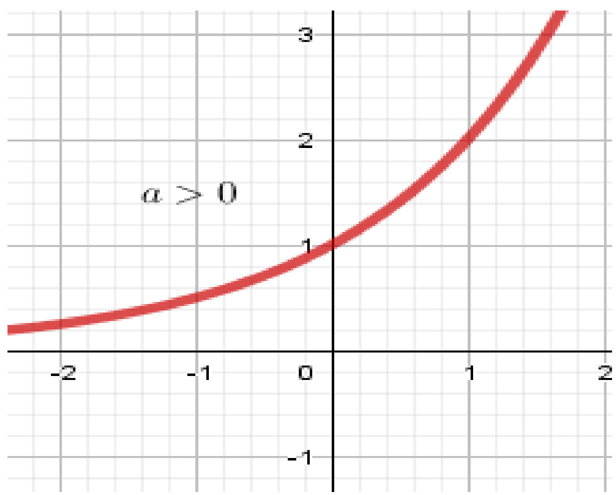
Trigonometric Functions

See Appendix D.

Exponential Functions

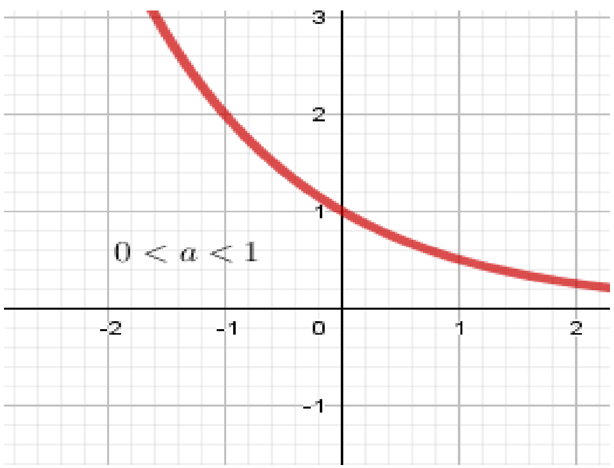
are of the form of $f(x) = a^x$,
a is a base > 0 , $a \neq 1$.

① $a > 1$, $y = 2^x, y = e^x$



Domain=
Range=
Increasing or decreasing !!
even or odd ?

② $0 < a < 1$, $y = \left(\frac{1}{2}\right)^x$



Domain=
Range=
Increasing or Decreasing
even or odd ?

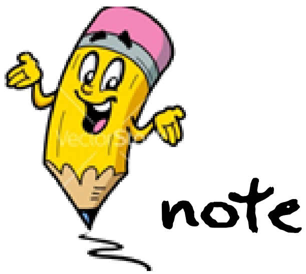
Logarithmic Function

$$f(x) = log_a\, x$$

العدد → x

القوة → $f(x)$: The power to which we raise a to get x

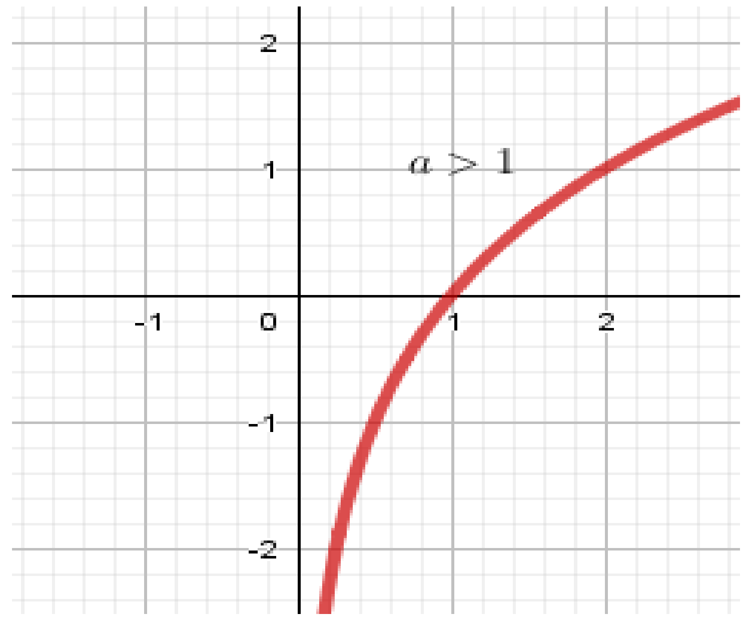
الأساس → a: The base a is a +ve constant $\neq 1$



① $y = log_e\, x = \ln x$

② log functions are the inverse functions of the exponential functions.

③ $log_a 1 = \cdots$
$log_a\, a = \cdots$



Domain=
Range=

Increasing or decreasing?
even or odd ?

Example 6

Classify the following functions as one of the types of functions that we have discussed.

(1) $f(x) = 5^x$ ....

(2) $g(x) = x^5$ ....

(3) $h(x) = \frac{1+x}{1-\sqrt{x}}$ ....

(4) $u(t) = 1 - t + 5t^4$ ....

Exercise 2

(1) $y = \pi^x$ ....

(2) $y = x^\pi$ .... + ...

(3) $y = \tan t - \cos t$ ....

homework